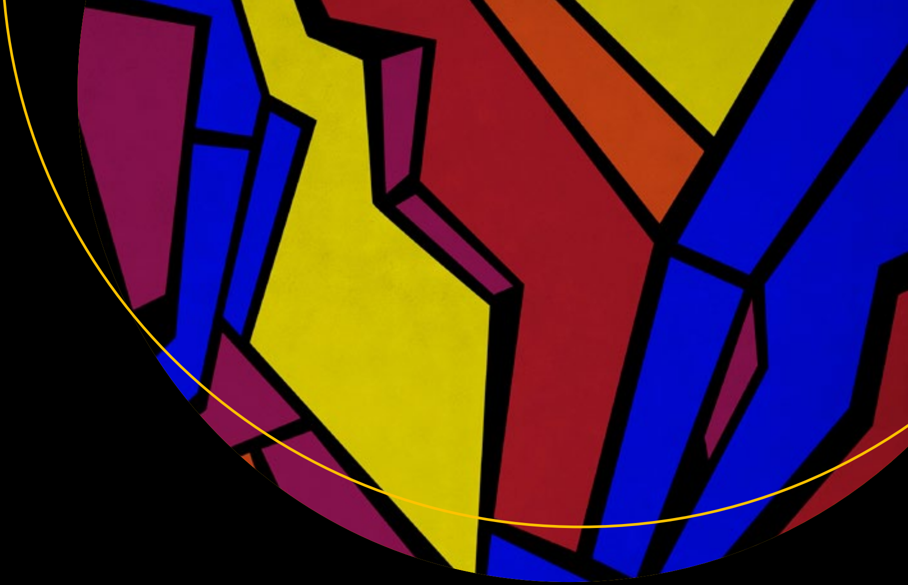


Beginning MATLAB and Simulink

From Novice to Professional

—

Sulaymon Eshkabilov

Apress®

Beginning MATLAB and Simulink

From Novice to Professional

Sulaymon Eshkabilov

Apress®

Beginning MATLAB and Simulink: From Novice to Professional

Sulaymon Eshkabilov
Fargo, United States

ISBN-13 (pbk): 978-1-4842-5060-0 ISBN-13 (electronic): 978-1-4842-5061-7
https://doi.org/10.1007/978-1-4842-5061-7

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publisher can accept any legal responsibility for any errors or omissions that may be made. The publisher makes no warranty, express or implied, with respect to the material contained herein.

Managing Director, Apress Media LLC: Welmoed Spahr
Acquisitions Editor: Steve Anglin
Development Editor: Matthew Moodie
Coordinating Editor: Mark Powers

Cover designed by eStudioCalamar

Cover image from rawpixel.com

Distributed to the book trade worldwide by Springer Science+Business Media New York, 233 Spring Street, 6th Floor, New York, NY 10013. Phone 1-800-SPRINGER, fax (201) 348-4505, e-mail orders-ny@springer-sbm.com, or visit www.springeronline.com. Apress Media, LLC is a California LLC and the sole member (owner) is Springer Science + Business Media Finance Inc (SSBM Finance Inc). SSBM Finance Inc is a **Delaware** corporation.

For information on translations, please e-mail editorial@apress.com; for reprint, paperback, or audio rights, please email bookpermissions@springernature.com.

Apress titles may be purchased in bulk for academic, corporate, or promotional use. eBook versions and licenses are also available for most titles. For more information, reference our Print and eBook Bulk Sales web page at http://www.apress.com/bulk-sales.

Any source code or other supplementary material referenced by the author in this book is available to readers on GitHub via the book's product page, located at www.apress.com/9781484250600. For more detailed information, please visit http://www.apress.com/source-code.

Printed on acid-free paper

To the memory of my father.
To my mother.
To my wife, Nigora, after 25 wonderful years together.

Table of Contents

About the Author

Sulaymon Eshkabilov, PhD, is currently a visiting professor at the Department of Agriculture and Biosystems, North Dakota State University, USA. He obtained an ME diploma from the Tashkent Automobile Road Institute, an MSc from Rochester Institute of Technology, NY, USA, and a PhD from Cybernetics Institute of Academy Sciences of Uzbekistan in 1994, 2001, and 2005, respectively. He was an associate professor at the Tashkent Automobile Road Institute from December 2006 until January 2017. He held visiting professor and researcher positions at Ohio University, USA, during the 2010/2011 academic year and at Johannes Kepler University, Austria from January-September, 2017. He teaches the following courses: "MATLAB/Simulink Applications for Mechanical Engineering and Numerical Analysis" and "Modeling of Engineering Systems" to undergraduate students, "Advanced MATLAB/Simulink Modeling" seminar/class, "Control Applications," "System Identification," and "Experimentation and Testing with Analog and Digital Devices" to graduate students.

His research interests are mechanical vibrations, control, mechatronics, and system dynamics. He developed simulation and data analysis models for vibrating systems, autonomous vehicle control, and studies of mechanical properties of bones. He has authored two books devoted to MATLAB/Simulink applications for mechanical engineering students and numerical analysis. He has worked as an external academic expert in the European Commission to assess academic projects in 2019.

About the Technical Reviewers

Dr. Joseph Mueller specializes in control systems and trajectory optimization. For his doctoral thesis, he developed optimal ascent trajectories for stratospheric airships. His active research interests include robust optimal control, adaptive control, applied optimization and planning for decision support systems, and intelligent systems to enable autonomous operations of robotic vehicles. Prior to joining SIFT in early 2014, Dr. Mueller worked at Princeton Satellite Systems for 13 years. In that time, he served as the principal investigator for eight small business innovative research contracts for NASA, the Air Force, the Navy, and MDA. He has developed algorithms for optimal guidance and control of both formation flying spacecraft and high altitude airships and developed a course of action planning tool for DoD communication satellites. In support of a research study for the NASA Goddard Space Flight Center in 2005, Dr. Mueller developed the Formation Flying Toolbox for MATLAB, a commercial product that is now used at NASA, ESA, and several universities and aerospace companies around the world. In 2006, Dr. Mueller developed the safe orbit guidance mode algorithms and software for the Swedish Prisma mission, which has successfully flown a two-spacecraft formation flying mission since its launch in 2010. Dr. Mueller also serves as an adjunct professor in the Aerospace Engineering & Mechanics Department at the University of Minnesota, Twin Cities campus.

Karpur Shukla is a research fellow at the Centre for Mathematical Modeling at FLAME University in Pune, India. His current research interests focus on topological quantum computation, nonequilibrium, and finite-temperature aspects of topological quantum field theories, and applications of quantum materials effects for reversible computing. He received an M.Sc. in physics from Carnegie Mellon University, with a background in theoretical analysis of materials for spintronics applications as well as Monte Carlo simulations for the renormalization group of finite-temperature spin lattice systems.

Acknowledgments

I would like to express my special gratitude to the technical reviewers of this book—Dr. Joseph Mueller and Mr. Karpur Shukla—for their very thorough work while reviewing the context and code of the book. Without their critical insights and corrections many points along the way, the book would not be as good as it is now. I would also like to express my special gratitude to Mark Powers for his timely and well planned correspondence throughout this book project.

My cordial gratitude goes to my mother, for her limitless support and love. Up until the last point of this book, she has kept checking my progress.

I would like to thank my wife, Nigora. Without her great support, I would not be able to take up the challenging task of writing this book. I have spent so much time over the weekends in my office writing and editing the book. In addition, I would like to thank our children—Anbara, Durdona, and Dovud—for being such delightful people and for being the inspiration for writing this book.

Introduction

This book is aimed at beginner-level learners of MATLAB/Simulink packages. It covers the essential, hands-on tools and functions of the MATLAB and Simulink packages, and explains them via interactive examples and case studies. The main principle of the book is “learning by doing” and it progresses from simple to complex. It contains dozens of solutions and simulation models via M/MLX files/scripts and Simulink models, which help you learn the programming and modeling essentials. Moreover, there are many recommendations for avoiding pitfalls related to programming and modeling aspects of MATLAB/Simulink packages.

“Beginning MATLAB and Simulink” explains various practical issues of programming and modeling in parallel by comparing programming tools of MATLAB and blocks of Simulink. After studying this book, you'll be proficient at using MATLAB/Simulink packages. You can apply the source code and models from the book's examples as templates to your own projects in data science, numerical analysis, modeling and simulation, or engineering.

Essential learning outcomes of the book include:

- Getting started using MATLAB and Simulink
- Performing data analysis and visualization with MATLAB
- Programming essentials of MATLAB and core modeling aspects of Simulink, and how to associate scripts and models of MATLAB and Simulink packages
- Developing GUI models and standalone applications in MATLAB
- Working with integration and numerical root-finding methods in MATLAB and Simulink
- Solving differential equations in MATLAB and Simulink
- Applying MATLAB for data analysis and data science projects

The book contains eight logically interlinked chapters.

- Chapter 1 is dedicated to introducing the MATLAB environment and MATLAB recognized data types, including numeric, cell, structure, character, logical, function handle, and table.
- Chapter 2 covers most of the essential programming tools and functions, such as `for … end` and `while … end` loop operators, `if…elseif…else…end` condition operators, symbol referencing, and most common errors and warnings. It also covers M-file debugging tools and options in MATLAB. Moreover, this chapter addresses MATLAB-specific programming tools, including function files, function handles, and display operators, such as `disp()`, `display`, `fprintf`, and `sprint`.
- Chapter 3 covers GUI development and how to write and edit GUI model callback functions.
- Chapter 4 addresses the issues of how to develop MEX files, C/C++ code, and standalone applications from the existing M-files.
- Chapter 5 is dedicated to Simulink modeling essentials. The examples in this chapter cover matrix operations, computing function values, modeling, and solving ordinary differential equations. Moreover, it covers issues around how to associate Simulink models with MATLAB scripts.
- Chapter 6 is devoted to data visualization issues, such as building 2D and 3D plots and animated plots in MATLAB.
- Chapter 7 is dedicated to matrix algebra and array operations. It addresses solving systems linear equations, eigen-value problems, matrix decompositions, matrix and vector operations, and conversions of arrays and strings via examples solved in MATLAB and Simulink in parallel.
- Chapter 8 covers some essential aspects of solving ordinary differential equations analytically and using MATLAB's built in functions and commands.

All of the source code (scripts, M/MLX/MAT files, Simulink models, SLX/MDL files, C code, MEX-files, and installation `*.exe` files) discussed in the book are available to readers via a Download Source Code button on the book's `apress.com` product page and accessible via the Download Source Code link located at `www.apress.com/9781484250600`.

Note The scripts in the book may not always be the best solutions to the given problems, but this is done intentionally in order to emphasize methods used to improve them. In some other cases, it is found to be the most appropriate solution to the author's best knowledge. Should I spot better alternative solutions to exercises, I will publish them via MathWorks' MATLAB Central User Community's file exchange, via my file exchange link there (under my name).

No matter how hard we worked to proofread this book, it is inevitable that there will be some typographical errors that might slip through and will appear in print.

September 2019 Sulaymon Eshkabilov

CHAPTER 1

Introduction to MATLAB

The MATLAB package is employed in wide ranges of engineering and scientific computing applications and is associated with the dynamic system simulation package called Simulink. The package has a few advantages and remarkable strengths, such as user-friendly and intuitive programming syntax, high-quality numerical algorithms for various numerical analyses, powerful and easy-to-use graphics, simple command syntax to perform computations, and many add-ons as toolboxes and real and complex vectors and matrices, including sparse matrices as fundamental data types.

There are diverse application areas of the package, i.e. simulation of various systems such as vehicle performances, mapping of the human genome, financial analysis in emerging economies, microbiology applications in diagnosis and treatment of small organisms, dynamic simulations of large ships in down-scaled laboratory models, simulation of the next generation network audio products, teaching computer programming to undergraduates with real-time laboratory tests and measurements, and image processing for underwater archeology and geology.

In this chapter, we discuss some essential key features of the graphical user interface (GUI) of MATLAB, how to use the help tools and library sources, how to adjust the format options and accuracy and precision settings, how to create various variables and variable structures, and how to employ the M/MLX editors to write and edit scripts/programs.

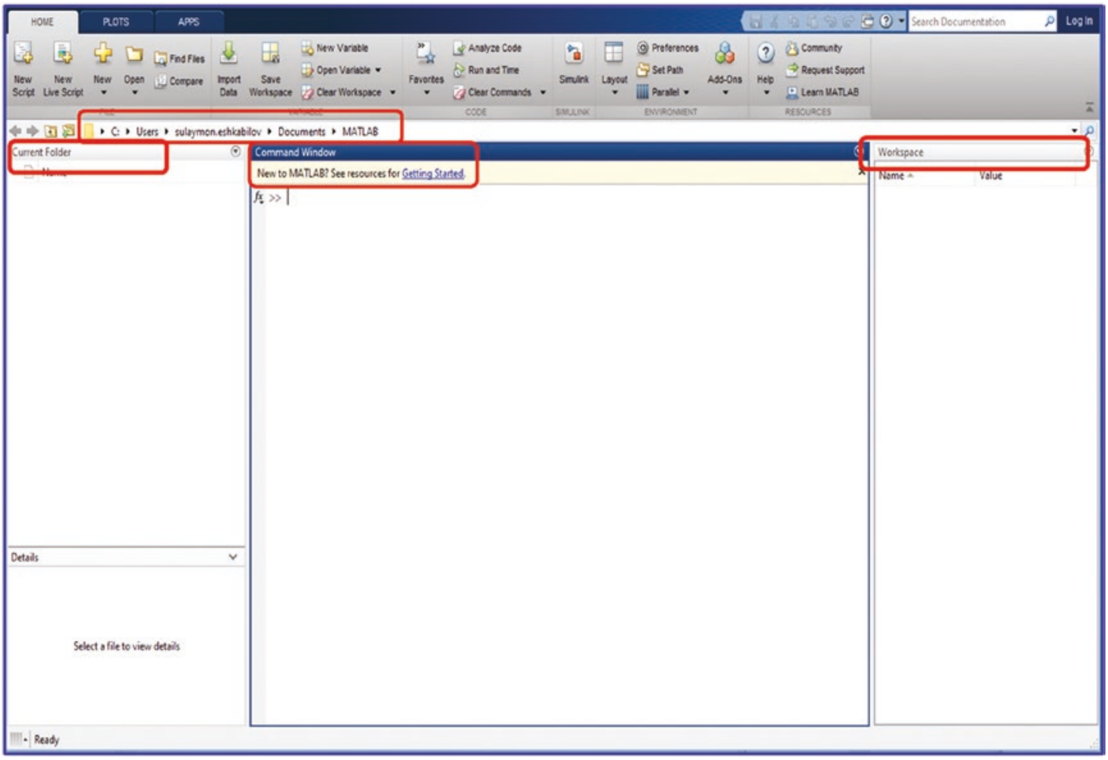
MATLAB's Menu Panel and Help

The MATLAB application can be launched from the Windows operating system by clicking the icon/shortcut from the desktop window or choosing Start ➤ All Programs ➤ MATLAB. As MATLAB loads, the user's last preserved data, files, entries, last 20 commands (by default), and menu bar and tools with the last preferences all appear.

S. Eshkabilov, *Beginning MATLAB and Simulink*, https://doi.org/10.1007/978-1-4842-5061-7_1

MATLAB's graphical user interface (GUI) tools and windows are customizable. Users can easily manipulate, customize, and change preferences of the package according to their needs. Figure 1-1 shows the default main window of MATLAB 2018a. It must be noted that the package's GUI menu and tools have changed over the years in an effort to make the package more user friendly and the tools more intuitive. The default window has the main menu tools, a Current Directory indicator, and Command, Workspace, and Command History windows. These windows can be docked/undocked or opened in a separate window, closed or removed from the main window, or dragged from one pane to another and maximized or minimized.

This main desktop window is shown in Figure 1-1.



New to MATLAB? See resources for Getting Started.

fx >>

Select a file to view details

__Figure 1-1.__ Default MATLAB desktop window (R2018b)

The main components of the package's graphical user interface (GUI) tools are as follows:

- Tools and toolbars are grouped into three menus—HOME, PLOTS, and APPs (see Figure 1-2). The HOME tab contains all the main tools. It's where you can create and delete new files and variables, import data, access the code analyzer, launch Simulink, change the MATLAB package desktop layout change options, set the path, and access add-ons and help options.

- The Current Directory window is in the left pane by default. This window shows all the files in the directory and the folder directory.

- The Command Window is in the central pane (by default). All commands and/or (small) scripts/codes can be entered directly after fx >>. By clicking on fx >> (see Figure 1-4) in the Command Window, you can view all built-in functions and MATLAB toolboxes. This option is only available starting from MATLAB version R2008a. All installed toolboxes can also be viewed or accessed by clicking on the APPS tab (see Figure 1-2), which is available in later versions of MATLAB, starting with the R2010a version.

- The Workspace pane of the default desktop window shows the current entries and saved variables during the session. These entries are saved temporarily until the MATLAB application is closed. All essential attributes and properties of entries/variables (variable names, values, and types) are displayed in the workspace.

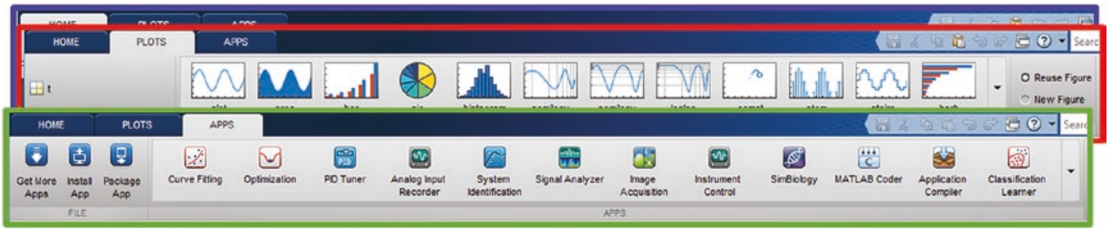
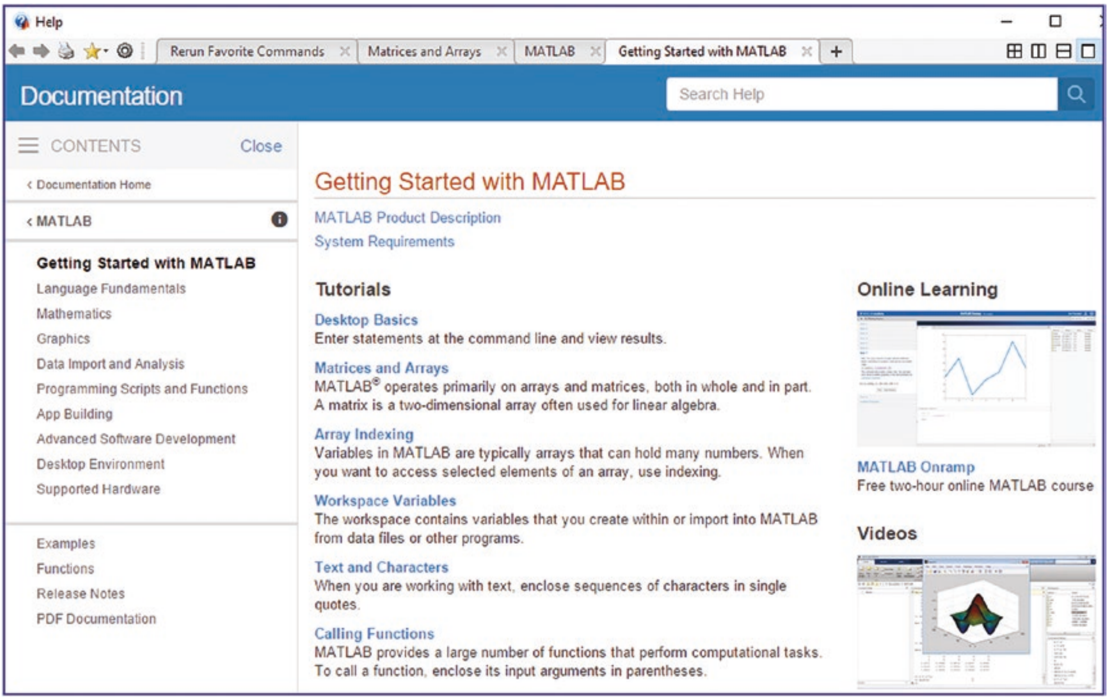


Figure 1-2. MATLAB main menu: The HOME, PLOTS, and APPS tabs (MATLAB R2018b)

One of the most essential strengths of this package is its help documentation, which includes help libraries and tools. They can be accessed using GUI tools and entries from the Command Window. It is a good idea to start using the package by exploring the Getting Started options, which you can access by clicking the Getting Started hyperlinked text, above the Command Window. This brings up the Help Library documentation, as shown in Figure 1-3, which contains the essential documentation and help tools, such as examples and function syntax.



‹ Documentation Home

‹ MATLAB

Getting Started with MATLAB
Language Fundamentals
Mathematics
Graphics
Data Import and Analysis
Programming Scripts and Functions
App Building
Advanced Software Development
Desktop Environment
Supported Hardware

Examples
Functions
Release Notes
PDF Documentation

Getting Started with MATLAB

MATLAB Product Description
System Requirements

Tutorials

Desktop Basics
Enter statements at the command line and view results.

Matrices and Arrays
MATLAB® operates primarily on arrays and matrices, both in whole and in part. A matrix is a two-dimensional array often used for linear algebra.

Array Indexing
Variables in MATLAB are typically arrays that can hold many numbers. When you want to access selected elements of an array, use indexing.

Workspace Variables
The workspace contains variables that you create within or import into MATLAB from data files or other programs.

Text and Characters
When you are working with text, enclose sequences of characters in single quotes.

Calling Functions
MATLAB provides a large number of functions that perform computational tasks. To call a function, enclose its input arguments in parentheses.

MATLAB Onramp
Free two-hour online MATLAB course

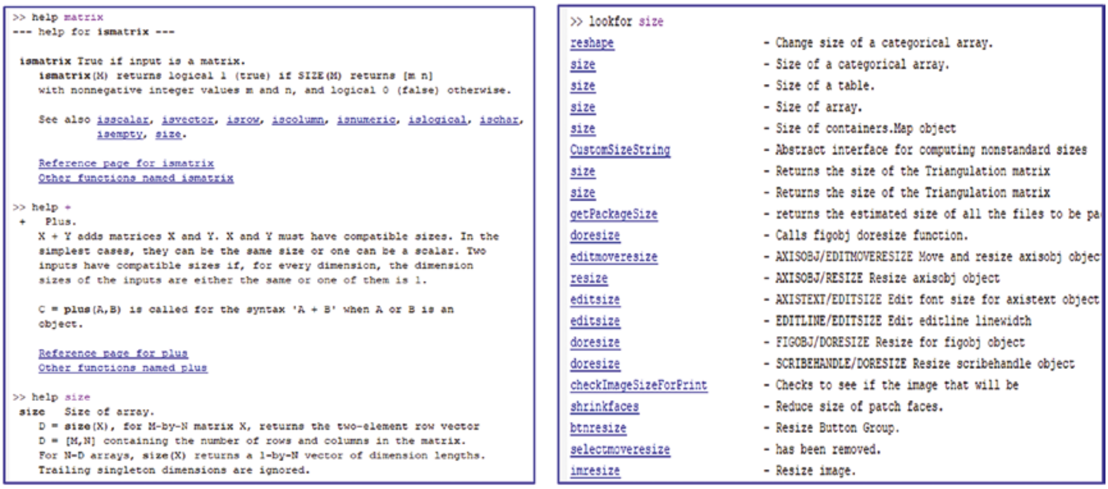
Figure 1-3. *Getting Started with MATLAB and Help Library documentation*

Note Before discussing the help options, I need to highlight one important point concerning comments. In MATLAB, users can write necessary hints and help remarks as comments within the M/MLX-files and in the Command Window. Comments need to start with an % sign. There are other options for adding comments, which I will discuss later when writing M/MLX-files.

There are a few other hands-on ways to obtain help. For example, to get quick help about MATLAB's built-in function tools and commands, including user-generated functions, users can type the following in the Command Window:

```
>> doc size;        % extended help on the command SIZE
>> helpwin size;    % help shown in a separate window on the command SIZE
>> help clear       % quick help on how to use the command CLEAR
>> help matrix;     % quick help how to use the command ISMATRIX
>> help +           % quick help on "+"
>> help size;       % quick help on SIZE
>> lookfor size     % extensive search for a list of functions and files
                      containing  the command SIZE.
```

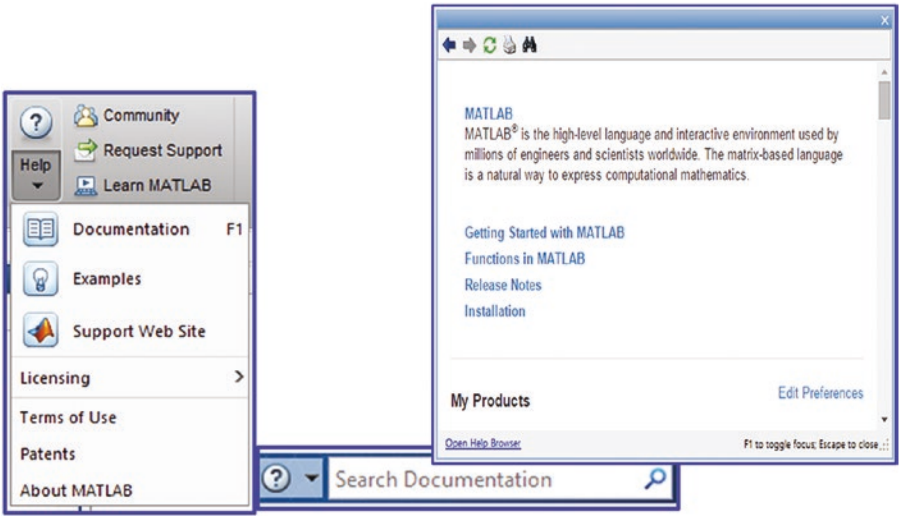
Figure 1-4 and Figure 1-5 show some of the results of quick and extensive help. In addition, the application has GUI tools and broad online library resources, product descriptions, video tutorials, and open public forums on the MathWorks website.



```
>> help matrix
--- help for ismatrix ---

 ismatrix True if input is a matrix.
    ismatrix(M) returns logical 1 (true) if SIZE(M) returns [m n]
    with nonnegative integer values m and n, and logical 0 (false) otherwise.

    See also isscalar, isvector, isrow, iscolumn, isnumeric, islogical, ischar,
             isempty, size.

    Reference page for ismatrix
    Other functions named ismatrix

>> help +
 +   Plus.
    X + Y adds matrices X and Y. X and Y must have compatible sizes. In the
    simplest cases, they can be the same size or one can be a scalar. Two
    inputs have compatible sizes if, for every dimension, the dimension
    sizes of the inputs are either the same or one of them is 1.

    C = plus(A,B) is called for the syntax 'A + B' when A or B is an
    object.

    Reference page for plus
    Other functions named plus

>> help size
 size   Size of array.
    D = size(X), for M-by-N matrix X, returns the two-element row vector
    D = [M,N] containing the number of rows and columns in the matrix.
    For N-D arrays, size(X) returns a 1-by-N vector of dimension lengths.
    Trailing singleton dimensions are ignored.
```

```
>> lookfor size
reshape                        - Change size of a categorical array.
size                           - Size of a categorical array.
size                           - Size of a table.
size                           - Size of array.
size                           - Size of containers.Map object
CustomSizeString               - Abstract interface for computing nonstandard sizes
size                           - Returns the size of the Triangulation matrix
size                           - Returns the size of the Triangulation matrix
getPackageSize                 - returns the estimated size of all the files to be pa
doresize                       - Calls figobj doresize function.
editmoveresize                 - AXISOBJ/EDITMOVERESIZE Move and resize axisobj objec
resize                         - AXISOBJ/RESIZE Resize axisobj object
editsize                       - AXISTEXT/EDITSIZE Edit font size for axistext object
editsize                       - EDITLINE/EDITSIZE Edit editline linewidth
doresize                       - FIGOBJ/DORESIZE Resize for figobj object
doresize                       - SCRIBEHANDLE/DORESIZE Resize scribehandle object
checkImageSizeForPrint         - Checks to see if the image that will be
shrinkfaces                    - Reduce size of patch faces.
btnresize                      - Resize Button Group.
selectmoveresize               - has been removed.
imresize                       - Resize image.
```

Figure 1-4. *Getting help*



MATLAB

MATLAB® is the high-level language and interactive environment used by millions of engineers and scientists worldwide. The matrix-based language is a natural way to express computational mathematics.

F1 to toggle focus; Escape to close

Figure 1-5. *Getting help*

You can search for help in MATLAB using the Command Window and the `help`, `lookfor`, `doc`, `docsearch`, and `helpwin` commands for the `clock` function , for example:

1. Quick help can be obtained from the Command Window with the `help` command. In this case, help is displayed from MATLAB's built-in commands/functions as well as within the user's function files. This is a quick way to obtain help.

```
>> help clock
```

2. Extensive (detailed) help, with examples, is displayed in the Help Library window with the following commands, but only if such a function file (e.g., `clock`) exists.

```
>> doc  clock
>> docsearch clock
```

3. You can access an extended list of M-files containing a keyword from the Command Window by using the following help command. Note that this option is much slower than the other two search options, due to its exhaustive search for the keyword.

```
>> lookfor clock
```

4. You can view the function file explanation in the Help Library by using the following command, but only if such a function file (e.g., `clock`) exists.

   ```
   >> helpwin clock
   ```

5. All extended tips, examples, and command syntax can be viewed from the Help Library (displayed in Figure 1-3), which can be accessed by clicking on the Help menu options.

6. You can use the F1 keyboard key to open the Help Library and documentation.

7. By clicking on the Help menu from the main panel, you can access various help resources from MathWorks, such as its Help Library resources, web resources, demo examples, updates, trials, and so forth.

There are a number of hands-on help resources available online, including MathWorks website academia, the user community's published scripts and file exchanges[1], and the MATLAB answers forum [2], where learners/users/developers post their questions and seek answers, or conversely post their answers to posted questions. It also includes function files, Simulink models, online forums, tutorials of numerous universities [3], and personal web pages of professors and researchers [4], just to name a few.

The MATLAB Environment

Let's start working with the MATLAB environment by making some changes to its layout using its GUI tools, such as Layout, Preferences, and Set Path, which are located on the HOME tab. To make changes to the layout (see Figure 1-6) from the HOME tab's main menu, you have to click on the Layout drop-down menu (1). Many options and different windows are available to choose from. The Desktop window consists of Command, Command History, Current Directory, and Workspace windows if there are tick marks before those window names. You can separate or drag around any of these windows by clicking the title bar and dragging the window to the new location.

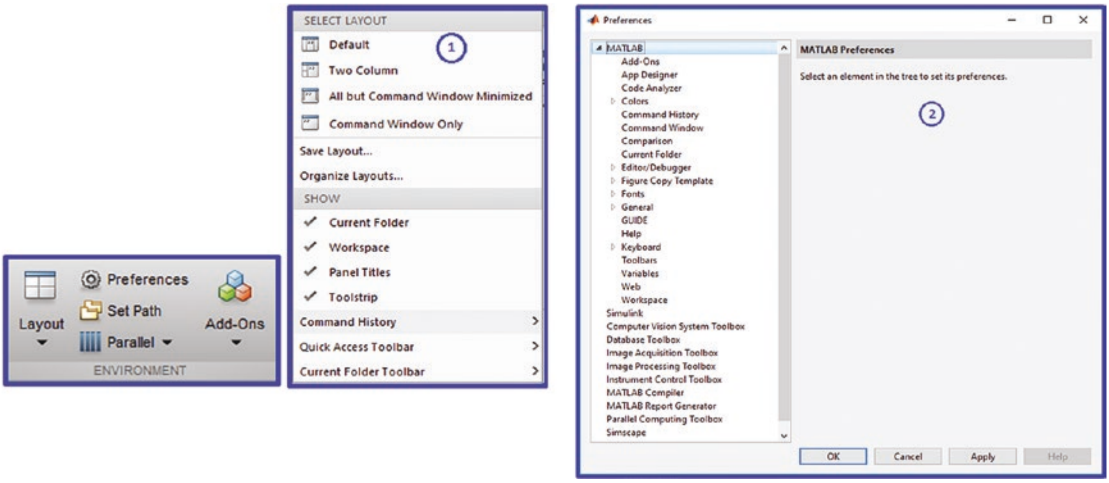


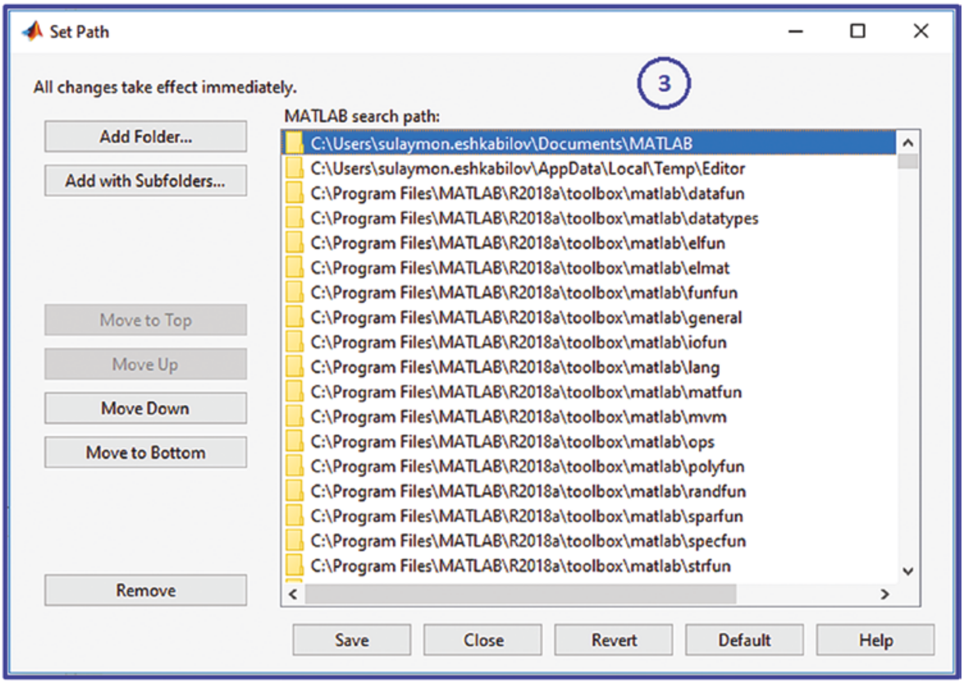
Figure 1-6. *Changes in Layout (1) and Preferences (2)*

To make changes to the Preferences window, you either click on Preferences from the main menu or type the following in the Command Window and press Enter:

```
>> preferences
```

The Preferences window shown in Figure 1-6 will pop up. The directories/paths to the current directory can also be altered and new paths can be added, as shown in Figure 1-7.

You can modify many options/tools, including the GUI quick access tools and Editor/Command window displays, from the Preferences window. To display the tips and highlight the current line in the Editor window, for example, you choose Preferences ➤ Editor/Debugger ➤ Display and then check Enable Data-Tips in Edit Mode. Many tools can be added to the main menu as shortcuts. Some of these key customizations in Preferences can also be attained by issuing commands from the Command Window.



All changes take effect immediately.

MATLAB search path:

```
C:\Users\sulaymon.eshkabilov\Documents\MATLAB
C:\Users\sulaymon.eshkabilov\AppData\Local\Temp\Editor
C:\Program Files\MATLAB\R2018a\toolbox\matlab\datafun
C:\Program Files\MATLAB\R2018a\toolbox\matlab\datatypes
C:\Program Files\MATLAB\R2018a\toolbox\matlab\elfun
C:\Program Files\MATLAB\R2018a\toolbox\matlab\elmat
C:\Program Files\MATLAB\R2018a\toolbox\matlab\funfun
C:\Program Files\MATLAB\R2018a\toolbox\matlab\general
C:\Program Files\MATLAB\R2018a\toolbox\matlab\iofun
C:\Program Files\MATLAB\R2018a\toolbox\matlab\lang
C:\Program Files\MATLAB\R2018a\toolbox\matlab\matfun
C:\Program Files\MATLAB\R2018a\toolbox\matlab\mvm
C:\Program Files\MATLAB\R2018a\toolbox\matlab\ops
C:\Program Files\MATLAB\R2018a\toolbox\matlab\polyfun
C:\Program Files\MATLAB\R2018a\toolbox\matlab\randfun
C:\Program Files\MATLAB\R2018a\toolbox\matlab\sparfun
C:\Program Files\MATLAB\R2018a\toolbox\matlab\specfun
C:\Program Files\MATLAB\R2018a\toolbox\matlab\strfun
```

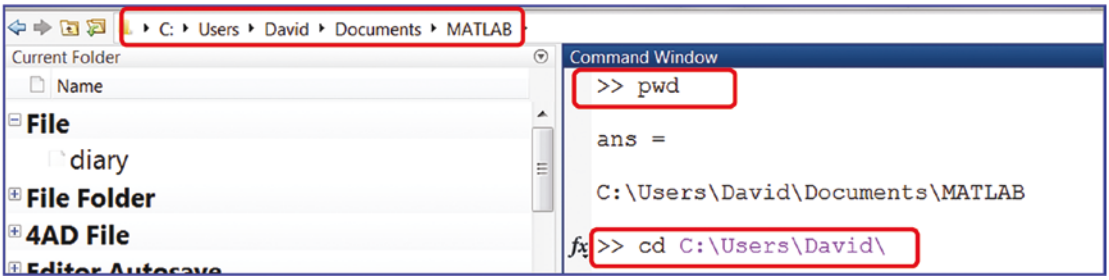
Figure 1-7. *Setting a path (3), adding a folder, and removing a path*

Working in the Command Window

Work in MATLAB generally starts in the Command Window, but before you type any command, it is worth noting the current directory. The current directory address can be viewed directly from the main window (see Figure 1-8) or by typing this command in the Command Window:

```
>> pwd
```

If required, you can change the current directory using the `>> cd` command or by clicking on the path's directory (see Figure 1-8) with left mouse button and typing the directory address (C: ➤ Users ➤ David ➤ Documents ➤ MATLAB ➤).



```
>> pwd

ans =

C:\Users\David\Documents\MATLAB

>> cd C:\Users\David\
```

Figure 1-8. Viewing and changing the current working directory

In addition, you can add a few new paths to work within the working directory. For instance, to add a path to an existing folder, use the following.

```
>> addpath('C:\Users\David\Documents\MATLAB');
```

The `addpath()` command might also be helpful in scripts, to read or load data from a specific folder or directory. For short commands and calculations and/or to view attributes of the available variables in the current directory, use the Command Window. However, for series of commands and longer scripts, it is more efficient to use script editors, such as the M-file and MLX-file editors.

The MATLAB application has a few files that are recognizable by their extensions. They are .M, .MLX, .MAT, .BI, and .FIG. M-files are used to write programs/scripts/function files. MLX-files (Live M-files) are used to write programs/scripts/function files and see the computation results within the MLX-file editor window. MAT-files are used to save all types of variables available in the workspace and can be accessed easily from M/MLX-files and the Command Window. Among these files, .BI files are used for built-in files of MATLAB and .FIG files are used to save figure windows in MATLAB. In addition, the Simulink application has three types of files—mdl, .slx, and .slxc. They are used to build and simulate Simulink models and can also be recalled/simulated from MATLAB without opening them. I will discuss the essential features of these files and how to use them in later sections of the book.

While using the Command Window for simple calculations and data generation and processing is sufficient, the keyboard's up-arrow key can be used to avoid having to retype the previously entered commands and entries. For example, if you entered `>> A1 = [1, 2, 4, -5, 6]; B = A+2` in the Command Window and then needed to make changes to these entries, you could use the up-arrow key after typing this: `>> A`. MATLAB will automatically recall your previous entry.

Command Window and Variables

MATLAB is case-sensitive and all of its built-in commands are lowercase. When you perform computations, you assign a name to the result of the computation. The assigned name is the variable name. The result of the computation is saved in the MATLAB workspace under the given variable names. For example, `>> A =13; B = A*2` means that the variable called `A` is equal to `13` and a variable called `B` is equal to `2` multiplied by `A`.

Using Variables

Variable names must start with a letter and can be combined with any integer numbers, such as 0, 1, 2, ... 9 and (_) an underscore sign. No other symbols can be used for variable names. The maximum length of the variable name is 63 characters. MATLAB treats the variables called `a` and `A` as two different variables because of its ANSI/ASCII symbol conversions. Now let's get started working in the Command Window by entering and assigning variable names, performing simple some basic arithmetic operations, and making changes to the output data formats.

```
>> A=3; B=-2; C=1/2; D = -1.5; ABCD=A^2+B/C+D;
>> ABCD              % ";" is missed and the content of the variable ABCD
                     is displayed
ABCD =
  3.5000
>> sqrt(ABCD)  % if a variable name is not assigned, "ans" is a default
               name by MATLAB
ans =
    1.8708
>> ans+1  % if a variable name is not assigned, "ans" will substitute the
          previous "ans".
ans =
   2.8708
```

The results of user entries are displayed in the Command Window and can be altered using these display format options: `format long`, `format short`, `format long eng`, `format bank`, `format hexadecimal`, `format rational`, etc. Here are some examples of format options. When you change the display format types, the actual variable values do not change.

```
>> A=3; B=-2; C=1/2; D = -1.5;
>> ABCD=A^2+B/C+D;
>> format bank
>> ABCD
ABCD =
          3.50
>> format long eng
>> ABCD
ABCD =
    3.50000000000000e+000
>> sqrt(ABCD)
ans =
    1.87082869338697e+000
>> format short
>> sqrt(ABCD)
ans =
    1.8708
>> format bank
>> sqrt(ABCD)
ans =
          1.87
>> format long eng
>> sqrt(ABCD)
ans =
    1.87082869338697e+000
>> format compact
>> sqrt(ABCD)
ans =
    1.87082869338697e+000
>> format rat
>> sqrt(ABCD)
ans =
    1738/929
```

```
>> format hex
>> sqrt(ABCD)
ans =
   3ffdeeea11683f49
```

Note MATLAB is case sensitive so it recognizes the variables called a and A as two different variables.

After entering a few starting letters of a variable name or built-in command/function name in the Command Window, you can use the TAB key from the keyboard to access matching commands/functions, including your developed function files. For example, if you typed `>> AB` and then pressed TAB, the rest of `ABCD` variable calculation expression would appear as an option.

Another useful feature of the Command Window is using the keyboard's up-arrow (↑) to recall previously typed variables or commands. You simply type a few starting letters of any previously typed commands or function names and then press the up-arrow (↑). For example, `>> f↑` recalls the previously typed-in command, `>> format long`. Moreover, the up-arrow (↑) can be associated with the TAB key to recall previously entered commands in the Command Window.

The values and attributes of all variables entered in the Command Window will be saved in the workspace until you clean up the workspace by deleting the variables using the `clear`, `clearvars`, or `clear all` command or by using the right and left mouse button options to select the variables and delete them. In addition, all of the variables and their attributes are saved in the workspace until the MATLAB package is closed.

The three most commonly used commands for housekeeping in MATLAB are:

- `clc` for cleaning up the Command Window and starting with a blank Command Window
- `clear` and `clearvars` for removing all variables saved in the workspace
- `clear all` for removing all variables as well as temporarily compiled and saved machine codes of M-files, breakpoints, and debug settings

All of these commands can also be used with M-files and MLX-files. It must be noted that the `clear all` command is not recommended with M-files and MLX-files, because it will decrease the efficiency of code/script and create unwanted behaviors when you declare the `clear global` command. Let's look at some ways to employ these commands efficiently.

```
>> A=3; B=-2; C=1/2; D = -1.5; ABCD=A^2+B/C+D;
>> clear        % Removes all of the entries in the workspace and
                 workspace becomes all blank
>> clearvars      % The same as "clear"
>> clear variables % The same as "clearvars"
>> clear all % The same as "clear" and also removes already compiled codes
   as well
>> clear  A B C D  % Removes variables: A B C D and leaves ABCD untouched
>> A=3; B=-2; C=1/2; D = -1.5; ABCD=A^2+B/C+D;
>> clearvars  -except A B C   % Removes all variables except for: A B C
```

MATLAB can use a wildcard * asterisk for variables and filenames. For example, to remove all variables whose names start with the letter "A", you would use the following command.

```
>> A=3; B=-2; C=1/2; D = -1.5; ABCD=A^2+B/C+D; ABC=A^2+B*C;
>> clear  A*    % Removes all variables staring with A, i.e. A, ABCD and
                ABC are removed
>> clearvars A*  % The same as: clear A*
```

All entered and saved variables can be viewed from the workspace window by typing the `who` or `whos` command in the Command Window. For example:

```
 >> A=3; B=-2; C=1/2; D = -1.5; ABCD=A^2+B/C+D; ABC=A^2+B*C;
>> who
  A   B   C   D   ABCD   ABC
>> whos
Name      Size            Bytes  Class     Attributes
  A       1x1                 8  double
  ABC     1x1                 8  double
  ABCD      1x1                     8  double
  B         1x1                     8  double
  C         1x1                     8  double
  D         1x1                     8  double
```

From these examples, it is clear that MATLAB reads every entry as an array/matrix. For example, scalar is read by MATLAB as an array of size 1-by-1. This attribute of MATLAB is logically linked to its name, MATrix LABoratory. MATLAB's default storage (memory allocation) is double precision, which is the maximum available space allocated. However, for memory efficiency and faster calculation purposes, other storage formats can also be used. MATLAB supports single precision or integer type and int8 ... 64, uint8 ...64 storage format types. Table 1-1 shows how data can be saved in every storage class type and the conversion function used in MATLAB for each type.

Note MATLAB's default storage type is double. However, that can be changed to single precision or integer types—int8 …64, uint8 …uint64—by specifying or converting the values of the variables/data.

Table 1-1. *Data Storage Format Types in MATLAB*

Class	Range of Values	Conversion Function
Signed 8-bit integer	-2^7 to 2^7-1	int8
Signed 16-bit integer	-2^15 to 2^15-1	int16
Signed 32-bit integer	-2^31 to 2^31-1	int32
Signed 64-bit integer	-2^63 to 2^63-1	int64
Unsigned 8-bit integer	0 to 2^8-1	uint8
Unsigned 16-bit integer	0 to 2^16-1	uint16
Unsigned 32-bit integer	0 to 2^32-1	uint32
Unsigned 64-bit integer	0 to 2^64-1	uint64

When you have numerical data and a floating point format, the double precision storage gives you the largest storage space for higher accuracy. Double precision can save up to 16 decimal digits. The single precision storage for floating point-formatted data is more memory efficient and less accurate than the double precision, which is the default storage format in MATLAB. If only integers are used in your calculations and data processing, it is more appropriate to use int8…64 or uint8 … 64, depending on the largest value of your data. Here are some examples of how to specify storage type while saving the values of the declared variables.

```
>> F01=127;Fint_08=int8(F01), Fnew1 = Fint_08+1
Fint_08 =
 int8
 127
Fnew =
 int8
 127
>> F16=65535; Fint_16=uint16(F16), Fnew2 = Fint_16+1
Fint_16 =
 uint16
 65535
Fnew2 =
 uint16
 65535
```

In calculations of the `Fnew1` and `Fnew2` variables from the `int8` and `uint16` formatted variable values, the allocated storage space cannot accommodate any more values (i.e., for int8 maximum allocated storage space is $127 = 2^7 - 1$ and for uint16 it is $65535 = 2^{16} - 1$). Therefore, errors in the calculations have taken place. Note that in such cases, MATLAB does not show an error. Figure 1-9 shows all the data storage types and data formats supported in MATLAB.

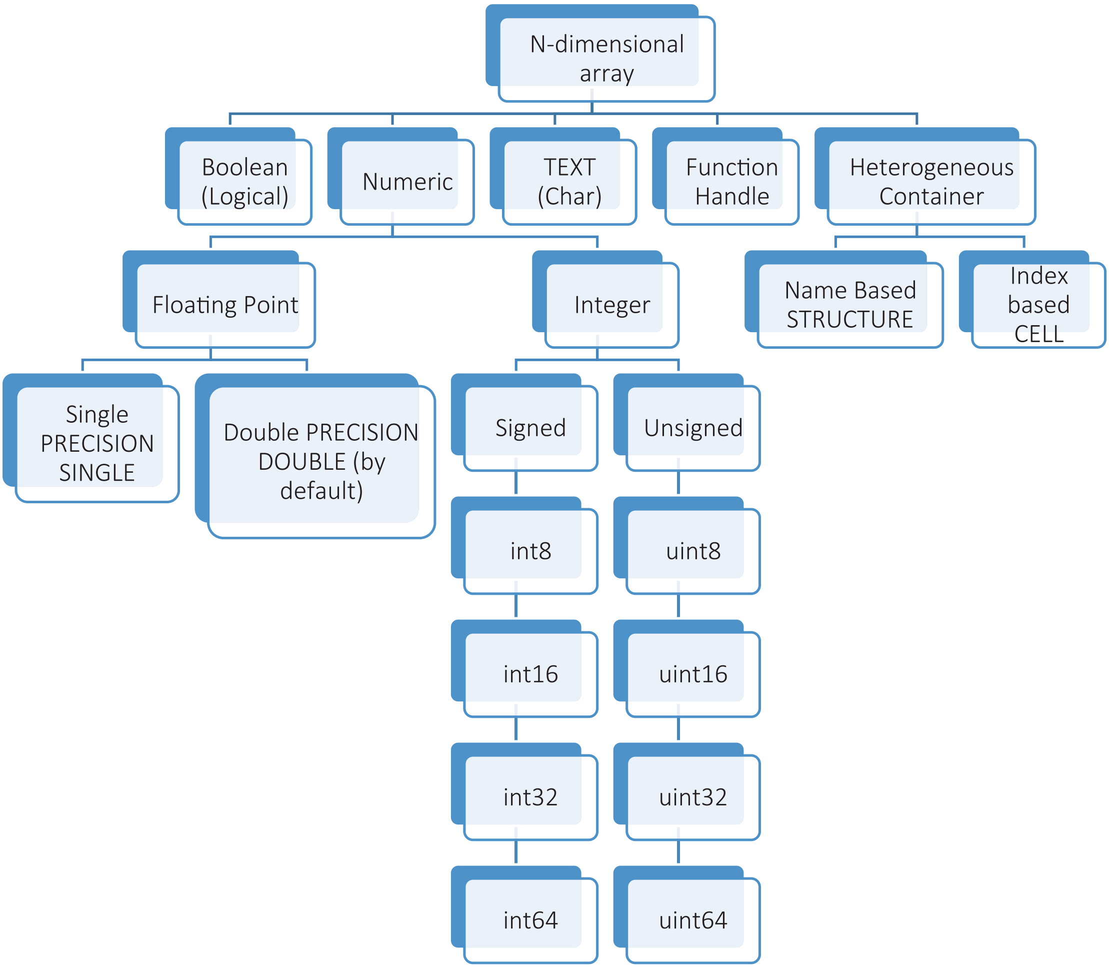


Figure 1-9. *Different types of data storage options supported in MATLAB*

The Command Window's history pane is a good way to review all of the entries that will be kept unless you delete them. You can also change the setting in the Preferences window to clear the history of entries after ending the session.

Finally, to exit from MATLAB or quit a work session, issue one of these commands from the Command Window:

```
>> exit
>> quit
```

Or use Ctrl+Q from the keyboard to quit.

An alternative way to exit is by clicking close (the x) in the upper-right corner of the main window. This will close the whole package.

When to Use the Command Window

Use the Command Window in these instances:

- To perform short calculations
- To view error and warning messages from commands or/and after executing M-files, MLX-files, and SLX/MDL Simulink models
- To view variable attributes saved in the workspace and files in the current directory
- To view the contents of MATLAB compatible files
- To execute MATLAB files, such as M-files, MLX-files, SLX/MDL-files, and MAT-files
- To get hands-on, quick help with MATLAB commands/functions and user-created function files
- To make adjustments to the display formats of numerical data
- To add/remove a path/directory
- To create/delete/save variables and files

Let’s look at several examples to show other operations you can perform in the Command Window.

1) To view and analyze common errors and interpret error messages:

```
>> F16=65535; Fint_16=uint16(F16); Fnew2 = Fint_16+1;
>> Fnew+2 % The variable Fnew does not exist in the workspace
Undefined function or variable 'Fnew'.
```

```
>> clar F16 % Typo error: "clar" instead of "clear"
Undefined function or variable 'clar'.
>> CLear % Typo error: "CLear" instead of "clear". Note: MATLAB
case-sensitive
Undefined function or variable 'CLear'.
Did you mean:
>> clear % MATLAB automatically suggests closest command's correct
syntax
>> B=-2; C=1/2; BC=B/.C; % Illegal operation: B/.C instead of B/C;
 B=-2; C=1/2; BC=B/.C;
                  ↑
Error: Unexpected MATLAB operator.
>> B=-2; C=1/2; BC=B /*C; % Illegal operation: B/*C instead of B/C;
 B=-2; C=1/2; BC=B /*C;
                   ↑
Error: Invalid use of operator.
>> % Let's create a two-row matrix containing two elements, viz.
B, C in the     >> % first row and F16 in the second row.
>> BCF = [B, C; F16] % Number of elements in row 1 does not match
with the ones in row 2
Error using vertcat
Dimensions of arrays being concatenated are not consistent.
>> % Let's try to create a row matrix with elements separated with
"," and   >> % space and "."
>> BCF = [B, C. F16] % Error is a misused "." instead of "," but
not dot indexing as shown
Dot indexing is not supported for variables of this type.
>> BCF = [B, C, F16] % This is the anticipated correct command.
```

2) To save the variables saved in the workspace in a *.mat file:

```
>> save MYdata.mat % Saves all variables residing in the workspace
in MYdata.mat file
>> save('MYdata.mat') % The same as above
>> save MYdata.mat F16 Fnew2 % Saves the variables F16, Fnew2 in
MYdata.mat file
```

```
>> save('MYdata.mat', 'F16 ', 'Fnew2 ') % The same as above
>> save MYdata.mat F* % Saves all variables whose name starts with
F (in the workspace)
```

3) To obtain quick help:

```
>> help format
 format Set output format.
 format with no inputs sets the output format to the default
 appropriate
 for the class of the variable. For float variables, the default is
 format SHORT. ...
>> help dir
 dir List directory.
 dir directory_name lists the files in a directory. Pathnames and
 asterisk wildcards may be used. A single asterisk in the path touching
>> help what
 what List MATLAB-specific files in directory.
 The command what, by itself, lists the MATLAB specific files found ...
>> help which
--- help for which ---
 which Locate functions and files.
 which ITEM displays the full path for ITEM. ITEM can include a partial
 path, complete path, relative path, or no path. If ITEM includes a
 partial path or no path, ...
```

4) To view MATLAB compatible files:

```
>> type QQQ.txt % Note: the file QQQ.txt was available in the
current directory

CY     bBb    88
AH     AAAA+  98
CWW    AAAA+  98
...
>> type MYfile.mlx % Note: the file MYfile.mlx was available in
the current directory
```

```
N=13;
M=randi(N,9);
stairs(M, 'bd-')
```

```
>> type myfun.m % Note: the file myfun.m was available in the
current directory
```

```
function f=myfun(x)
f=[2*x(1)-x(2)-exp(-x(1));
 -x(1)+2*x(2)-exp(-x(2))];
End
```

5) To create, open, and execute MATLAB files (M-files, MLX-files, MDL/MLX-files, and MAT-files):

```
>> edit TRY1.m % To create a new M-file called TRY1.mlx
>> edit MYfile.mlx % To create a new MLX-file called MYfile.mlx
>> open('TRY1.m') % To open the file if it is residing in the
current directory
>> run('\...\TRY1.m') % Directory and a file name is needed, if it
is outside of the current dir.
>> TRY1 % To execute the file if it is residing in the current directory
>> open('MYfile.mlx') % To open the file if it is residing in the
current directory
>> MYfile % To execute the file if it is residing in the current
directory
>> load MYdata.mat % Load contents of MYdata.mat (existing in the
current directory)
>> load('MYdata.mat') % The same as above
```

6) To delete files in the current directory or variables residing in the workspace.

Warning Be careful when using these `delete` commands, because they delete files that cannot be recovered.

```
>> delete TRY1.m % Deletes the file TRY1.m residing in the current
directory
>> delete MYfile.mlx % Deletes the file MYfile.mlx residing in the
current directory
>> delete QQQ.txt % Deletes the file QQQ.txt residing in the
current directory
>> delete *.txt % Deletes all *.txt files in the current directory
>> delete *.mlx % Deletes all *.mlx files in the current directory
>> delete DA*.txt % Deletes all *.txt files whose name starts
with DA...
>> delete *.asv % Deletes all *.asv files (autosave) of MATLAB in
the current directory
```

7) To view the current directory, change a directory, create a new directory, and remove a directory from the MATLAB path:

```
>> MD = pwd % Shows the current directory and assigns to a
character type of variable: MD
>> cd C:\Users\sulaymon.eshkabilov\Documents\MATLAB % Change to
this directory
>> cd('C:\Users\sulaymon.eshkabilov\Documents\MATLAB') % The same
as above
>> mkdir MYBook % Creates a new folder (directory) inside the
current directory
>> mkdir('MYBook') % The same as above
>> mkdir c:\Users\sulaymon.eshkabilov\BOOK % The same as above
with a full path
>> addpath C:\Documents % Adds this path (C:\Documents) to the
MATLAB's search
>> addpath('C:\Documents') % The same as above
>> rmdir('MYtask') % Removes the directory (folder: MYtask)
including its contents from the hard disk
>> rmdir c:\Users\sulaymon.eshkabilov\TASK % Removes the
directory: TASK
```

Note MATLAB supports wildcards (the asterisk *) when deleting and saving files and variables in the current directory and workspace. For example, `>> delete M*.mat` deletes all the `*.mat` files whose names start with `M`. The `>> save MYdata.mat B*` command saves all variables whose names start with `B`. The `>> clearvars A*` command clears all variables whose names start with `A`.

Many of the operations performed in the Command Window, such as performing calculations and analyses and viewing variables or file contents, can also be done other ways. For example, most of the operations carried out in the Command Window can also be done via GUI tools. You can create new variables New Variable, delete them Clear Workspace ▾, or open them Open Variable ▾.

Similarly, you can create a MATLAB file with New ▾ or an M-file with New Script or open an existing MATLAB file with Open ▾. You can delete the files via the right and left mouse button options. You can also view the current directory or change it using ⇦ ⇨ ▸ C: ▸ Users ▸.

One of the most essential functions of the Command Window that cannot be done easily with GUI tools is viewing the error and warning messages obtained while and after executing M-files, MLX-files, and MDL/SLX-files. This is essential for good programming. Another good use of the Command Window is to obtain quick help with MATLAB commands/functions.

Different Variables and Data Sets in MATLAB

MATLAB supports several different data types—numeric, character, logical, table, cell, structure, and function handle. The flowchart in Figure 1-10 shows the hierarchy of all data types supported in MATLAB that can be used for data storage. Note that the function handle can also take vectors (row or column vectors), not only scalar numbers.

As stated, MATLAB reads every entry (numerical and character types) as an array. If the storage type is not specified, the default storage type is always double. Let's look at several examples of how to generate various data types supported in MATLAB:

- Numerical data
- Logical arrays
- Character arrays/variables

- Table arrays
- Cell arrays
- Structure arrays
- Function handles
- Classes and graphic handles

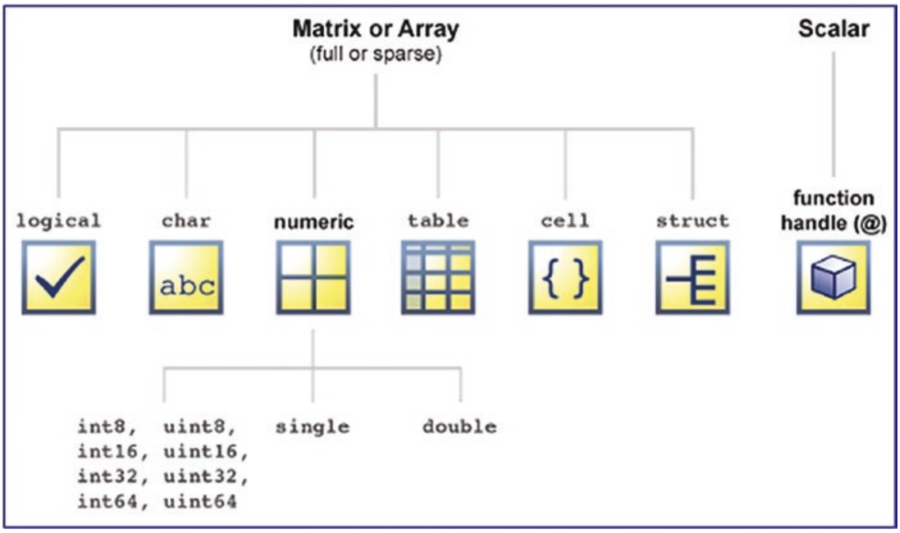


Figure 1-10. *Types of data (array) sets supported in MATLAB*

While demonstrating how to generate these arrays, all of the created variable/arrays types will be preserved until the end of this section. Therefore, all variables/arrays are created once and preserved from the examples. Note that in some of the examples that generate random matrices, we employ MATLAB's random number generators, which will create different random numbers every time they are called. However, in order to have permanent random values for variables and arrays, we have to set up the seed value of the random number generator `rng()`. With the fixed seed value, the random number generators (`rand()`, `randi()`, `randn()`, and so forth) will generate permanent/fixed random values every time they are called.

Numerical Data/Arrays

There are many different ways to create and generate numerical arrays. For example, you can use direct entries from the Command Window by typing all the numerical values of an array and you can generate numerical entries using built-in functions and commands. Data can be imported from another file (.mat, .txt, .dat, .xls, .jpeg, .au, etc.) or generated using existing data or imported data in MATLAB. Data can also be computed using specific mathematical expressions and arithmetic/matrix operations.

```
>> format short      % To display numerical elements of arrays in a short
                     format
>> A = 2; B = -3.25; C=(A+B)^2; % Entries and arithmetic operations.
>> ABC=[A, B, C] % Use of Existing Data: 1-by-3 array
ABC =
    2.0000   -3.2500    1.5625>> Drow=[1, 2, 3, -4] % Just entry: row
                        array. Comma "," element separator in it.
Drow =
     1     2     3    -4>> Erow=[-2 -1 0 Drow] % Entry and Use of existing
                      data: Row Array. Space is a separator.
Erow =
    -2    -1     0     1     2     3    -4>> Fcol=[1; 2; 3; -4] % Entry:
                                        Column Array. Elements are
                                        separated with ";".
Fcol =
     1
     2
     3
    -4>> DE = [2*Drow; (3/4)*Drow] % Use of Existing Data and arithmetic
    operations: 2-by-4 array
DE =
    2.0000    4.0000    6.0000   -8.0000
    0.7500    1.5000    2.2500   -3.0000
```

An alternative way to generate numerical arrays/data is to employ built-in array generator functions of MATLAB.

```
>> Aone=ones(2, 6) % 2-by-6 array generated with elements of 1
Aone =
     1     1     1     1     1     1
     1     1     1     1     1     1>> Bzero=zeros(5, 6) % 5-by-6 array
                                   generated with elements of 0
Bzero =
 0 0 0 0 0 0
 0 0 0 0 0 0
 0 0 0 0 0 0
 0 0 0 0 0 0
 0 0 0 0 0 0
>> Ceye=eye(6) % 6-by-6 eye matrix array generated
Ceye =
 1.00 0 0 0 0 0
 0 1.00 0 0 0 0
 0 0 1.00 0 0 0
 0 0 0 1.00 0 0
 0 0 0 0 1.00 0
 0 0 0 0 0 1.00

>> Seed = 1; rng(Seed);    % Seed value of the random number generator
                           rng() is set up in order to
                           % generate fixed random numbers
>> Dr1 =rand(9, 5)     % 9-by-5 array of uniform distributed random numbers
                       generated
Dr1 =
    0.4170    0.5388    0.1404    0.0391    0.6865
    0.7203    0.4192    0.1981    0.1698    0.8346
    0.0001    0.6852    0.8007    0.8781    0.0183
    0.3023    0.2045    0.9683    0.0983    0.7501
    0.1468    0.8781    0.3134    0.4211    0.9889
    0.0923    0.0274    0.6923    0.9579    0.7482
    0.1863    0.6705    0.8764    0.5332    0.2804
    0.3456    0.4173    0.8946    0.6919    0.7893
    0.3968    0.5587    0.0850    0.3155    0.1032>> Dr2 =randn(4, 6) %
    4-by-6 array of normally distributed random numbers generated
```

```
Dr2 =
   -0.1551   -1.1714   -0.5581   -2.0187   -0.4852    0.0407
    0.6121   -0.6856   -0.0285    0.1997    0.5943    0.2830
   -1.0443    0.9262   -1.4763    0.4259   -0.2765    0.0636
   -0.3456   -1.4817    0.2589   -1.2700   -1.8576    0.4334>> Dr3
   =randi([-5, 5], 5,5) % 5-by-5 array of integer numbers ranging between
   -5 ... 5
>> Dr3 =randi([-5, 5], 5,5)
Dr3 =
     1    -1     2    -3     1
     4    -4     4     4    -4
    -4     5     1    -1     5
    -4    -2     3     5    -1
     3     3    -2     2     1
```

Note that I have set up the seed value of the random number generator in order to generate the permanent values from the random number generators `rand()`, `randn()`, and `randi()`.

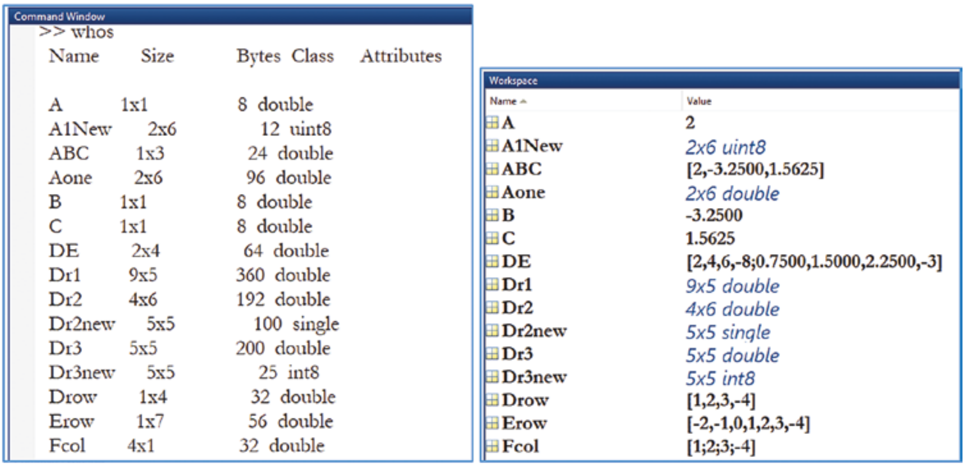
Moreover, there are many other standard matrix/array generators built into MATLAB, e.g. `pascal()`, `krylov()`, `leslie()`, `cauchy()`, `clement()`, `lesp()`, `poisson()`, `neumann()`, etc. To get help with these array generators, type the following in the Command Window:

```
>> help gallery
>> doc gallery
```

It must be noted that all of these generated numerical entries/arrays are saved in double precision format. Changing the storage format type of any these variables is quite simple and straightforward.

```
>> Dr3new=int8(Dr3) % Dr3new is Saved in int8
Dr3new =
  5×5 int8 matrix
    1   -1    2   -3    1
    4   -4    4    4   -4
   -4    5    1   -1    5
   -4    -2     3     5    -1
    3     3    -2     2     1>> Dr2new=single(Dr3) % Dr2new is Saved in a
                            single precision
Dr2new =
  5×5 single matrix
     1     -1      2     -3      1
     4     -4      4      4     -4
    -4      5      1     -1      5
    -4     -2      3      5     -1
     3      3     -2      2      1>> A1New=uint8(Aone) % A1new is Saved in uint8
A1New =
  2×6 uint8 matrix
    1    1    1    1    1    1
    1    1    1    1    1    1
```

Now it is time to check the attributes/properties of the numerical entries created in the Command Window and saved in the workspace. You do this by using the `whos` command, as shown in Figure 1-11.



```
>> whos
  Name        Size            Bytes  Class     Attributes

  A           1x1                 8  double
  A1New       2x6                12  uint8
  ABC         1x3                24  double
  Aone        2x6                96  double
  B           1x1                 8  double
  C           1x1                 8  double
  DE          2x4                64  double
  Dr1         9x5               360  double
  Dr2         4x6               192  double
  Dr2new      5x5               100  single
  Dr3         5x5               200  double
  Dr3new      5x5                25  int8
  Drow        1x4                32  double
  Erow        1x7                56  double
  Fcol        4x1                32  double
```

Name	Value
A	2
A1New	2x6 uint8
ABC	[2,-3.2500,1.5625]
Aone	2x6 double
B	-3.2500
C	1.5625
DE	[2,4,6,-8;0.7500,1.5000,2.2500,-3]
Dr1	9x5 double
Dr2	4x6 double
Dr2new	5x5 single
Dr3	5x5 double
Dr3new	5x5 int8
Drow	[1,2,3,-4]
Erow	[-2,-1,0,1,2,3,-4]
Fcol	[1;2;3;-4]

Figure 1-11. *Created numerical data types and variables residing in the workspace*

The most used data type in MATLAB is the numerical array. Therefore, it is essential to learn how to work with arrays of different sizes (many rows and many columns). In order to work with arrays, you should understand how to properly locate addresses of array elements, rows, and columns. Arrays are read as rows by columns. For example, >> DE(2, 1) means we are taking the element of DE residing on the second row and the first column, i.e. 0.7500. Likewise, >> DE(1, 4) means we have selected the element of DE residing on the first row and fourth column, i.e. -8.

```
>> DE
DE =
    2.0000    4.0000    6.0000   -8.0000
    0.7500    1.5000    2.2500   -3.0000
>> DE(2,1)
ans =
    0.7500
>> DE(1,4)
ans =
    -8
```

Thus, an element in any array can be located with respect to its row and column. The colon operator : is used to select all elements along rows or columns or both. For example, >> DE(1,:) selects all elements residing in the first row of DE, i.e. [2, 4, 6 -8]. Likewise, DE(:, 3) selects all elements residing in the third column of DE, i.e., [6; 2.25].

```
>> DE(1,:)
ans =
     2     4     6    -8
>> DE(:,3)
ans =
    6.0000
    2.2500
```

You can use the colon operator to select all element of matrices or arrays, For example, >> DE(:, :) is the equivalent of DE. The end keyword enables you to select elements up to the last one. For example, >> DE(1, 1:end) is equivalent to >> DE(1, :).

```
>> DE(1, 1:end)
ans =
     2     4     6    -8
>> DE(1, :)
ans =
     2     4     6    -8
```

As another example of how to select elements of matrices, in Dr3new, the element residing in the second row and first column is 0. An alternative way to locate any element in any given array is by the order count. Elements in arrays are counted on a column basis. For example, the order of the element number 2 in Dr3new will be 0 and element number 6 in Dr3new is 2. Note that I used the random number generator to create Dr3new and thus, your Dr3new will differ. Again, let's look at several examples of manipulating arrays based on Dr3new and Dr1.

```
>> Dr3new
Dr3new =
  5×5 int8 matrix
    3   -5    0    1   -3
    3   -5    1    5    3
    5   -5    4    1   -1
    2   -3   -4   -5    4
   -4    4   -2    3    3
>> Dr3new(2,1)
ans =
  int8
   3>> Dr3new(2)
ans =
  int8
   3>> Dr3new(6)
ans =
  int8
   -5>>  Dr3new(2,:)=0 % This makes a second row of elements equal to 0
Dr3new =
  5×5 int8 matrix
    3   -5    0    1   -3
    0    0    0    0    0
    5   -5    4    1   -1
    2   -3   -4   -5    4
   -4    4   -2    3    3>> Dr3new(:,5)=1 % This makes fifth column of
                           elements equal to 1
```

```
Dr3new =
  5×5 int8 matrix
    3   -5    0    1    1
    0    0    0    0    1
    5   -5    4    1    1
    2   -3   -4   -5    1
   -4    4   -2    3    1>> Dr3new(end,:)=-5 % This makes the last row of
                          elements equal to -5
Dr3new =
  5×5 int8 matrix
    3   -5    0    1    1
    0    0    0    0    1
    5   -5    4    1    1
    2   -3   -4   -5    1
   -5   -5   -5   -5   -5>> Dr2new(1:5, 4:5)=2+2 % This makes the last two
                         columns of elements equal to 4
Dr2new =
  5×5 single matrix
     3    -5     0     4     4
     3    -5     1     4     4
     5    -5     4     4     4
     2    -3    -4     4     4
    -4     4    -2     4     4>> Dr1(9, :)=[] % The last row elements are
                               removed
>> Dr1(9,:) =[]
Dr1 =
    0.4479    0.4916    0.4142    0.1393    0.6237
    0.9086    0.0534    0.0500    0.8074    0.7509
    0.2936    0.5741    0.5359    0.3977    0.3489
    0.2878    0.1467    0.6638    0.1654    0.2699
    0.1300    0.5893    0.5149    0.9275    0.8959
    0.0194    0.6998    0.9446    0.3478    0.4281
    0.6788    0.1023    0.5866    0.7508    0.9648
    0.2116    0.4141    0.9034    0.7260    0.6634>> Dr1(:, 4)=[ ] % The
                                                  fourth column elements are
                                                  removed
```

```
Dr1 =
    0.4479    0.4916    0.4142    0.6237
    0.9086    0.0534    0.0500    0.7509
    0.2936    0.5741    0.5359    0.3489
    0.2878    0.1467    0.6638    0.2699
    0.1300    0.5893    0.5149    0.8959
    0.0194    0.6998    0.9446    0.4281
    0.6788    0.1023    0.5866    0.9648
    0.2116    0.4141    0.9034    0.6634
```

In addition, you can create a new array from the elements of existing arrays:

```
>> NewDr = [Dr1(1:3,:), Dr2new(1:3, 2:4)] % Some elements of Dr1 and Dr2new
are taken
NewDr =
  3×7 single matrix
  Columns 1 through 6
    0.4170    0.5388    0.1404    0.6865   -1.0000    2.0000
    0.7203    0.4192    0.1981    0.8346   -4.0000    4.0000
    0.0001    0.6852    0.8007    0.0183    5.0000    1.0000
  Column 7
    4.0000
    4.0000
    4.0000>> NewDr2 = [Dr3new(:,:); Dr2new(2:end, :)] % All elements of
    Dr3new and some from Dr2new
NewDr2 =
  9×5 int8 matrix
     1   -1    2   -3    1
     0    0    0    0    1
    -4    5    1   -1    1
    -4   -2    3    5    1
    -5   -5   -5   -5   -5
     4   -4    4    4    4
    -4    5    1    4    4
    -4   -2    3    4    4
     3    3   -2    4    4
```

Note in these examples that the colon is one of the essential operators in managing and manipulating matrix and array elements. For example, NewDr(2, :) is equivalent to NewDr(2, 1:end). Both select all elements along row 2. Likewise, NewDr(:,:) is equal to NewDr(1:end, 1:end). They both select all elements starting from the first one, up to the last one.

These are a few examples of creating arrays in the Command Window. As stated, numerical arrays can be imported from other data files, such as *.dat, *.txt, *.xls, *.xlsx, and *.csv, as well as image files, such as *.jpg/jpeg, *.tiff, *.png, etc.

NaN (Not-a-Number)

While working and processing different data sets and analyzing experimental data, it is quite common to work with NaN (not-a-number) values. NaN is the result of 0/0 or of importing character strings as numerical data sets. NaN is also the result of inf-inf (∞ - ∞) or when trying to interpolate beyond a given range. So, how do you handle NaN values in numerical arrays? There are a few ready-to-use functions/tools in MATLAB that handle NaNs. Let's take a look at the following examples:

```
% Given:
A_var = [-8 10 NaN 9 4 -4 -7; 9 NaN 9 4 -10 9 0; -8 10 NaN 5 -10 -1 NaN]

A_var =

 -8 10 NaN 9 4 -4 -7
  9 NaN 9 4 -10 9 0
 -8 10 NaN 5 -10 -1 NaN
```

How do you compute the summation of the given numerical array, A_var? Note that sum() computes the sum of the columns of a matrix, if the matrix has more than one row and column. If the matrix is a row matrix, it computes the sum of all the row elements.

```
>> sum(A_var)

ans =

 -7 NaN NaN 18 -16 4 NaN
```

Sometimes, you may need to remove NaN from your data. For instance, say you are analyzing measured data with some missing points (NaN). You need to remove the NaN from the data. How do you address this problem?

One solution is to remove all NaN components of the array `A_var` by replacing them with `0`. Otherwise, the summation will not give numerical results. You can substitute all NaN components using element-wise substitution, one by one. Or you can do it with a single command by recalling the indexes of all the NaN components.

```
>> A_var(5)=0 % Element by element change

A_var =

 -8 10 NaN 9 4 -4 -7
  9 0 9 4 -10 9 0
 -8 10 NaN 5 -10 -1 NaN

>> A_var([7 9 21])=[0, 0, 0] % All at once or % A_var([7 9 21])=0

A_var =

 -8 10 0 9 4 -4 -7
  9 0 9 4 -10 9 0
 -8 10 0 5 -10 -1 0
```

Now the summation can be performed:

```
>> sum(A_var)
ans =
 -7 20 9 18 -16 4 -7
```

This answer is correct. This approach is quite straightforward, but with very large data sets, it will become tedious and time consuming or might even be impossible.

Note You can assign new values to selected elements/components of arrays element by element or all at once by specifying the indexes (e.g., `A_var([7 9 21)=[0 0 0])` of the elements/components.

Here is a second solution. The nansum() MATLAB built-in function handles the summation of numerical arrays with NaN components (elements).

```
>> A_var = [-8 10 NaN 9 4 -4 -7; 9 NaN 9 4 -10 9 0; -8 10 NaN 5 -10 -1 NaN]

A_var =

 -8 10 NaN 9 4 -4 -7
  9 NaN 9 4 -10 9 0
 -8 10 NaN 5 -10 -1 NaN

>> nansum(A_var)

ans =

 -7 20 9 18 -16 4 -7
```

Moreover, there are several other MATLAB functions that compute mean values, standard deviations, covariance values, etc., of numerical data arrays with NaN elements.

Note You can use the following MATLAB functions to compute maximum, mean, median, minimum, standard deviation, and variance values of a numerical array containing NaN elements: nanmax(), nanmean(), nanmedian(), nanmin(), nanstd(), and nanvar(). They work by ignoring all of the NaN elements in the given array.

There's also a third way, by using logical indexing. It's widely employed when working with large data sets, in order to sum the data, compute the mean or average values, plot the values, and perform other arithmetical matrix/array operations.

```
>> Index1=isnan(A_var) % Finds which elements of A_var are NaN and sets
them equal to 1
Index1 =
 3×7 logical array
 0 0 1 0 0 0 0
 0 1 0 0 0 0 0
 0 0 1 0 0 0 1
>> A_var(Index1)=0 % Assigning all NaN elements to "0"
```

```
A_var =
 -8 10 0 9 4 -4 -7
  9 0 9 4 -10 9 0
 -8 10 0 5 -10 -1 0
>> sum(A_var)
ans =
 -7 20 9 18 -16 4 -7
```

Here, the isnan() function identifies which elements of A_var are NaN and which ones are not. The new logical array called Index contains 1s and 0s. The 1s represent NaN elements and the 0s represent all other numerical elements.

There is also an alternative logical indexing function introduced in recent versions of MATLAB called ismissing(). It's used to identify missing data elements in numerical arrays.

```
Index2=ismissing(A_var) %Identifies all missing elements hidden behind NaN
Index2 =
 3×7 logical array
 0 0 1 0 0 0 0
 0 1 0 0 0 0 0
 0 0 1 0 0 0 1
>> A_var(Index2)
ans =
 NaN
 NaN
 NaN
 NaN
>> A_var(Index2)=0
A_var =
 -8 10 0 9 4 -4 -7
  9 0 9 4 -10 9 0
 -8 10 0 5 -10 -1 0
```

The logical indexing approach is very efficient and flexible and it can be applied to many other cases as well. For instance, you can easily identify all negative elements of a given data set or all elements within certain value ranges. Let's take the following numerical array (of 7 by 7) generated by randi() and separate out all the elements greater than 3 but smaller than 9, then equate all of them to 5.

```
>> B_var = randi([0, 25], 7) % This creates uniform distributed integers
within [0, 25] of 7-by-7 size
B_var =
 25 1 17 21 1 11 12
 18 13 13 2 10 0 8
 13 2 25 3 13 25 24
 12 21 16 4 10 4 23
 1 21 20 10 17 2 1
 17 18 11 21 16 9 19
 1 3 11 20 7 5 6
>> IndexB = find(B_var>3 & B_var<9)
IndexB =
 25
 35
 39
 42
 44
 49
>> B_var(IndexB)=5
B_var =
 25 1 17 21 1 11 12
 18 13 13 2 10 0 5
 13 2 25 3 13 25 24
 12 21 16 5 10 5 23
 1 21 20 10 17 2 1
 17 18 11 21 16 9 19
 1 3 11 20 5 5 5
```

Note Logical indexing is a very powerful and efficient tool in identifying certain elements of numerical data sets/arrays/matrices according to their values and then assigning them new values.

Character Types of Variables

MATLAB can recognize characters based on ASCII/ANSI character symbols in the form of numerical arrays. For example:

```
>> Ach1 = 'matlab' % Character type of variable
Ach1 =
 ' matlab '
>> Bch2 = ' mathworks ' % Character type of variable
Bch2 =
 ' mathworks '
>> Cch3= ' matlab belongs to mathworks ' % Character type of variable
Cch3 =
 'matlab belongs to mathworks'
>> Dch4 = 'www.mathworks.com' % Character type of variable
Dch4 =
 ' www.mathworks.com '
```

These are character type variables, but when the arithmetical operations are performed on these variables, they become numerical arrays.

```
>> format short
>> Aa1 =Ach1+0
Aa1 =
 109    97   116   108    97    98
>>Ba2 = Bch2+0
Ba2 =

 109    97   116   104   119   111   114   107   115    32
>> Ca3=Cch3+0
Ca3 =
 Columns 1 through 12
   109    97   116   108    97    98    32    98   101   108   111   110
  Columns 13 through 24
   103   115    32   116   111    32   109    97   116   104   119   111
  Columns 25 through 28
   114   107   115    32>> Da4 =Dch4+0
```

```
Da4 =
 Columns 1 through 12
   119   119   119    46   109    97   116   104   119   111   114   107
  Columns 13 through 17
   115    46    99   111   109
```

So, behind these numbers are the characters according to ASCII/ANSI standards. It is possible to get the character representations of the new variables using the MATLAB function char().

```
>> char(Aa1)
ans =
 'matlab'
>> char(Ba2)
ans =
 'mathworks'
>> char(Ca3)
ans =
 'matlab belongs to mathworks'
>> char(Da4)
ans =
 'www.mathworks.com'
```

The following variables reside in the workspace:

```
 A 1x1 8 double
 A1New 2x6 12 uint8
 ABC 1x3 24 double
 Aa1 1x6 48 double
 Ach1 1x6 12 char
 Aone 2x6 96 double
 B 1x1 8 double
 Ba2 1x9 72 double
 Bch2 1x9 18 char
 Bzero 5x6 240 double
 Ca3 1x27 216 double
 Cch3 1x27 54 char
 Ceye 6x6 288 double
```

```
DE 2x4 64 double
Da4 1x17 136 double
Dch4 1x17 34 char
Dr1 8x4 256 double
Dr2 4x6 192 double
Dr2new 5x5 100 single
Dr3 5x5 200 double
Dr3new 5x5 25 int8
Drow 1x4 32 double
Erow 1x7 56 double
Fcol 4x1 32 double
NewDr 3x7 84 single
NewDr1 3x7 84 single
NewDr2 9x5 45 int8
ans 1x17 34 char
```

Note that these variables are kept and used in the coming sections to generate logical, table, cell, and structure types of array variables.

Function Handle

The function handle is a special MATLAB data type that's used to store a link to an expression or a function. By calling a function handle, we invoke the expression or function stored within that specific function handle. The function handle is one of the most useful features in MATLAB. For instance, function handles are used in various simulations and calculations of functions and mathematical expressions, in solving various equations and problems, and in user interface developments. They are also widely employed in solving differential equations. The syntax is rather intuitive and can be in two forms:

- `Function_handle_name =@MYfunction`
- `Function_handle_name=@(variable1, variable2, ...) ([expression1, expression2, ...])`

Let's look at several examples of how to generate function handles:

```
F1 = @MY_function;
```

Where MY_function is a function file, function expression, or another function handle.

For example:

```
function x = MY_function(a, b, c)
% MY_function.m is a function file that solves the quadratic equation
w.r.t. % a  user entries for a, b, c and outputs the found solutions.

D= b^2-4*a*c;
x1 = (-b+sqrt(D))/(2*a);
x2 = (-b-sqrt(D))/(2*a);
x = [x1; x2];
end
```

We can test the function handle called F1 with the following command:

```
>> x = F1( 1, 2, 3)
x =
  -1.0000 + 1.4142i
  -1.0000 - 1.4142i
```

Here the function handle called F1 calls the function file called MY_function.m and executes it with the user-specified input data for the a, b, and c variables.

Note that more detailed explanations of features of the function files (e.g., MY_function.m) and how to create them is found in Chapter 2, “Programming Essentials”.

$$f(x, a_1, a_2, a_3) = a_1x^2 + a_2x + a_3;$$

```
>> f =@(x,a1,a2,a3)(a1*x^2+a2*x+a3)
f =
 function_handle with value:
 @(x,a1,a2,a3)(a1*x^2+a2*x+a3)
```

Moreover, the function handles can be used to define a function of functions. For example, $H = 2e^{\sin(x)}$ can be expressed in three ways with the following function of functions:

```
>> ff1 = @(x) sin(x); ff2 = @(ff1)exp(ff1); ff3 = @(ff2)2*ff2;     % 1 - Way
>> gg1 = @(x)sin(x); gg2 = @(x)exp(gg1(x)); ff3 = @(x) 2*gg2(x);   % 2 - Way
>> hh3=@(x)2*exp(sin(x));                                          % 3 - Way
```

It's quite straightforward to perform computations from function handles:

```
>> ff3(ff2(ff1(pi)))
ans =
    2.0000
>>  gg3(pi)
ans =
   2.0000e+00
>> hh3(pi)
ans =
   2.0000e+00
>> x=1.3; a1=2; a2=-3; a3=13; f (x, a1, a2, a3)
ans =
 12.48
>> x=1.3; a1=[2, 3]; a2=[-3, 4]; a3=[11,13]; f2(x, a1, a2, a3)
ans =
   10.4800   23.2700
>> x=1:3; a1=2; a2=-3; a3=11; f2(x, a1, a2, a3)
Error using  ^  (line 51)
Incorrect dimensions for raising a matrix to a power.
Check that the matrix is square and the power is a
scalar. To perform elementwise matrix powers, use
'.^'.
Error in @(x,a1,a2,a3)(a1*x^2+a2*x+a3)
```

In the last part, the expression of $f(x,a_1,a_2,a_3)$ needs to be fixed for element-wise matrix operations.

```
>> f =@(x,a1,a2,a3)(a1*x.^2+a2*x+a3)
f =
  function_handle with value:
    @(x,a1,a2,a3)(a1*x.^2+a2*x+a3)>> x=1:3; a1=2; a2=-3; a3=11;
    f(x, a1, a2, a3)
ans =
    10    13    20
```

An alternative version of the function handle is the inline function. It's similar to the function handle. Note that the inline function will be removed in future releases of MATLAB.

```
>> F1=inline('a1*x^2+a2*x+a3', 'a1', 'a2', 'a3', 'x')
F =
 Inline function:
 F (a1,a2,a3,x) = a1*x^2+a2*x+a3
```

Now, the inline functions can be evaluated with specific values of variables—x, a1, a2, and a3.

```
>> x=1.3; a1=2; a2=-3; a3=13; F(a1, a2, a3, x)
ans =
   12.4800
```

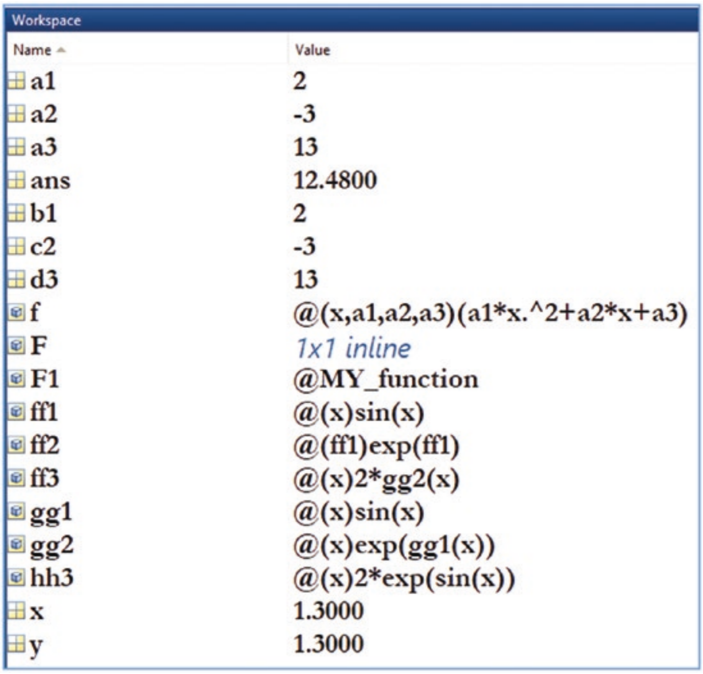
It is important while employing the function handles to follow the order of the variables. In other words, while calling them, you need to follow the sequence of the input variable values. For example:

```
>> x=1.3; a1=2; a2=-3; a3=13;
>> f =@(x,a1,a2,a3)(a1*x.^2+a2*x+a3);
>> f(x, a1, a2, a3)
ans =
   12.4800
>> f( a1, a2, a3, x)
ans =
   15.3000
```

The function handles can take different predefined input variables:

```
>> f =@(x,a1,a2,a3)(a1*x.^2+a2*x+a3);
>> y=1.3; b1=2; c2=-3; d3=13;
>> f(y,b1,c2,d3)
ans =
   12.4800
```

Figure 1-12 shows the list of variables and function handles created in this section that reside in the workspace.



Workspace

Name	Value
a1	2
a2	-3
a3	13
ans	12.4800
b1	2
c2	-3
d3	13
f	@(x,a1,a2,a3)(a1*x.^2+a2*x+a3)
F	1x1 inline
F1	@MY_function
ff1	@(x)sin(x)
ff2	@(ff1)exp(ff1)
ff3	@(x)2*gg2(x)
gg1	@(x)sin(x)
gg2	@(x)exp(gg1(x))
hh3	@(x)2*exp(sin(x))
x	1.3000
y	1.3000

Figure 1-12. *Variables residing in the workspace*

Broader applications and uses of function handles are discussed in other chapters of the book—in Chapter 2, "Programming Essentials," and Chapter 8, "Ordinary Differential Equations".

Logical Arrays

Logical arrays are very important for programming and Boolean operations. The local answers are 0s and 1s. 0 means that a statement or condition is not true and 1 means that a statement is true. There are a few ways that logic arrays can be generated in MATLAB. You can apply the comparative analysis within numerical arrays or verify them or define their types. The following Boolean logical arrays are based on existing variables created in the previous sections.

```
>>A = 2; B = -3.25; C=(A+B)^2; ABC=[A, B, C];
>> isnumeric(A)
ans =
 logical
 1
```

```
>> Ach1 = 'matlab';
>> isstr(Ach1)
ans =
 logical
 1
>> ABC_logic=ABC>=2
ABC_logic =
 1×3 logical array
 1 0 0
>> Seed = 1; rng(Seed);   % Setting up the random number generator to
generate permanent numbers.
>> Dr1 = rand(4, 5)
>> Dr1 = rand(4, 5)
Dr1 =
    0.0500    0.9446    0.1393    0.9275    0.8833
    0.5359    0.5866    0.8074    0.3478    0.6237
    0.6638    0.9034    0.3977    0.7508    0.7509
    0.5149    0.1375    0.1654    0.7260    0.3489
>> Dr1Logic=Dr1>0.5 & Dr1<=0.71
>> Dr1Logic=Dr1>0.5 & Dr1<=0.71
Dr1Logic =
  4×5 logical array
   0   0   0   0   0
   1   1   0   0   1
   1   0   0   0   0
   1   0   0   0   0
```

Note that we set the seed value of the random number generator `rng()` in order to generate permanent element values with the uniform random number generator `rand()`.

You can use the evaluated logical arrays to index arrays and find out which values meet set conditions and which ones do not. For example, in the previous examples, `ABC_logic` states that the first element of `ABC` meets the set condition and is greater than or equal to 2. Similarly, the `Dr1Logic` array states that three elements in row 2 and one element in row 5 are greater than 0.5 and less than or equal to 0.71. Now you can determine which elements meet the set conditions by using indexing operations.

```
>> Index=find(Dr1>0.5 & Dr1<=0.71)
>> Index=find(Dr1>0.5 & Dr1<=0.71)   % Index numbers indicate which element
                                       has met the condition.
Index =
     2
     3
     4
     6
    18>> Dr1(Index)                 % The element meeting the set conditions
ans =
    0.5359
    0.6638
    0.5149
    0.5866
    0.6237
```

An alternative way of finding these true values is the following:

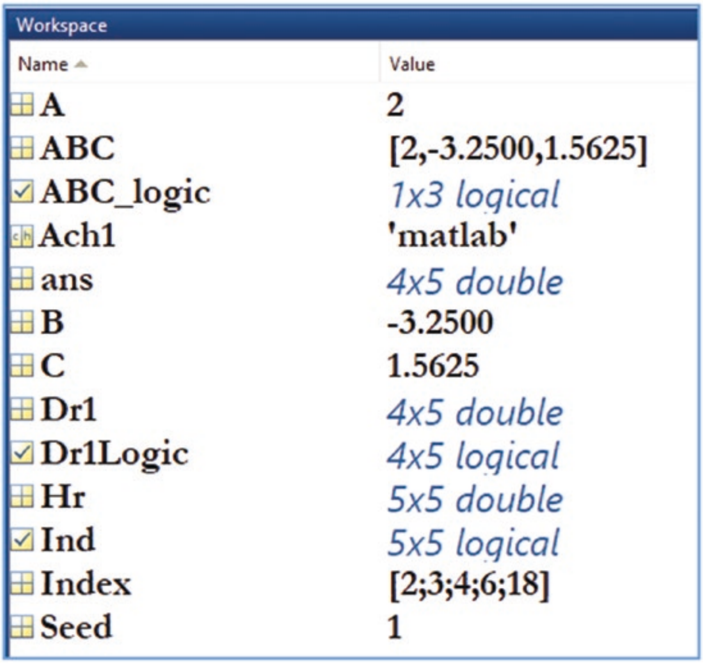
```
>> Dr1Logic.*Dr1
ans =
         0         0         0         0         0
    0.5359    0.5866         0         0    0.6237
    0.6638         0         0         0         0
    0.5149         0         0         0         0
```

As another example, let's find the "0" elements in an array generated by the random integer number generator, in order to generate integer numbers within [-2 2], filling the 5-by-5 square matrix:

```
>> Seed = 1; rng(Seed);   % Setting up the random number generator to
                            generate permanent numbers.
>> Hr=randi([-2, 2], 5)
```

```
Hr =
     0    -2     0     1     2
     1    -2     1     0     2
    -2    -1    -1     0    -1
    -1    -1     2    -2     1
    -2     0    -2    -2     2>> Ind=(~Hr)    % Locates which elements of
                                                 Hr are equal to "0"
Ind =
  5×5 logical array
   1   0   1   0   0
   0   0   0   1   0
   0   0   0   1   0
   0   0   0   0   0
   0   1   0   0   0
```

In this section, we generated the numerical and logic matrices and character strings, as shown in Figure 1-13.



Workspace

Name ▲	Value
A	2
ABC	[2,-3.2500,1.5625]
ABC_logic	1x3 logical
Ach1	'matlab'
ans	4x5 double
B	-3.2500
C	1.5625
Dr1	4x5 double
Dr1Logic	4x5 logical
Hr	5x5 double
Ind	5x5 logical
Index	[2;3;4;6;18]
Seed	1

Figure 1-13. *Variables residing in the workspace*

More uses/examples of logical arrays and indexing are discussed in other sections of the book.

Table Arrays

The table arrays are a newly introduced tool in MATLAB and thus, in older versions of MATLAB, table arrays cannot be generated. Table arrays are particularly useful for preparing reports and displaying/presenting the simulation/analysis results obtained in arrays of several columns and rows, each of which represents certain variables. They are used to collect heterogeneous data and metadata into a single container in a tabular data format. Table Arrays can accommodate variables of different types, sizes, units, etc. They are often used to store experimental data, with rows representing different observations and columns representing different measured variables. They display the arrays in a more informative and tabulated format by indicating names of the columns and rows by respective assigned names. Let's look at several examples of creating Table Arrays with the `table()` command by using the existing variables created in the previous sections.

```
>> A1new = [1 2 3; 3 4 5; 7 8 9];
>> A1Tab=table(A1new)
A1Tab =
  3×1 table
       A1new

    ___________

    1    2    3
    3    4    5
    7    8    9>> B1new = A1new/5;
>> B1Tab=table(B1new)
B1Tab =
  3×1 table
          B1new

    _________________

    0.2    0.4    0.6
    0.6    0.8      1
    1.4    1.6    1.8
```

In these cases, the Table Arrays `A1Tab` and `B1tab` are created as column tables from `A1New` and `B1New`, respectively. It is also possible to obtain/create tables from the existing arrays (arrays, cells, and structures) by using `array2table()`, `cell2table()`, and `struct2table()` commands:

```
>> A1Tab2=array2table(A1New) % Column names are not specified
>> A1Tab2=array2table(A1new)
A1Tab2 =
  3×3 table
    A1new1    A1new2    A1new3

    ______    ______    ______
      1         2         3
      3         4         5
      7         8         9   >> >> Ach1Tab=array2table(A1new,
                              'variablenames',{'a','b','c'})
Ach1Tab =
  3×3 table
    a    b    c

    _    _    _
    1    2    3
    3    4    5
    7    8    9 >> C1new = [A1new, B1new]
C1new =
    1.0000    2.0000    3.0000    0.2000    0.4000    0.6000
    3.0000    4.0000    5.0000    0.6000    0.8000    1.0000
    7.0000    8.0000    9.0000    1.4000    1.6000    1.8000
>> Dr1Tab=array2table(C1new, 'variablenames', {'v1', 'v2', 'v3', 'v4',
'v5', 'v6'})
Dr1Tab =
  3×6 table
    v1    v2    v3    v4     v5     v6

    __    __    __    ___    ___    ___
    1     2     3     0.2    0.4    0.6
    3     4     5     0.6    0.8      1
    7     8     9     1.4    1.6    1.8
```

It is also possible to convert Table Arrays into arrays and cell arrays by using the `tabel2array()` and `table2cell()` commands, respectively. Understanding and working with Table Arrays will be of great help not only when you're preparing reports but also when you're importing data using the Import wizard and manipulating various data sets from external files (e.g. `.txt`, `.xls`, `.dat`, etc.) into the MATLAB workspace.

Cell Arrays

Cell arrays are useful for accommodating various types of arrays (numerical, character, logical, table, and function handle) in different cells of one cell type variable by preserving all attributes of each variable, unchanged. They might be very handy to carry or pass various data sets inside one variable. The cell arrays contain indexed data containers—cells accommodating lists of text, character strings, combinations of text and numerical data, and numerical arrays, function handles, structure arrays, and tables. One of the most essential features of cell arrays is that they require curly brackets to be used in specifying cell addresses. Another important feature of cell arrays is that data read by MATLAB will be in cell array mode.

Let's look at several examples that create cell arrays with different types of arrays and generate new ones.

```
>> Acell = cell(5,5) % Creates an empty cell of size 5-by-5
>> Acell = cell(3,4)
Acell =
  3×4 cell array
    {0×0 double}    {0×0 double}    {0×0 double}    {0×0 double}
    {0×0 double}    {0×0 double}    {0×0 double}    {0×0 double}
    {0×0 double}    {0×0 double}    {0×0 double}    {0×0 double}% Each cell
                                                    of the cell array Acell
                                                    will be filled with the
                                                    Arrays
>> A0 = 13; A1new = [1 2 3; 3 4 5; 7 8 9];
>> Acell{1,1}=A0  % Cell (1, 1) is filled with A0
```

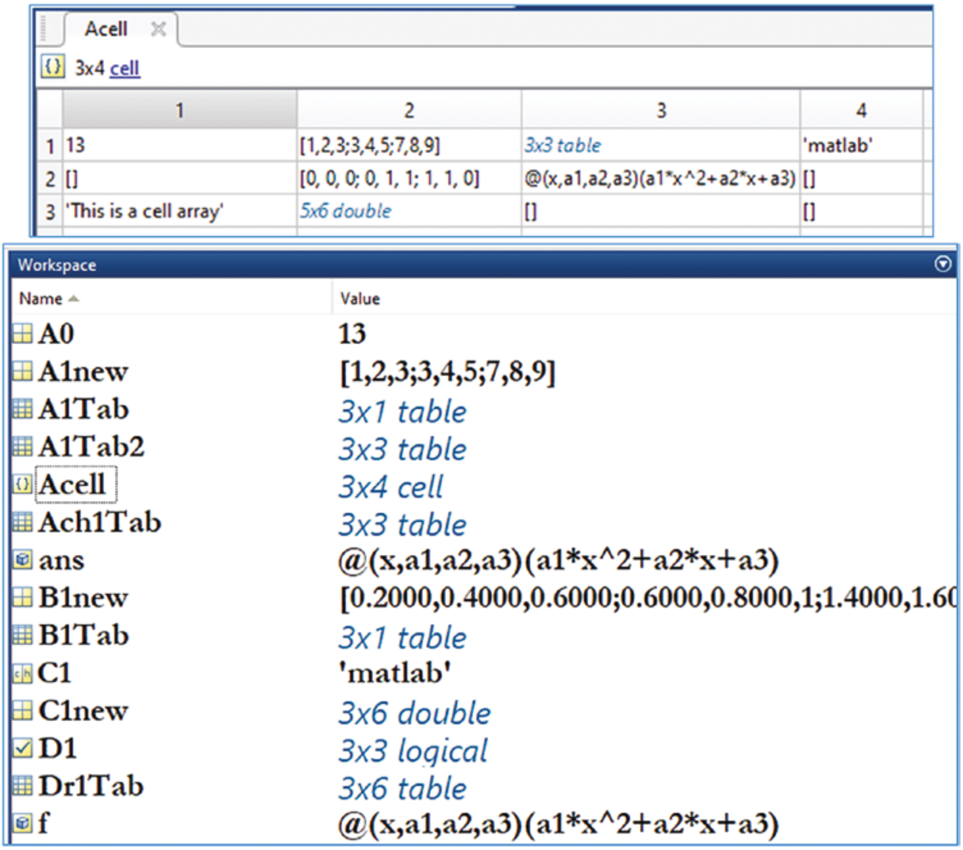
```
Acell =
  3×4 cell array
    {[      13]}    {0×0 double}    {0×0 double}    {0×0 double}
    {0×0 double}    {0×0 double}    {0×0 double}    {0×0 double}
    {0×0 double}    {0×0 double}    {0×0 double}    {0×0 double}
>> Acell{1,2}=A1new  % Cell (1, 2) is filled with A1new
Acell =
  3×4 cell array
    {[      13]}    {3×3 double}    {0×0 double}    {0×0 double}
    {0×0 double}    {0×0 double}    {0×0 double}    {0×0 double}
    {0×0 double}    {0×0 double}    {0×0 double}    {0×0 double} >>
                                                    Ach1Tab=array2table
                                                    (A1new, 'variablenames',
                                                    {'a','b','c'});
>> Acell{1,3}=Ach1Tab  % Cell (1, 3) is filled with Ach1Tab
Acell =
  3×4 cell array
    {[      13]}    {3×3 double}    {3×3 table }    {0×0 double}
    {0×0 double}    {0×0 double}    {0×0 double}    {0×0 double}
    {0×0 double}    {0×0 double}    {0×0 double}    {0×0 double} >>
                                                    C1 = 'matlab';
>> Acell{1,4}=C1  % Cell (1, 4) is filled with C1
Acell =
  3×4 cell array
    {[      13]}    {3×3 double}    {3×3 table }    {'matlab'  }
    {0×0 double}    {0×0 double}    {0×0 double}    {0×0 double}
    {0×0 double}    {0×0 double}    {0×0 double}    {0×0 double} >>
                                                    D1 = A1new>3 & A1new<9
D1 =
  3×3 logical array
   0   0   0
   0   1   1
   1   1   0 >> Acell{2,2}=D1  % Cell (2, 2) is filled with D1 logical
           array
```

```
Acell =
  3×4 cell array
    {[      13]}    {3×3 double }    {3×3 table }    {'matlab'  }
    {0×0 double}    {3×3 logical}    {0×0 double}    {0×0 double}
    {0×0 double}    {0×0 double }    {0×0 double}    {0×0 double} >> f=@(x,
                                                     a1, a2, a3)
                                                     (a1*x^2+a2*x+a3);
>> Acell{2,3}=f  % Cell (2, 3) is filled with f function handle
Acell =
  3×4 cell array
    {[      13]}    {3×3 double }    {3×3 table      }    {'matlab'  }
    {0×0 double}    {3×3 logical}    {function_handle}    {0×0 double}
    {0×0 double}    {0×0 double }    {0×0 double     }    {0×0 double}
                                                          >> Acell{3,1}=
                                                          'This is a cell
                                                          array' % Entered
                                                          character strings
                                                          into Cell array:
                                                          Cell (3, 1)
Acell =
 3×4 cell array
 {[               13]}    {3×3 double }    {3×3 table      }
       {'matlab'  }
 {0×0 double          }    {3×3 logical}    {function_handle}
       {0×0 double}
 {'This is a cell ...'}    {0×0 double }    {0×0 double     }
       {0×0 double}>> Acell{3,2}=rand(5,6) % Entered numerical array into
       Cell array: Cell(3,2)
Acell =
  3×4 cell array
 {[               13]}    {3×3 double }    {3×3 table      }    {'matlab'  }
 {0×0 double          }    {3×3 logical}    {function_handle}    {0×0 double}
 {'This is a cell ...'}    {5×6 double }    {0×0 double     }    {0×0 double}
```

To access and view the contents of the cell arrays, use the following commands.

```
>> Acell{1,1}
ans =
    13>> Acell{1,4}
ans =
    'matlab'
>> Acell{2,3}
ans =
  function_handle with value:
    @(x,a1,a2,a3)(a1*x^2+a2*x+a3)>> Acell{7}
ans =
  3×3 table
    a    b    c

    _    _    _
    1    2    3
    3    4    5
    7    8    9
>> Acell{8}
ans =
  function_handle with value:
    @(x,a1,a2,a3)(a1*x^2+a2*x+a3)
```

By double-clicking on the cell variable name in the workspace, the contents of the cell can be viewed, as shown in Figure 1-14.



	1	2	3	4
1	13	[1,2,3;3,4,5;7,8,9]	3x3 table	'matlab'
2	[]	[0, 0, 0; 0, 1, 1; 1, 1, 0]	@(x,a1,a2,a3)(a1*x^2+a2*x+a3)	[]
3	'This is a cell array'	5x6 double	[]	[]

Name	Value
A0	13
A1new	[1,2,3;3,4,5;7,8,9]
A1Tab	3x1 table
A1Tab2	3x3 table
Acell	3x4 cell
Ach1Tab	3x3 table
ans	@(x,a1,a2,a3)(a1*x^2+a2*x+a3)
B1new	[0.2000,0.4000,0.6000;0.6000,0.8000,1;1.4000,1.6
B1Tab	3x1 table
C1	'matlab'
C1new	3x6 double
D1	3x3 logical
Dr1Tab	3x6 table
f	@(x,a1,a2,a3)(a1*x^2+a2*x+a3)

Figure 1-14. *Variables in the workspace and contents of the Acell cell array*

You can change the contents of the cell array by double-clicking in each cell and entering the new values.

Structure Arrays

Structure arrays can accommodate all of the previously created array and entry (variable) types, namely, all types of numeric, logical, character, table, cell, and function handles. They can store data not only of different types but also of different sizes. One of the more important aspects of structure arrays is that they are suitable to code generation. Moreover, they are very useful in programming, data processing, data acquisition, and reading from Simulink models.

Structure arrays store data in different fields or field names, which you can access by their names. For example:

```
% Person 1: Name - SE; DOB - June 6, 1982; Profession - professor
WHO(1).Name = 'SE';
WHO(1).DOB = '06.06.1982';
WHO(1).Profession = 'professor';
% Person 2: Name - NE; DOB - Dec 12, 1992; Profession - designer
WHO(2).Name = 'NE';
WHO(2).DOB = '12.12.1982';
WHO(2).Profession = 'designer';
```

The structure variable WHO contains information about two people. The data contains their names, date of birth, and profession.

```
>>WHO =
  1×2 struct array with fields:
    Name
    DOB
    Profession
>> WHO(1)
ans =
  struct with fields:
          Name: 'SE'
           DOB: '06.06.1982'
    Profession: 'professor'
>> WHO(2)
ans =
  struct with fields:
          Name: 'NE'
           DOB: '12.12.1982'
    Profession: 'designer'
```

Note that we saved only one variable that is in a structure array form. If you want to have access to a specific field of the created structure, you call that field name along with the structure name:

```
>> WHO(1).Name   % To access to the name field of a person 1
```

```
ans =
    'SE'
>> WHO(1).Profession   % To access to the profession field of a person 1
ans =
    'professor'
>> WHO(2).DOB    % To access to the DOB field of a person 2
ans =
    '12.12.1982'
```

Besides this approach of creating structure arrays, there are several other ways to set up or create them. Let's look at some examples. Another way is to create an empty structure with the `struct()` command. The empty structure will be filled with variables with the `struct('FieldName', VALUE)` command syntax. Note that the field names (variable names) cannot contain empty spaces or symbols, except for the underscore sign _.

```
>>  Astr1 = struct()
Astr1 =
  struct with no fields.
>> % Now we can assign/set up values and data fields inside the created
empty structure: Astr1.
>> a = 2.1; A = [1, 2; 3, 4]; B = A<a; f = @(x)(a*x^2+x-a); C{1} =rand(3);
C{2} = 'matlab';
>> D = 'mathworks.com';
>> F = table(magic(3));
>>  Astr1 = struct('Number', a, 'Matrix', A, 'Logic', B, 'F_Handle', f,
'Cell', C, 'Char', D, 'Table', F)

Astr1 =

  1×2 struct array with fields:
    Number
    Matrix
    Logic
    F_Handle
    Cell
    Char
    Table
```

Another way to create structure arrays is to assign their fields individually, as shown in the beginning with the example of creating the structure variable WHO.

```
>> Bstr.Number = a; Bstr.Matrix = A; Bstr.Logic = B; Bstr.F_Handle=f; Bstr.
Cell=C; Bstr.Char = D;
>> Bstr.Table = F;
>> Bstr
Bstr =
  struct with fields:
      Number: 2.1000e+00
      Matrix: [2×2 double]
       Logic: [2×2 logical]
    F_Handle: @(x)(a*x^2+x-a)
        Cell: {[3×3 double]  'matlab'}
        Char: 'mathworks.com'
       Table: [3×1 table]
```

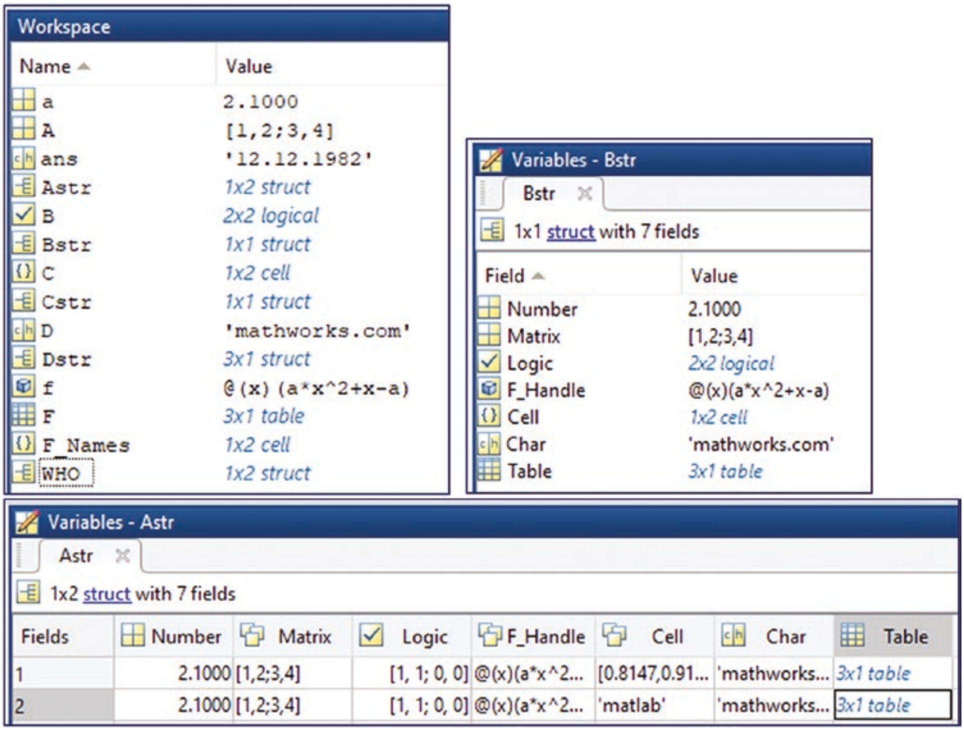
Now we can compare the two different ways we created the structure variables Astr and Bstr. Another way to create structure arrays is conversions; in other words, you convert existing cell type variables or table type variables into structure type variables with the cell2struct() and table2struct() commands.

```
>> % Cell variable C contains: Matrix C and String character 'matlab'
>> C{1} =rand(3); C{2} = 'matlab';
>> F_Names = {'Matrix', 'Char'};    % Field names/Headers
>> Cstr = cell2struct(C, Headers, 2) % 2 means two cells embedded
Cstr =
  struct with fields:
    Matrix: [3×3 double]
      Char: 'matlab'
```

Now let's see how to convert a table array into a structure array with the table2struct() command.

```
>> F = table(magic(3));
>> Dstr = table2struct(F)
Dstr =
  3×1 struct array with fields:
    Var1
```

Figure 1-15 shows the list of variables saved in the workspace and the contents of the `Astr` and `Bstr` structure arrays.



Workspace

Name ▲	Value
a	2.1000
A	[1,2;3,4]
ans	'12.12.1982'
Astr	1x2 struct
B	2x2 logical
Bstr	1x1 struct
C	1x2 cell
Cstr	1x1 struct
D	'mathworks.com'
Dstr	3x1 struct
f	@(x)(a*x^2+x-a)
F	3x1 table
F_Names	1x2 cell
WHO	1x2 struct

Variables - Bstr

Bstr

1x1 struct with 7 fields

Field ▲	Value
Number	2.1000
Matrix	[1,2;3,4]
Logic	2x2 logical
F_Handle	@(x)(a*x^2+x-a)
Cell	1x2 cell
Char	'mathworks.com'
Table	3x1 table

Variables - Astr

Astr

1x2 struct with 7 fields

Fields	Number	Matrix	Logic	F_Handle	Cell	Char	Table
1	2.1000	[1,2;3,4]	[1, 1; 0, 0]	@(x)(a*x^2...	[0.8147,0.91...	'mathworks...	3x1 table
2	2.1000	[1,2;3,4]	[1, 1; 0, 0]	@(x)(a*x^2...	'matlab'	'mathworks...	3x1 table

Figure 1-15. *Created variables and Astr and Bstr structure arrays*

This section has demonstrated via examples how to create seven different types of variables (arrays)—numerical arrays (scalar and array type variables), character strings, logical arrays, table arrays, cell and structure arrays, and function handles. You can remove from the workspace any of the created variables by using the `clear` or `clearvars` commands or using the right-mouse button options and Delete. From the Command Window, you can clear the variables from the workspace with the following commands.

```
>> clearvars a A  B% Removes variables a, A, B
>> clear ans C D % Removes variables ans, C, D
```

From the attributes of the variables in the workspace (see Figure 1-12), you can determine the variable type (scalar, array, logical, table, cell, structure, character, or function handle) and its storage type (double, single, uint8, or int8). Moreover, the symbols representing each variable shown in Figure 1-15 demonstrate some of the MATLAB supported data (array) sets shown in Figure 1-9.

It must be noted that many of these arrays can be converted from one type to another, as you have seen in some of the examples. For example, a cell array can be converted into a table array via the `cell2table()` function or similarly, a structure array can be converted into a table array via `struct2table()`. A table array can be converted into a cell and table array via `table2cell()` and `table2struct()`, respectively.

Complex Numbers

Two letters—`i` and `j` or `1i` and `1j`—are reserved for notating imaginary numbers. Therefore, it is advised not to use these letters to assign variable names. An alternative approach to assigning numbers is to multiply them by `sqrt(-1)`.

Precision

MATLAB's precision is not absolute.

For instance:

```
>> sin(pi)
ans =
 1.2246e-016
```

In MATLAB, sin(π) is not equal to 0. That is due to the fact that the number π is represented by the double precision number in MATLAB. We can demonstrate the precision issue by performing the calculations of the Pythagorean Theorem: $1 = \sin^2(t) + \cos^2(t)$.

```
t=0:pi/50:2*pi; F=1-(sin(t).^2+cos(t).^2); plot(t, F)
```

In these expressions, `t` is a time vector containing a row of elements, such as [0, π/50, … 2π]. Some values of `F` are zero and some are non-zero, even though they are very small numbers. The reason for this is that all of the trigonometric functions, including exponential and logarithmic functions, are approximated by a polynomial of degree 13 with only odd powers of the argument variable (in this example, `t`). For instance, $sin(t) \approx t - c_1t^3 + c_2t^5 + \ldots + c_6t^{13} = p(t)$. The computation algorithm for all of these functions is implemented based on `fdlibm`, a "Freely Distributable Math Library" developed by at Sun Microsystems by K. C. Ng and others (for more information, see `www.netlib.org/fdlibm`).

It must be noted that MATLAB's accuracy (precision) level depends on which data storage type is used to save data. For instance:

```
>> int8(128)*-5
ans =
 -128
```

The allocated data storage `int8` can hold up to $2^8 - 1$ integer numbers. All MATLAB supported data storage types are shown in Figure 1-15.

M-File and MLX–File Editors

In the context of this book, the terms code, script, and program are used interchangeably to refer to the M-files with the extension of `*.m` and MLX-files with the extension `*.mlx`, including function and executable files. Because of their extensions, these files are called M-files and MLX-files. In the previous examples, all of the operations and work are done in the Command Window. However, for programming and writing, editing and debugging, M-file and MLX-file editors will be of great help due to their helpful tools and hints in writing fast and more efficient codes/scripts/programs.

The overall functionality of M-files and MLX-files is similar except for one important feature. The MLX file editor window can display the outputs of calculations/simulations within the MLX editor window and indicate the most common command syntax related errors on its left-side output window. The M-file editor shows all errors in the Command Window after the M-file's execution. Moreover, the MLX-file editor can interactively show all inserted equations via the equation editor; inserted images and hyperlinked texts are right in the same window. The outputs from the both files will be shown in the workspace. Both files can be used interchangeably. Let's start reviewing M-file and MLX-file Editor windows and tools.

M-File Editor

The M-file editor window menu and GUI tools are sub-grouped in three tabs—EDITOR, PUBLISH, and VIEW, as shown in Figures 1-16, 1-17, and 1-18. Note that there are three main menu sub-groupings—HOME, PLOTS, and APPS—which belong to the main MATLAB window and were shown in initial sections.

In the M-file editor's main tools menu (see Figure 1-16), there are five subsections—FILE, NAVIGATE, EDIT, BREAKPOINTS, and RUN. All of the tools in each subsection are quite intuitive. For example, the FILE sub-section has GUI tools used to create a new file or open existing files, save the current file, find M-files, compare different versions of the M-files with the same names, and print the current M-file. Similarly, the NAVIGATE subsection tools help a user move the cursor within the current file and find keywords and, if necessary, substitute them with other words. The EDIT subsection tools allow you to insert a new section into the current file, add or remove a comment line or wrap comments and add indents to make the file more readable. The BREAKPOINTS subsection enables you to debug/edit the current M-file code. Finally, the RUN subsection has GUI tools that run different cell sections of the current M-file step by step and run the current M-file and measure the evaluation time in different sections of the file. It should be noted that when writing M-files, the EDITOR window tools are mainly used.

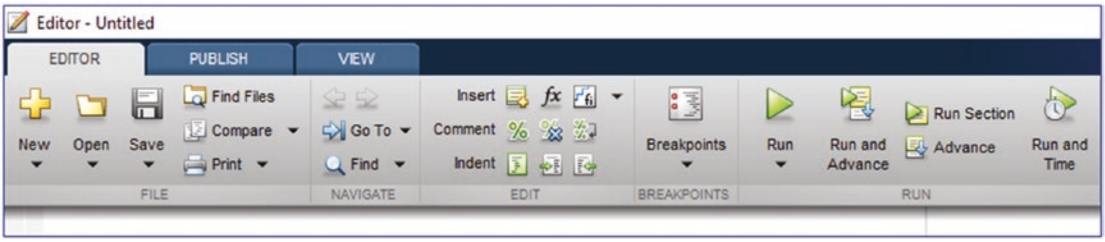


***Figure 1-16.** M-file editor's main tools menu*

The PUBLISH tools, shown in Figure 1-17, are used to generate report files in different file formats, such as HTML, DOC, PPT, etc. The PUBLISH window's GUI tools are very intuitive and similar to many document editing software applications.

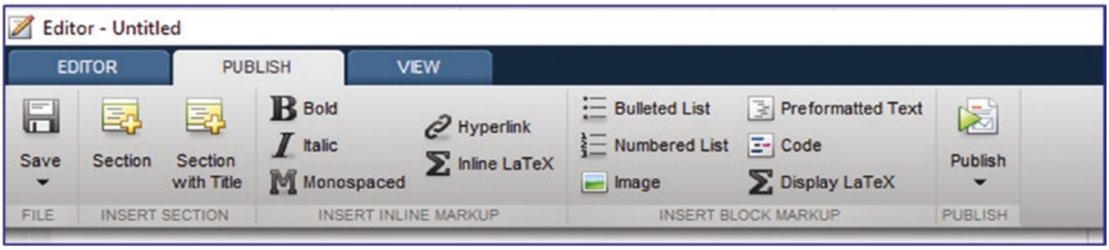
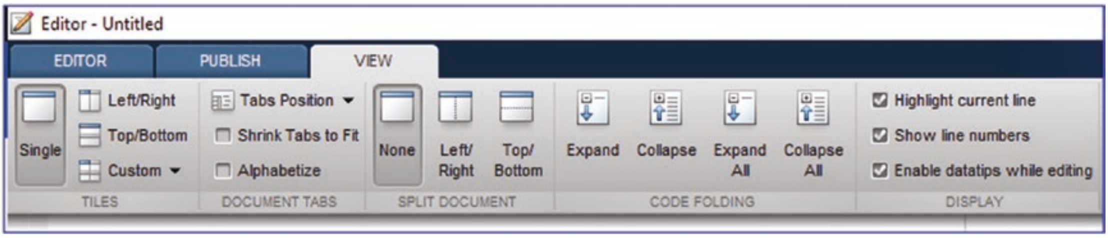


***Figure 1-17.** M-file editor's Publish tools menu*

The VIEW tools, shown in Figure 1-18, are used to display several windows of M-files and MLX-files (documents) within one window area. You can split the view side by side or top to bottom by using the GUI tools called TILES, DOCUMENT TABS, or SPLIT DOCUMENT. The tickmark options in the DISPLAY subsection are very handy to display data-tips, show line numbers, and highlight the current line while editing M-files.



□ Shrink Tabs to Fit
□ Alphabetize
☑ Highlight current line
☑ Show line numbers
☑ Enable datatips while editing

Figure 1-18. *M-file editor's view tools menu*

MLX-File Editor

The MLX-file (live editor) editor tab (shown in Figures 1-19, 1-20, and 1-21) contains many of the M-file editor tools along with several other different GUI tools. The LIVE EDITOR tab (see Figure 1-19) has one main different sub-section from the M-file editor, called TEXT, and it contains most of the functions of the PUBLISH tools of the M-file editor.

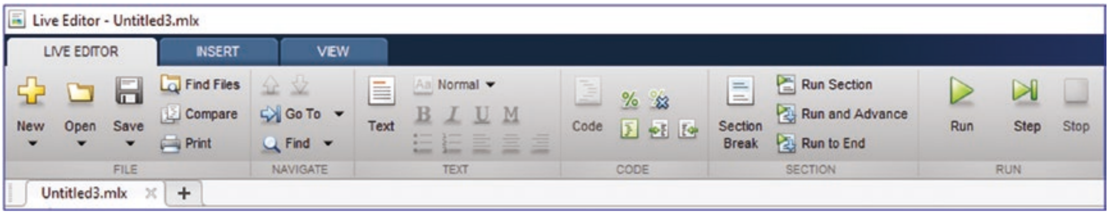


Figure 1-19. *MLX-file editor's main tools menu*

Moreover, the MLX-editor's INSERT tab (see Figure 1-20) has a few very handy tools to write/edit equations directly in the MLX-file and insert subsections with comments. You can insert control GUI tools, such as slider and drop-down boxes, within the current MLX-file content.

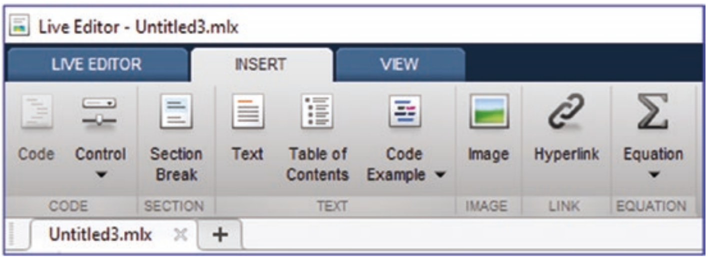


***Figure 1-20.** MLX-file editor's INSERT menu*

The VIEW window (shown in Figure 1-21) of the MLX-editor has some similar tools of the M-file editor's VIEW and some other tools, such as DOCUMENT TABS, DISPLAY, OUTPUT, and LAYOUT, by which a user can display script (code) line numbers (excluding the comment lines), data-tips, full screen view, clean up the output, and show or hide output.

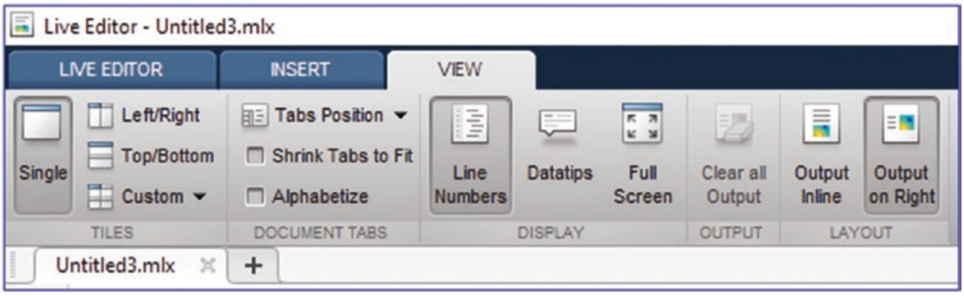


***Figure 1-21.** MLX-file editor's view tools menu*

M-files and MLX-files can be created in several different ways—by using GUI buttons New Script or New Live Script or by typing the command in the Command Window.

In order to demonstrate some of the highlighted tools and options of the M-file and MLX-file editors, let's look at the following example to demonstrate that MATLAB's precision is not absolute via the Pythagorean Theorem.

Example: $\alpha = -\frac{\pi}{2} \ldots \frac{\pi}{2}, \Delta\alpha = \frac{\pi}{100}; F(\alpha) = 1 - \left(\sin^2\alpha + \cos^2\alpha\right);$

First, the solution script of this simple example will created in the M-file editor and the results will be published. Subsequently, all of the simulations will be carried out within the MLX-editor to demonstrate similar and different features of the both editors.

The algorithm for solving this exercise is composed of the following six steps.

1. Insert some comments describing the given problem statement.
2. Define the input variable: $\alpha = -\frac{\pi}{2} \ldots \frac{\pi}{2}, \Delta\alpha = \frac{\pi}{100};$.
3. Perform the computation: $F(\alpha) = 1 - (\sin^2\alpha + \cos^2\alpha)$.
4. Define for which values of the input variable α the function values of $F(\alpha) = 0$.
5. Plot the simulation results: α vs. $F(\alpha)$.
6. Publish all of the obtained results, including the whole script.

This is the script written directly in the M-file editor.

```
% Step 1. Problem statement
% MATLAB's precision is not absolute.
% Pythagorean Theorem: F = 1 - (sin^2 (alpha) + cos^2(alpha));
% Input variable: alpha = -0.5*pi ... 0.5*pi

% Step 2. Define the input variable
alpha = -pi/2:pi/100:pi/2;

% Step 3. Perform the computation
F = 1-(sin(alpha).^2+cos(alpha).^2);

% Step 4. Define for which values of alpha, F(alpha) = 0.
Findex=find(F==0);
F0=F(Findex);

% Step 5. Plot the simulation results: alpha vs. F(alpha)
plot(alpha, F, 'b-', alpha(Findex), F0, 'ro'),
legend('\alpha vs F(\alpha)', 'F(\alpha) = 0')
title('Simulation of MATLAB"s precision via the Pythagorean Theorem '),
grid on
xlabel('\alpha'), ylabel('F(\alpha) = 1-(sin(\alpha)^2+cos(\alpha)^2)')

% Step 6. Publish all of the obtained results including the whole script
```

Note that in this script, we used % to insert comments (non-executable information) and \ to insert and display Greek letters.

% Comments

Comments are not executable and contain additional information for the users. The % sign is used to place comments and remarks or any additional information within M-files and MLX-files. The comments can be added on a separate line or after the command syntax as long as they start with %. Using a double sign (%%) followed by a space automatically makes the following comments bold. Moreover, inserting %% at the beginning of a line and leaving a blank space after it creates a cell mode in the script. We discuss in detail the cell mode options and advantages in Chapter 2. Note that there are several other functionalities of the % sign. It is used to format specifications for write, display, read purposes, which we discuss in Chapter 2.

This is slightly edited with the PUBLISH tools, such as bold **B** Bold for Steps 1, 2, ..6) under INSERT INLINE MARKUP and Publish (PUBLISH); see Figure 1-17. Note that to make selected lines of comments bold, you first select the line and then press the **B** Bold button. Note that in this script, we used the LaTEX commands to insert the Greek letters (α, π) and the equation $F = 1 - (sin^2\alpha + cos^2\,\alpha)$.

The M/MLX editors are compatible with most common LaTEX mathematical mode commands. The LaTEX compatible mathematical commands and symbols can be inserted for plot titles, axis labels, graphic notes, and so forth, which are discussed in examples in the following chapters (programming, plots, and ODEs). For example, to insert the expression: $x^2 + y^2 = R^2$, you enter the expression: `x^2+y^2=R^2`. Or to insert α, β, Ω, Ψ, you type `\alpha`, `\beta`, `\Omega`, `\Psi`. Moreover, to insert the equations with Greek letters, the notations need to start with $$ signs and end with $ (See Step 1. Problem statement and Step 5. Plot ...). For more information and help with various mathematical expressions and symbols to write in LaTEX math, type the following in the command window: `>> doc latex`

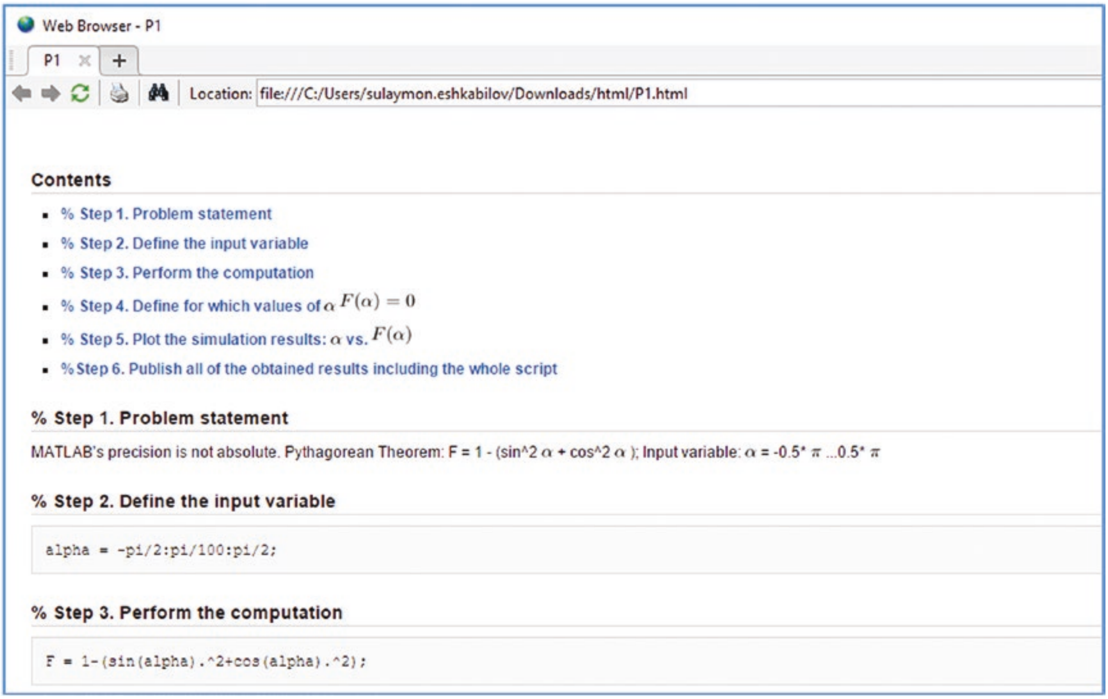
See [6] also. Note that %% at the beginning initializes automatic recognition of the LaTEX compatible mathematical commands within Step 1.

```
%%
%% *% Step 1. Problem statement*
% MATLAB's precision is not absolute.
% Pythagorean Theorem: F = 1 - (sin^2 $$\alpha$ + cos^2 $$\alpha$ );
% Input variable: $$\alpha$ = -0.5* $$\pi$ ...0.5* $$\pi$
%% *% Step 2. Define the input variable*
alpha = -pi/2:pi/100:pi/2;
```

```
%% *% Step 3. Perform the computation*
F = 1-(sin(alpha).^2+cos(alpha).^2);
%% *% Step 4. Define for which values of $$\alpha$ $$F(\alpha) =0$*
Findex=find(F==0); F0=F(Findex);
%% *% Step 5. Plot the simulation results: $$\alpha$ vs. $$F(\alpha)$*
plot(alpha, F, 'b-', alpha(Findex), F0, 'ro'),
legend('\alpha vs F(\alpha)', 'F(\alpha) = 0')
title('The Pythagorean Theorem '), grid on
xlabel('\alpha'), ylabel('F(\alpha) = 1-(sin^2(\alpha)+cos^2(\alpha))')
%% *%Step 6. Publish all of the obtained results including the whole script*
```

After writing the script in the editor, save the M-file with a valid filename (e.g., `P1.m`).

Now, to publish it as an HTML format, press the Publish Publish button under the PUBLISH tab (see Figure 1-17). After pressing the button, the script is executed automatically and its HTML-formatted report will be generated, as shown in Figure 1-19.


Web Browser - P1
P1
+
Location: file:///C:/Users/sulaymon.eshkabilov/Downloads/html/P1.html

Contents

- % Step 1. Problem statement
- % Step 2. Define the input variable
- % Step 3. Perform the computation
- % Step 4. Define for which values of α $F(\alpha) = 0$
- % Step 5. Plot the simulation results: α vs. $F(\alpha)$
- %Step 6. Publish all of the obtained results including the whole script

% Step 1. Problem statement

MATLAB's precision is not absolute. Pythagorean Theorem: F = 1 - (sin^2 α + cos^2 α); Input variable: α = -0.5* π ...0.5* π

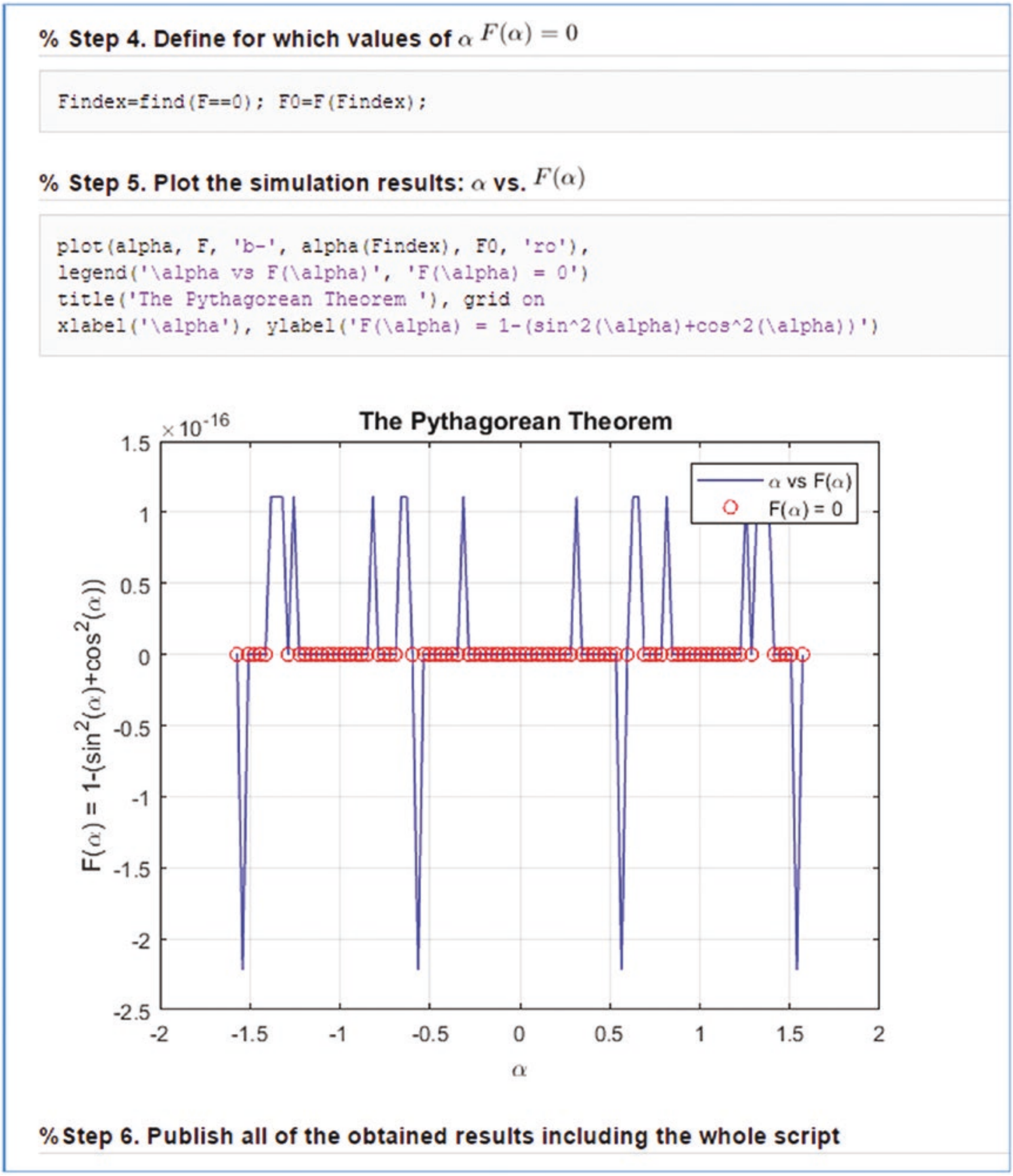
% Step 2. Define the input variable

```
alpha = -pi/2:pi/100:pi/2;
```

% Step 3. Perform the computation

```
F = 1-(sin(alpha).^2+cos(alpha).^2);
```

Figure 1-22. *The MATLAB-generated HTML-formatted report of the M-file called P1.m*



% Step 4. Define for which values of α $F(\alpha) = 0$

```
Findex=find(F==0); F0=F(Findex);
```

% Step 5. Plot the simulation results: α vs. $F(\alpha)$

```
plot(alpha, F, 'b-', alpha(Findex), F0, 'ro'),
legend('\alpha vs F(\alpha)', 'F(\alpha) = 0')
title('The Pythagorean Theorem '), grid on
xlabel('\alpha'), ylabel('F(\alpha) = 1-(sin^2(\alpha)+cos^2(\alpha))')
```

%Step 6. Publish all of the obtained results including the whole script

Figure 1-23. *The MATLAB-generated HTML-formatted report of the M-file called P1.m*

Note that the bold lines (Figure 1-19), which are the starting lines of cell modes preceded with %%, have been recognized by the M-file editor automatically and put in contents and hyperlinked, such as `% Step 1 ... Step 2 ... Step 6`.

Now let's try the same procedure with the MLX-file editor. Note that the comments are edited using the TEXT tab tools of the MLX-editor (shown in Figure 1-19). The parts of the script are entered as text (comments) by using Text. The Step 1 ... Step 2 ... Step 3... Step 6 lines are made bold with **B**.

`Step 1. Problem statement`

`MATLAB's precision is not absolute.`

`Pythagorean Theorem: F = 1 - (sin^2 (alpha) + cos^2(alpha));`

`Input variable: alpha = -0.5*pi ... 0.5*pi`

`Step 2. Define the input variable`

`Step 3. Perform the computation`

`Step 4. Define for which values of alpha, F(alpha) = 0.`

`Step 5. Plot the simulation results: alpha vs. F(alpha)`

`Step 6. Publish all of the obtained results including the whole script`

Note that there is an alternative way to make the chosen lines bold. That is to use %% followed by a blank space, as in the M-file editor. In this case, such editing is automatically detected as a header of the following section of the script. If you enter this in the Code section of MLX-editor:

```
%% Step 2. Define the input variable.
```

and press Enter, MATLAB automatically creates this bolded text header:

Step 2. Define the input variable.

Now, in between Step 2, Step 3, Step 5, and Step 6, the following executable commands are inserted by placing the cursor on the desired line and pressing the Code button from CODE sub-tab (see Figure 1-19). Finally, the complete code is obtained.

`Step 1. Problem statement`

`MATLAB's precision is not absolute.`

`Pythagorean Theorem: F = 1 - (sin^2 (alpha) + cos^2(alpha));`

`Input variable: alpha = -0.5*pi ... 0.5*pi`

Step 2. Define the input variable

```
alpha = -pi/2:pi/100:pi/2;
```

Step 3. Perform the computation

```
F = 1-(sin(alpha).^2+cos(alpha).^2);
```

Step 4. Define for which values of alpha, F(alpha) = 0.

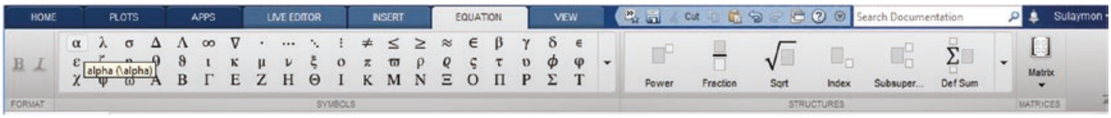
```
Findex=find(F==0);F0=F(Findex);
```

Step 5. Plot the simulation results: alpha vs. F(alpha)

```
plot(alpha, F, 'b-', alpha(Findex), F0, 'ro'),
legend('\alpha vs F(\alpha)', 'F(\alpha) = 0')
title('The Pythagorean Theorem'), grid on
xlabel('\alpha'), ylabel('F(\alpha) = 1-(sin(alpha)^2+cos(alpha)^2)')
```

Step 6. Publish all of the obtained results including the whole script

Save the file as an `*.mlx` file (e.g., `P2.mlx`). Insert the mathematical expressions with the Equation tools under the INSERT sub-tab (shown in Figure 1-20) and by using Σ Equation. When you press the Σ Equation button, the whole range of symbols, structures, and matrices will be opened as shown in the following image.



Now you bring the cursor where you want to insert the mathematical expressions and insert the symbols by selecting the necessary symbols, or by using LaTEX expressions. For example, `\alpha` gives α. Enter the expressions of the Pythagorean formulation and input the variable range, then finalize the script.

Step 1. Problem statement

MATLAB's precision is not absolute

The Pythagorean Theorem: $F = 1 - (\sin^2\alpha + \cos^2\alpha)$

Input variable: $\alpha = -0.5\alpha \ldots 0.5\alpha$

Step 2. Define the input variable

```
alpha = -pi/2:pi/100:pi/2;
```

Step 3. Perform the computation

```
F = 1-(sin(alpha).^2+cos(alpha).^2);
```

Step 4. Define for which values of α, $F(\alpha) = 0$

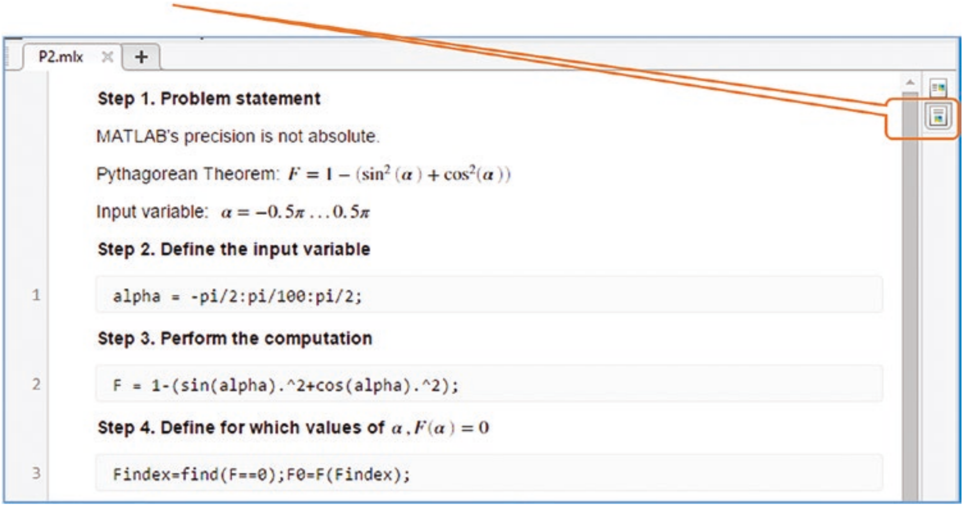
```
Findex=find(F==0);
F0=F(Findex);
```

Step 5. Plot the simulation results: α vs. $F(\alpha)$

```
plot(alpha, F, 'b-', alpha(Findex), F0, 'ro'),
legend('\alpha vs F(\alpha)', 'F(\alpha) = 0')
title('The Pythagorean Theorem '), grid on
xlabel('\alpha'), ylabel('F(\alpha) = 1-(sin^2(alpha) +cos^2(alpha))')
```

Step 6. Publish all of the obtained results including the whole script

After executing the saved MLX-file (by pressing the Run button shown in Figure 1-19), you'll get the script outputs via Output Inline in LAYOUT. See Figure 1-24 and Figure 1-25.



Step 1. Problem statement

MATLAB's precision is not absolute.

Pythagorean Theorem: $F = 1-(\sin^2(\alpha)+\cos^2(\alpha))$

Input variable: $\alpha = -0.5\pi \ldots 0.5\pi$

Step 2. Define the input variable

```
alpha = -pi/2:pi/100:pi/2;
```

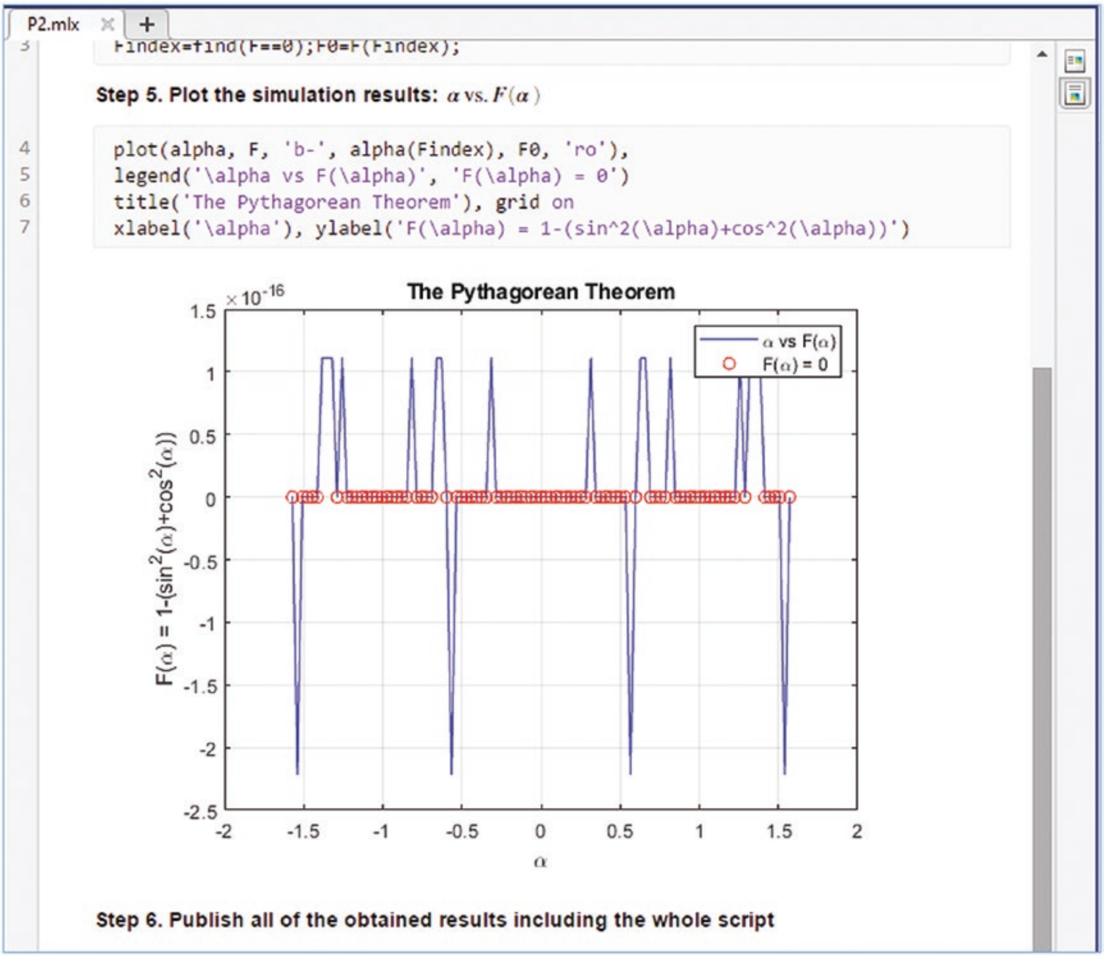
Step 3. Perform the computation

```
F = 1-(sin(alpha).^2+cos(alpha).^2);
```

Step 4. Define for which values of α , $F(\alpha) = 0$

```
Findex=find(F==0);F0=F(Findex);
```

Figure 1-24. *MLX editor output of P2.mlx*



```
Findex=find(F==0);F0=F(Findex);
```

Step 5. Plot the simulation results: α vs. $F(\alpha)$

```
plot(alpha, F, 'b-', alpha(Findex), F0, 'ro'),
legend('\alpha vs F(\alpha)', 'F(\alpha) = 0')
title('The Pythagorean Theorem'), grid on
xlabel('\alpha'), ylabel('F(\alpha) = 1-(sin^2(\alpha)+cos^2(\alpha))')
```

Step 6. Publish all of the obtained results including the whole script

Figure 1-25. MLX editor output of P2.mlx

Note that in the MLX editor, all of the executable and non-executable lines of the script are identified automatically and placed in separate sections. There are other salient issues about hints, warnings, and error messages from the M/MLX editors that we discuss in Chapter 2.

Closing the MATLAB Window

Quitting MATLAB is quite simple. There are several commands that can be used to complete the work in MATLAB and close down all its windows. You can type `>> quit` or `>> exit` or use the keyboard shortcut Ctrl+Q. You can also click the X in the upper-left corner of the main MATLAB window or call the `>> finishdlg` function from the Command Window.

Note that all variables residing in the workspace will be cleared upon exiting/quitting MATLAB. They will be lost and not be recovered by default the next time MATLAB is started. However, they can be saved to a `*.mat` file and loaded back to the workspace. The command history of entered commands is saved automatically and all of the entered commands in the Command Window can be accessed the next time you launch MATLAB. If you're interested in changing the number of commands that can be adjusted via MATLAB preferences, choose Preferences ➤ Command History.

Summary

This chapter introduced the MATLAB environment, including its settings, variables, common commands, and M-file/MLX-file GUI tools. You also learned about assigning variables and values from the Command Window and working in the M/MLX-file editor windows. In addition, the chapter explored data types, formats, and structures and covered how to use built-in MATLAB commands and functions. In particular, it covered help search options and most commonly used commands, including `help`, `helpwin`, `helpbrowser`, `doc`, `lookfor`, `clear`, `clear all`, `dir`, `pwd`, `cd`, `ls`, `save`, `load`, `clearvars`, `edit`, `format`, `char`, `size`, `who`, `whos`, `input`, `what`, and `exit`/`quit`.

References

[1] http://www.mathworks.com/matlabcentral/fileexchange/authors

[2] https://www.mathworks.com/matlabcentral/answers/

[3] http://ctms.engin.umich.edu/CTMS/index.php

[4] http://www.mit.edu/people/abbe/matlab/

[5] `https://www.mathworks.com/company/newsletters/articles/the-tetragamma-function-and-numerical-craftsmanship.html`

[6] `https://www.mathworks.com/help/matlab/creating_plots/greek-letters-and-special-characters-in-graph-text.html`

Exercises for Self-Testing

Exercise 1

Perform the following:

1. Find the *Vibrating Logo* demo from the pre-installed MATLAB demos and run it. Hint: `membrane`.
2. Locate *Product Overview* from the package's Help Library.
3. Change the font type and size of the Command Window.

Exercise 2

Do the following:

1. Use the Help Library to determine how to add a new path for search. Add a new path for search: `C:\Users\Public`. Hint: `addpath`.
2. Use the MATLAB help browser to find out how to create a new directory. Create a new directory called `my_new_dir` inside the `C:\Users\Public` directory. Hint: `mkdir`.

Exercise 3

Get help with the MATLAB `exp` (exponential) function using the Command Window. Use the `help`, `lookfor`, `doc`, and `help browser` commands and then compare the results of the four help options.

Exercise 4

Create and open an `*.m` file called `learn.m` in M-file editor window using the Command Window. (Hint: `>> edit ...`). Insert two commands in it that display the current date and time. Hint: `date`, `clock`.

Exercise 5

Create a shortcut (a set of favorite commands) that opens a new M-file named `My_Shortcut` and simultaneously closes all figure windows and clears the Command Window and Workspace from all previously entered data and commands.

Exercise 6

Change from the Preferences window a display of data formats using the Command Window. Make it hexadecimal format. Hint: `format`.

Exercise 7

Which commands are used to clean up the Workspace, Command Window, and History?

Exercise 8

Given $x = 2.25$, $y = 3.1$ and $z = 13.20$, use MATLAB to evaluate the following expressions:

$$A = \sqrt[y]{xyz - \frac{z}{xy}} + x^2 y^{\frac{3}{2}} z^{\frac{5}{4}};\ B = e^{-xyz} \ln\left(\frac{xy}{z}\right).$$

Use the Array Editor to change the assigned values for x, y and z and re-execute the expressions to compute A and B.

Exercise 9

Create a script and save it as `my_first_program.m`.

a) Your program should contain an input variable that is the length of a square side as a variable parameter in meters.

b) Your program should calculate the area of a square and the volume of a cube.

c) Your program should output the calculated results (area and volume) in the metric (m^2, m^3) and British (in^2, in^3) systems by using conversion, e.g., 1 *inch* = 25.4 *mm*.

d) Execute your script (`my_first_program.m`) from the Command Window.

e) Execute your script (`my_first_program.m`) from the M-file Editor Window.

Exercise 10

Do/answer the following:

- Which command displays what is stored in your MATLAB workspace?
- Create $P=\frac{\pi}{4}$ and save it in `uint64` format.
- Why is the value of `P` in `uint64` equal to 1? How do you fix this issue?

Exercise 11

Save all computation results from Exercises 9 and 10 in a MAT file called `my_FIRSTwork.mat` and clean up your MATLAB workspace, removing all variables except for `A` and `B` from Exercise 8.

Exercise 12

Given:

```
C1 = Hello. I'm from NYC. What is your name?
D 1= I'm Dan.
C2 = Nice to meet you
D 2= Nice to meet u 2
```

Using these entries (C1, C2, D1, and D2), create a conversation displayed in the Command Window in the form of this dialogue:

```
'Hello.' 'I'm from NYC.'
'What is your name?' 'I'm Dan.'
'Nice to meet you.' 'Nice to meet u 2.'
```

Exercise 13

Do the following:

1. Change display data formats for the Command Window and make it a *long e*.
2. Where are the preferences settings of MATLAB saved?
3. What is the main function of the M-file `finishdlg.m` and what commands does it contain?

Exercise 14

Do the following:

1. Write a command in the Command Window that creates and opens an M-file called `Ex14.m`.
2. Edit your M-file `Ex14.m` so that it contains a command that changes the display format type.
3. Edit your M-file `Ex14.m` so that it contains a command that changes the current directory.
4. Edit your M-file `Ex14.m` so that it contains a command that displays the current date and time.

Exercise 15

Given:

```
>> day_ONE='MatlabDAY'; day_DUE=day_ONE+1.0
day_DUE =
 78 98 117 109 98 99 69 66 90
```

Why is day_DUE numeric and equal to a 10-element row matrix?

What do these numbers represent?

Exercise 16

Given: `>> A=[1,2; -12.0, 3]; mat2str(A); SNA=ans+0; char(SNA)`

What is hidden behind the variable SNA?

Exercise 17

Create a structure composed of cell and numeric array elements called `Matlab` and `classes`:

1. Create a cell called `Matlab` that is composed of two sub-cells: `{'Day#1', 'Start'}`
2. Create a numeric array called `classes` that contains elements $[\pi;2\pi;\sqrt{\pi}+2i]$. Note that `i` represents an imaginary number.

Exercise 18

Create a function handle and inline function of the following mathematical expression:

$h(\theta, t) = 1.3 * e^{-ti\theta}$. Note that i represents an imaginary number.

Exercise 19

It is analytically proven that $cos2\alpha = 2\cos^2\alpha - 1$. Use MATLAB to compute the equality for different values $[0, \ldots \frac{\pi}{2}, \pi \ldots \frac{3\pi}{2}, 2\pi \ldots \frac{5\pi}{2}, 3\pi \ldots 5\pi]$ of α and define the values of α in which the accuracy of MATLAB calculations does not represent equality.

Exercise 20

Use MATLAB to compute the expression $\sqrt[5]{1.5 * 10^{24} - 10^{24} i}$ in a most accurate way. Note that it is in a fifth root.

Exercise 21

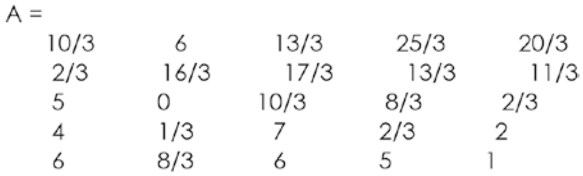
Why does the following output look “strange”?

```
>> A=[4/5, 'matlab'+0, sin(pi)]
A =
 Columns 1 through 3
 3fe999999999999a 405b400000000000 4058400000000000
 Columns 4 through 6
 405d000000000000 405b000000000000 4058400000000000
 Columns 7 through 8
 4058800000000000 3ca1a62633145c07
```

How do you fix this problem and make the results legible?

Exercise 22

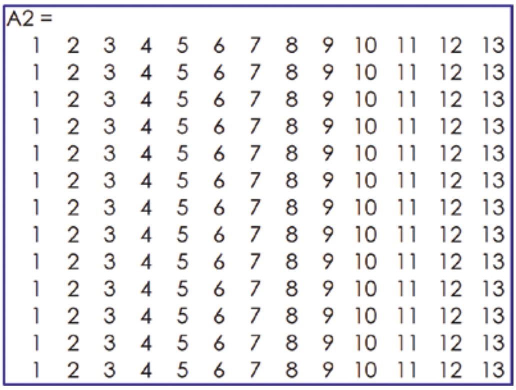
Create a 5-by-5 matrix called A by using `randi()` within [1, 20] divided by 3. Display A as rational numbers, as shown here. Note that your array numbers (in the numerator) of A will differ from the ones shown here. Why does your answer differ from the one shown here?



```
A =
    10/3      6      13/3     25/3     20/3
    2/3     16/3     17/3     13/3     11/3
    5        0      10/3      8/3      2/3
    4       1/3      7        2/3      2
    6       8/3      6        5        1
```

Exercise 23

Create the following array in the most efficient way (at least two different ways). Note the display format of the A2 elements.



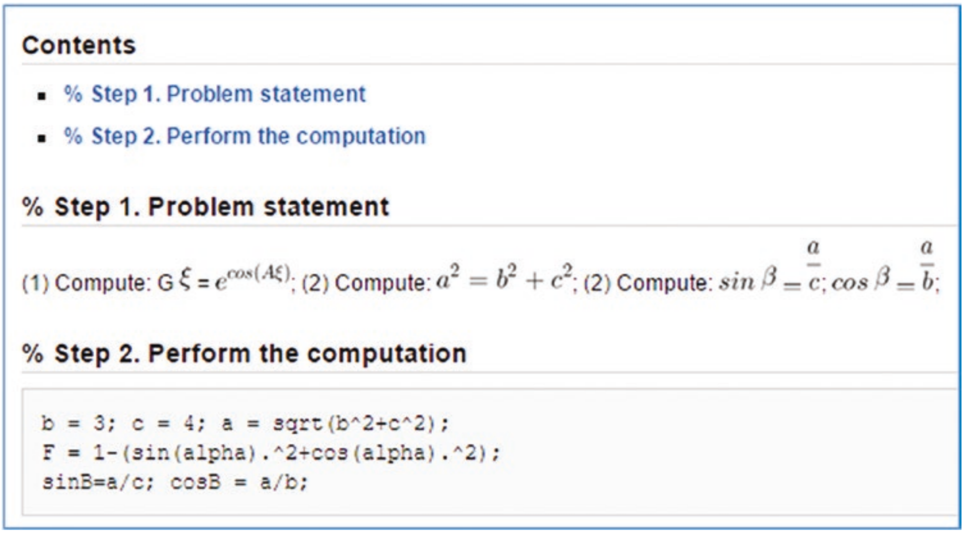
```
A2 =
    1    2    3    4    5    6    7    8    9   10   11   12   13
    1    2    3    4    5    6    7    8    9   10   11   12   13
    1    2    3    4    5    6    7    8    9   10   11   12   13
    1    2    3    4    5    6    7    8    9   10   11   12   13
    1    2    3    4    5    6    7    8    9   10   11   12   13
    1    2    3    4    5    6    7    8    9   10   11   12   13
    1    2    3    4    5    6    7    8    9   10   11   12   13
    1    2    3    4    5    6    7    8    9   10   11   12   13
    1    2    3    4    5    6    7    8    9   10   11   12   13
    1    2    3    4    5    6    7    8    9   10   11   12   13
    1    2    3    4    5    6    7    8    9   10   11   12   13
    1    2    3    4    5    6    7    8    9   10   11   12   13
    1    2    3    4    5    6    7    8    9   10   11   12   13
```

Exercise 24

Using `randi()`, create a 15 by 15 array with elements ranging from -125 to 127 and save it in the most memory-efficient way with the name `A3`.

Exercise 25

Create the following HTML report using M-file editor tools.



Contents

- % Step 1. Problem statement
- % Step 2. Perform the computation

% Step 1. Problem statement

(1) Compute: G $\xi = e^{cos(A\xi)}$; (2) Compute: $a^2 = b^2 + c^2$; (2) Compute: $sin\,\beta = \frac{a}{c}; cos\,\beta = \frac{a}{b};$

% Step 2. Perform the computation

```
b = 3; c = 4; a = sqrt(b^2+c^2);
F = 1-(sin(alpha).^2+cos(alpha).^2);
sinB=a/c; cosB = a/b;
```

Exercise 26

Given `A5 = [1 2 3; 4 5 6; 7, 8, 9]`.

1. Obtain `B5` from `A5` using two arithmetic operations: `B5 = [16, 9 4; 1 0 1; 4 9 16]`.
2. Obtain `C5` from `A5` and `B5` by using relational logic (`<, >`) and arithmetic operations (`+ 13`).

```
C5 = [13 13 13; 14 14 14; 14 13 13];
```

Exercise 27

Create three numerical (row matrix) arrays (variables called `AJ`, `IS`, and `LJ`) so that, when you subtract three from each and then perform one conversion operation, you obtain Al-Khwarizmi, Ibn Sina, and Lennart Johansson.

Exercise 28

Save all of your created variables (`A`, `A2`, `A3`, `A4`, `A5`, and `A6`) from Exercises 22-27 in a `*.mat` file with your last name, e.g. `Jones_HW2.mat`.

Exercise 29

Create a cell array (called `A`) containing three variables: `a=4/5`, `b='matlab'+0`, and `c=sin(`π`)`, and create a structure (called `B`) containing four variables: `a`, `b`, `c`, and `A`. Show how you access the `a`, `b`, and `c` variables residing inside `A` and `B`.

Exercise 30

Create the following variables and entries in the MLX File Editor:

Function handle F: $F(\omega_1, \omega_2, \theta) = \cos(\omega_1\theta) - \sin(\omega_2\theta)$

Identity matrix: $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Magic numbers: $M = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$

Multiply *the* matrix by 2 and subtract it from the M matrix. Call the new matrix MI:

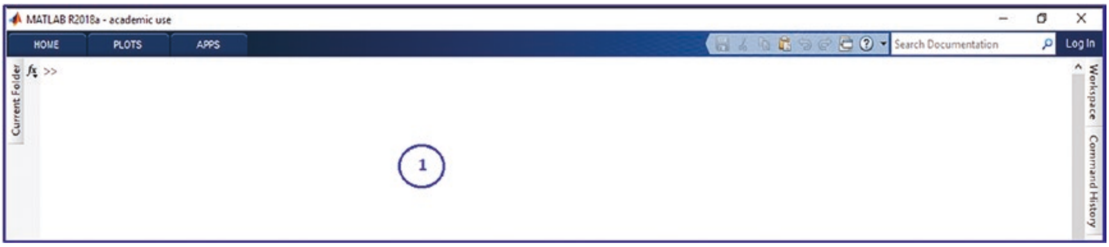
$$MI = \begin{bmatrix} 6 & 1 & 6 \\ 3 & 3 & 7 \\ 4 & 9 & 0 \end{bmatrix}$$

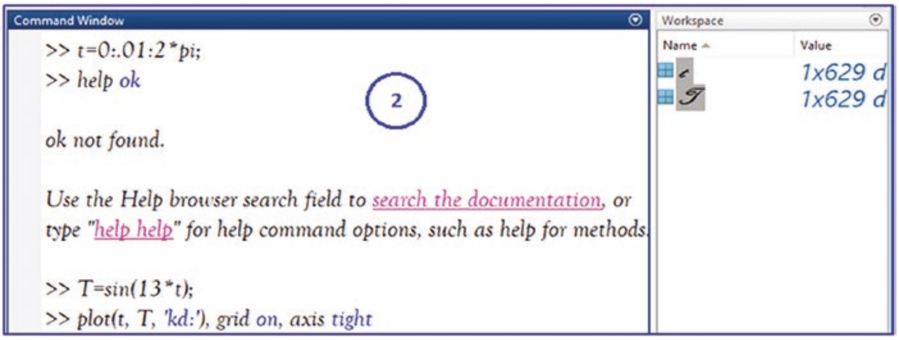
Locate array L by locating/comparing the elements of MI that are greater than 1 and less than 6: $L = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

```
Table array TM from M:
Cell Array CA containing F, I, M, MI, L, TM
Structure SA containing F, I, M, MI, L, TM, CA.
```

Exercise 31

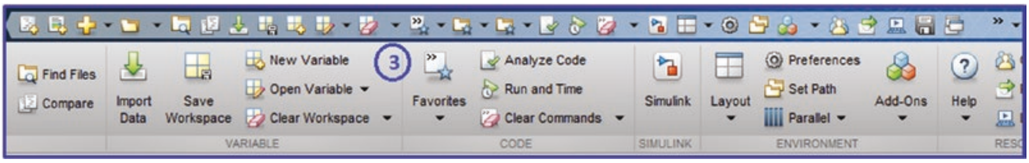
Explain how to obtain the following Layout (1), Preferences (2), and Quick Access (3) windows, as shown in the screenshots. Note: Fonts and Colors in (2).





```
>> t=0:.01:2*pi;
>> help ok

ok not found.

Use the Help browser search field to search the documentation, or
type "help help" for help command options, such as help for methods.

>> T=sin(13*t);
>> plot(t, T, 'kd:'), grid on, axis tight
```

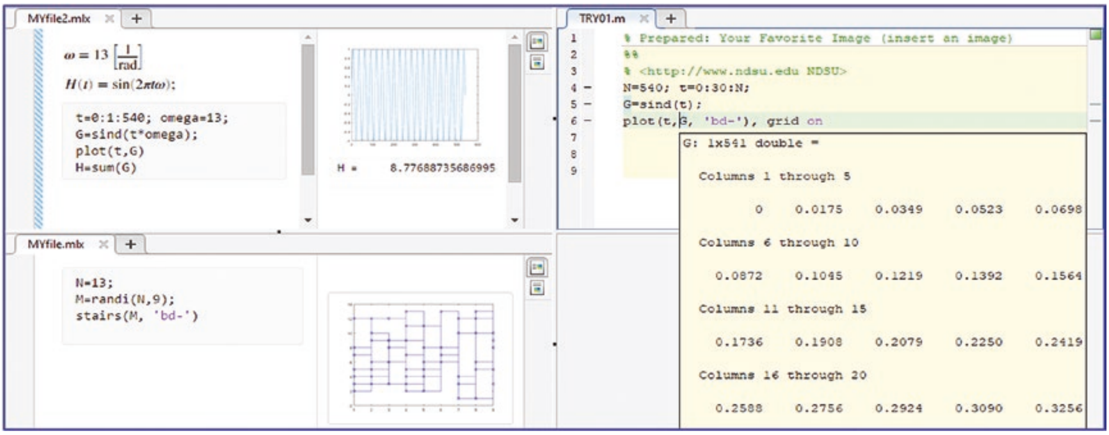
Name	Value
t	1x629 d
T	1x629 d



Exercise 32

Create these files (MLX/M-files) and explain how to display the results as shown in the figure in the Live Editor window.

Note that there are four windows displaying MLX and M-files; Equations, Greek letters, Plot figures, hyperlinks, data tips, and how to insert an image.



$\omega = 13 \left[\frac{1}{\text{rad}}\right]$

$H(t) = \sin(2\pi t\omega)$;

```
t=0:1:540; omega=13;
G=sind(t*omega);
plot(t,G)
H=sum(G)
```

```
H =     8.77688735686995
```

```
N=13;
M=randi(N,9);
stairs(M, 'bd-')
```

```
% Prepared: Your Favorite Image (insert an image)
%%
% <http://www.ndsu.edu NDSU>
N=540; t=0:30:N;
G=sind(t);
plot(t,G, 'bd-'), grid on
```

```
G: 1x541 double =

  Columns 1 through 5

         0    0.0175    0.0349    0.0523    0.0698

  Columns 6 through 10

    0.0872    0.1045    0.1219    0.1392    0.1564

  Columns 11 through 15

    0.1736    0.1908    0.2079    0.2250    0.2419

  Columns 16 through 20

    0.2588    0.2756    0.2924    0.3090    0.3256
```

Exercise 33

Answer the following questions:

1. MATLAB's default numerical format (int8, uint8, int16, uint16, ... single, or double) depends on the operating system on which MATLAB is installed. (True/False)

2. Where does MATLAB display a list of stored variables and associated attributes?

3. The MATLAB user interface is customizable. (True/False)

4. MATLAB supports cell arrays only if they are numerical data, not strings. (True/False)

5. It is possible to put a table array variable into a cell array. (True/False)

6. `A=1.1; B=[2, 3; 1 2]; C=B/A; format hex; C`

   We are changing the values of entries `C` by changing the display format. (True/False)

7. `>> D=uint8(255); D=255; D+1 =255; E = [12/14, 3/5; 1/3, 4/9]; format rat; E`

   We have changed the values of entries `D` and `E` by changing the display format. (True/False)

Exercise 34

Answer the following array size and representation related questions:

1. Given a cell array `ABBA` with 10 cells, which command will recall the elements residing in cell 3 of `ABBA`?

2. Given a 5-by-5 array (matrix) called `A, A(4:end, 3:4)`, it will produce a matrix of what size?

3. How do you create a linearly spaced data array: `a=(-13, -12, -11, ... 11, 12, 13)` and `b=(0, 1/13, 2/13, 3/13, ... 24/13, 25/13, 2)` without typing all the elements? Note how to obtain the rational format type of the array `b`.

Exercise 35

How do you get a logical array `c = (1 1 1 1 1 0 0 0 0 0 0)` from the array `a = (-5, -4, -3 ... 3, 4, 5)` whose elements are linear (equally spaced)?

Exercise 36

Given in the Command Window:

```
>> x=linspace(-13, 0), F(x)=2*x^2+2*x-1;
```

There are errors. Find errors and fix them. What is the size of `x` and `F` now?

Exercise 37

Write the answers to the following questions:

1. How do you change a working directory to: `C:\Users\Public` and add this path for searching?
2. Create a new directory called `MYdir` inside the `C:\Users\Public` directory.
3. How do you find out which variables are stored in your MATLAB workspace?

Exercise 38

Answer the following array related questions on equally spaced data points:

1. Change the last two columns (column 8 and 9) of `D2` (`D2=zeros(9); D2(6,:)=1:9;`) given above to have the elements: [e^0, e^1, e^2, …, e^8] *and* [tan(e^0), tan(e^1), tan(e^2), …, tan(e^8)], respectively in the most efficient way.
2. Generate these vector spaces in two different ways`: [-100, -90, -80, ... 100], [-100, -99, -98, ... 100]`.
3. How do you generate 500 data points equally spaced within $-\pi..\pi$]?

Exercise 39

Answer the following logical indexing and logical array related questions:

1. Given >> `A=magic(3); C1=le(A,7), C2=A<=7`

   What do `0` and `1` mean in all logical arrays for each individual case?

2. Given >> `A1=12.12; C='nan'; B=[1 2; 0, 3i]; D=B/0; AA1=isfinite(A1), CC1=isnan©, DD1=isinf(D), DD2=isnan(D)`

   What do `0` and `1` mean in all logical arrays for each individual case?

3. Given `>> AG=randi([-13, 25], 3, 2); BAG=(AG>0 & AG<13)`

   Why do your answers differ when you run these commands to define AG and BAG?

4. Given `>> GAB=find(AG>0 & AG<13), AG(GAB)`

   What numbers are behind GAB and how are they related to AG?

5. Given `>> 13>10; -1.2<=7.8; -11+13>=3`

   Why are we getting 0s and 1s?

6. Given `>> AA=randi([-13, 13], 10, 2); AA(AA<=0); % OR >> IN=(AA<=0); AA(find(IN));`

   What does IN represent with reference to [AA]?

7. Given `>>B = randi([0, 13], 5); k=find(B>=3 & B<=5)`

   Which numbers are in k with reference to B?

Exercise 40

Create an `* .m` file called `ARRAY_1.m` and write the following:

1. Write the command to clean up the workspace and Command Window of MATLAB, and display the current date and time in the Command Window.

2. Create array A and write commands to generate arrays: A1 (1-by-10) with the operator `:` A2 (10-by-1) with `linspace()`, A3 (2-by-10) with `eye()`.

3. Create array B and write commands to create arrays: B1 (5-by-6) with `randi()` elements ranging between [-1....1], B2 (5-by-6) with `rand()`, and B3 (5-by-10) with `randn()`.

4. Create array C and write the commands to generate arrays: C1 (5-by-10) with `magic()` and `repmat()`, C2 (6-by-10) with `eye()`, and C3 (10-by-10) with `ones()`.

5. Write the commands to perform all possible (arithmetic array) operations (+, -, , /, ., ./, ^, and .^) with `A1`, `A2`, and `A3` (at least three operations) and call new matrices: `A1new1`, `A1new2`, `A1new3`, `A2new1`, `A2new2`, `A2new3`, `A3new1`, `A3new2`, and `A3new3`. Hints: use `transpose()` and `rot90()` while performing arithmetic array operations.

6. Write the commands that perform all possible (arithmetic array) operations (+, -, , /, ^, ., ./, , sum, and mean) with `B1`, `B2`, and `B3` (at least three operations) and call new matrices: `B1new1`, `B1new2`, `B1new3`, `B2new1`, `B2new2`, `B2new3`, `B3new1`, `B3new2`, and `B3new3`. Hint: Use `fliplr()` while performing arithmetic array operations.

7. Create `AB1`, `AB2`, and `AB3` matrices from `A1`, `A2`, `A3`, and `B1`, `B2`, and `B3`. Also, use part of any the `A1`, `A2`, `A3` and `B1`, `B2`, `B3` arrays. Note that every `AB1`, `AB2`, `AB3`, `ABC4`, and `ABC5` array should contain some elements from arrays `A` and `B`. Hint: Use `flipud()` and `repmat()` while creating the arrays `AB1`, `AB2`, and `AB3`.

8. Create `ABC1`, `ABC2`, and ABC3 matrices by combining/concatenating the previously created arrays: `A1, A2, A3` and `B1, B2, B3` and `C1, C2, C3`. You should also use part of any `A1, A2, A3` and `B1, B2, B3` and `C1, C2, C3` arrays. Note that every `ABC1`, `ABC2`, `ABC3` array should contain some elements from `A, B`, and `C` matrices from Parts 1, 2, and 3.

Exercise 41

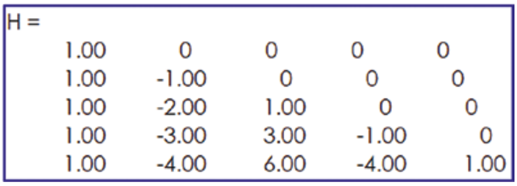
Create an `*.mlx` file called `Ex41.mlx` and write in it the following:

1. The command that creates a cell array called `Q1` with six empty column cells.

2. The command that gives binary representations of `12321` and `987654321`, and a command writing these numbers, including their binary representations in cells 1, 2, 3, and 4 of Q1, respectively.

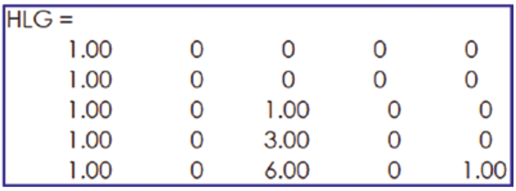
3. The command that converts the binary representations of `12321` and `987654321` into numerical arrays by using MATLAB's conversion commands—`str2num( )` or `str2double( )`—and the command that writes the two converted numerical arrays in cells 5 and 6 of Q1, respectively.

4. The command that generates the following array using `pascal()` (a MATLAB built-in matrix function):



```
H =
      1.00       0        0        0        0
      1.00   -1.00        0        0        0
      1.00   -2.00     1.00        0        0
      1.00   -3.00     3.00    -1.00        0
      1.00   -4.00     6.00    -4.00     1.00
```

And by applying logical indexing (logical array) and element-wise matrix multiplication operations, generate the following array:



```
HLG =
      1.00      0       0       0       0
      1.00      0       0       0       0
      1.00      0    1.00       0       0
      1.00      0    3.00       0       0
      1.00      0    6.00       0    1.00
```

5. The command that creates a structure array called `S5` and the command that writes its Q1, H, and HLG.

6. Explain why the converted numbers (in Step 3) of the binary representations (of `12321` and `987654321`) differ from the original decimal numbers, i.e. `12321` and `987654321`?

Exercise 42

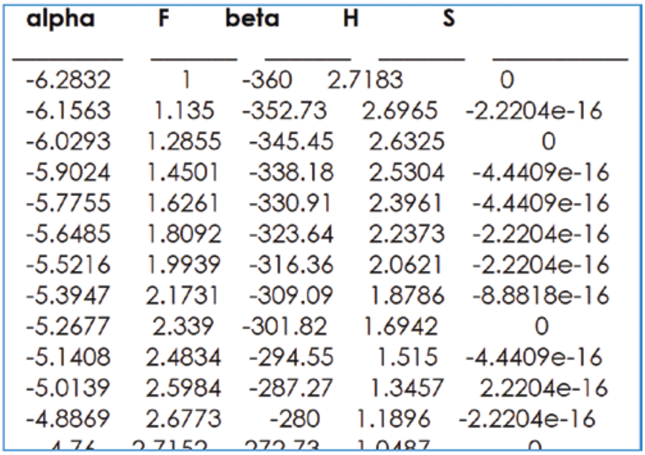
Create an `*.mlx` file (called `Ex42.mlx`) and add all necessary comments, such as questions, equations, explanations, and other relevant remarks. Also write in these commands:

1. Generate two column arrays with 202 equally spaced data points in two ways: $= -2\pi \ldots 2\pi$; $\beta = -360^0 \ldots 360^0$.

2. Compute these three equations (take the values of α, β from Step 1): $F(\alpha) = e^{\sin(\alpha)}$; $H(\beta) = e^{\cos(\beta)}$; $S = 1 - (sin^2\alpha + cos^2\,\beta)$. Note: β is given in degrees not in radians and thus, do not forget to convert it into radians. Also, insert the equations by using the equation editor of `*.mlx`.

3. Create an array (called `Solution`) of five columns containing: α, $F(\alpha)$, β, $H(\beta)$, S

4. Create a table of arrays. The table of variables should be called `TVall` and has to be in the following format.



alpha	F	beta	H	S
-6.2832	1	-360	2.7183	0
-6.1563	1.135	-352.73	2.6965	-2.2204e-16
-6.0293	1.2855	-345.45	2.6325	0
-5.9024	1.4501	-338.18	2.5304	-4.4409e-16
-5.7755	1.6261	-330.91	2.3961	-4.4409e-16
-5.6485	1.8092	-323.64	2.2373	-2.2204e-16
-5.5216	1.9939	-316.36	2.0621	-2.2204e-16
-5.3947	2.1731	-309.09	1.8786	-8.8818e-16
-5.2677	2.339	-301.82	1.6942	0
-5.1408	2.4834	-294.55	1.515	-4.4409e-16
-5.0139	2.5984	-287.27	1.3457	2.2204e-16
-4.8869	2.6773	-280	1.1896	-2.2204e-16
-4.76	2.7152	-272.73	1.0487	0

5. Find all of the positive values of `F`, `S`, and `H`, and the corresponding α, β values and save all of them in a cell array variable called `FSH_pos`.

6. Find all the absolute zero values of `S` and the corresponding α, β values. Save them in an array called `ABS_0` with three columns of the found `S` α, β values.

7. Create a structure of arrays called `ABFSH_struct` containing: `SOLUTION`, `TVall`, `FSH_pos`, and `ABS_0` from Steps 3, 4, 5, and 6.

8. Clear all the variables in the workspace except for α, β, `F`, `H`, `S`, `SOLUTION`, `TVall`, `FSH_pos`, and `ABFSH_struct` from the previous steps. Save these variables in an `*.mat` file called `Ex42.mat`.

Exercise 43

Create an `*.m` file (called `Ex43.m`) that performs the following operations:

1. Clears the workspace and Command Window from all entries.
2. Closes all open figure windows.
3. Creates a new directory called `C:\Documents\Ex43`.
4. Changes the current directory of MATLAB to a newly created directory: `C:\Documents\Ex43`.
5. Computes $t = [0, 3\pi]$ with $\Delta t = \pi/200$, $f_1(t) = 3\sin\left(\frac{3t}{2}\right)$, $f_2(t) = 2\cos\left(\frac{2t}{5}\right)$
6. Saves the computation results $(t\, f_1(t), f_2(t))$ in a `.mat` file called `Ex43.mat`.

Exercise 44

Create an `*.m` file (called `Ex44.m`) that performs the following operations:

1. Changes the current directory of MATLAB to the MATLAB's root directory (Hint: `matlabroot`).
2. Removes the directory created in Exercise 43: `C:\Documents\Ex43`.
3. Displays the MATLAB's root directory in the Command Window.
4. Creates two function handles: $\mathbf{r} = \frac{1}{2}\mathbf{a}t^2 + v_{\boldsymbol{o}}t + \mathbf{r}_{\boldsymbol{o}}$ and $A = P\left(1 + \frac{r}{n}\right)^{nt}$.
5. Saves the current path and the function handles in a structure array called `EX44`.

CHAPTER 2

Programming Essentials

This chapter covers the most essential and widely used programming tools, operators, and control statements in MATLAB, modeling essentials in Simulink, graphical user interface development and its tools, and the development of MATLAB executable files and standalone applications.

The chapter also contains a number of plain examples that demonstrate efficient ways and tools of programming and modeling in MATLAB/Simulink to save computation time. It explains how to create short and compact code/scripts, and it includes a few essential hints and approaches in writing robust programs and scripts.

Throughout the book, the key terms—script, code, program, M-file, MLX-file, and function file—are used frequently to refer to the programs written and executed in MATLAB. The terms used in the context of the book—code, script, and program, such as M/MLX-files and function files—are meant to be source code that's legible to users, not the machine code understood by a computer.

Writing M/MLX-Files

The previous chapter discussed arrays, characters, cells, and structure, table, logic arrays via examples. It included a general overview of M/MLX-file editors. Scripts (M/MLX-files) can also be written in Notepad or WordPad editors. They become M-files as soon as they are named with file extension of `*.m` or `*.mlx` extension. They can also be opened and edited using the Notepad editor.

As discussed in the previous chapter, the M/MLX-file editors have many easy-to-use tools and intuitive options that you can employ while writing scripts and debugging them. From the M/MLX-file editor, you can use cell modes, set up breakpoints, evaluate selected lines of cells or scripts, automatically detect errors such as misspelled command names and missing brackets in algebraic operations, detect all used and unused variables, improve script performance, display warning hints about unnecessarily assigned variables, set up variable values within loops, use the profile viewer, and so forth.

S. Eshkabilov, *Beginning MATLAB and Simulink*, https://doi.org/10.1007/978-1-4842-5061-7_2

All of these tools are helpful in avoiding syntax errors while writing scripts. In addition, there are many other tools that help you save time and effort when writing and publishing reports in HTML (hypertext) or PDF format, for example.

Before starting to write a script, let's look at some essential preparation steps. These are important to know and consider before writing actual code in any programming language. The process of writing a script/program starts with pen and paper. It's composed of the steps shown in the flowchart in Figure 2-1.

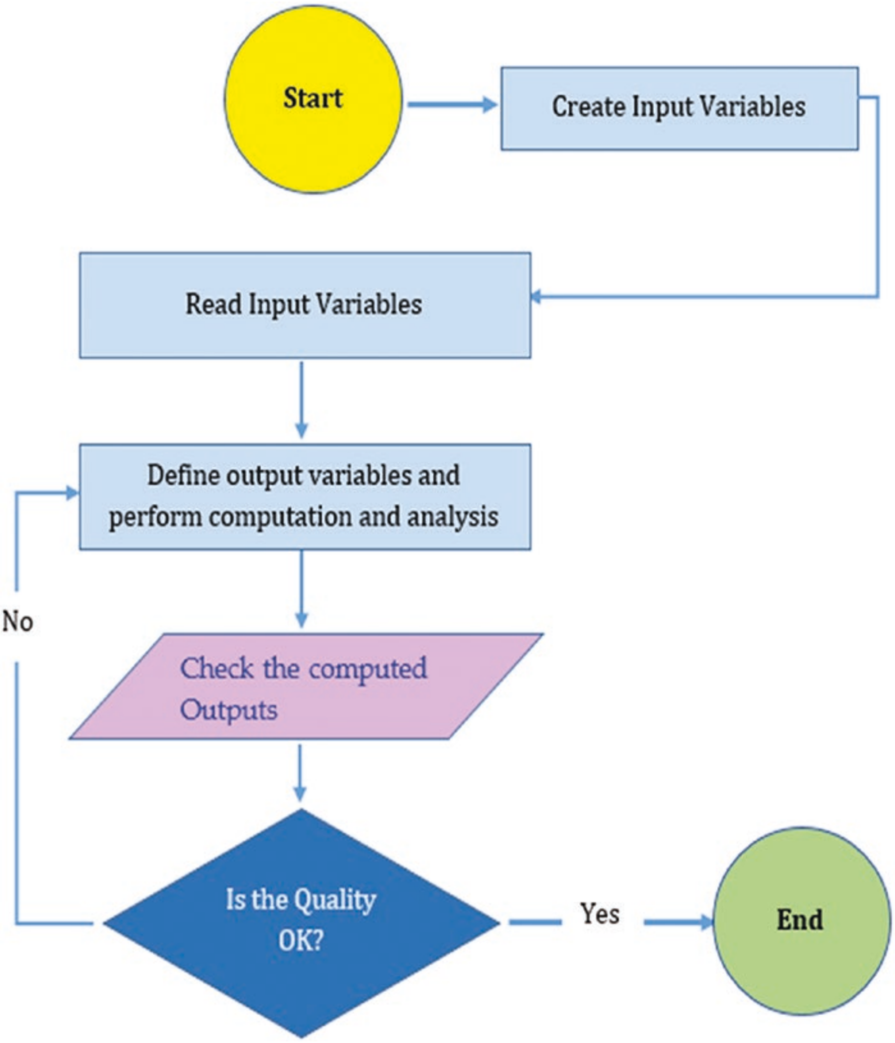

Start
Create Input Variables
Read Input Variables
Define output variables and perform computation and analysis
Check the computed Outputs
Is the Quality OK?
No
Yes
End

Figure 2-1. *General process of script/program writing*

This flowchart illustrates these steps:

1) Clarify the problem statement.
2) Clarify/create/declare the input variables, such as `var1`, `var2`, `var3` ... `varN`.
3) Read the values `var1`, `var2`, `var3`, ... `varN`.
4) Define the output variables, such as `out1`, `out2`, `out3`, ... `outN`.
5) Perform the computation, evaluation, and analysis operations.
6) Check the quality/correctness of the obtained results/output variables.
7) If the quality/correctness of the achieved results is not adequate, go back to Step 3. Repeat Step 3-5 until the expected quality/correctness is attained.
8) End and report on the results.

The described algorithm is a very generalized process and might also include considering objectives, input/output data types, sources, evaluation/computation operations, etc. Once the algorithm is well defined, you begin coding in the given programming language. When writing a program/script, the most time-consuming part is carrying out verification operations in Steps 3-5. This is called *debugging* and it helps you locate errors or flaws in the calculation/evaluation and analysis operations from Step 4.

Debugging is the process of correcting the syntax of the program with respect to the programming language, correcting the calculation/computation operations with respect to the given problem statement and, if required, adjusting the precision of the output. Debugging is not so straightforward in many instances and MATLAB's M/MLX-file editors have a few helpful tools that detect general errors made in your scripts. At the same time, many errors cannot be detected without executing and analyzing the obtained output. There is no single approach to finding all possible bugs (flaws and mistakes) made when writing code. One of the most common ways to see if the code is performing as anticipated is to use test examples with known output and then verify the output.

Throughout this book, considerable attention is given to debugging. It must be noted that it is impossible to write code without any flaws. Therefore, debugging is a "must have" step. Moreover, you'll hone and build your programming skills when you write code that solves different problems and make errors that you can then find and fix.

Note Always start writing your scripts with simple and plain operations/commands that perform the most essential parts of the exercise. Once the essential parts of the exercise are complete, you can add more detail. It's good programming to move from the simple to the complex.

How to Create an M/MLX-File

There are a few ways to create a new script (in this case, an M/MLX-file).

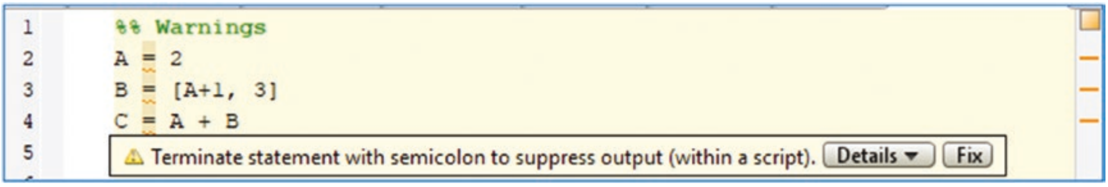
- By typing `>> edit` in the Command Window and pressing the Enter key from the keyboard.
- By pressing Ctrl+N keys from the keyboard.
- By clicking on New Script for an M-file.
- By clicking on New Live Script for an MLX-file.
- By clicking on New and selecting from the drop-down options Script Ctrl+N for M-file, Live Script for MLX-file, Function for a function file with an `*.m` file extension, or Live Function for a function file with an `*.mlx` file extension.
- By collecting the commands typed in the Command Window via these commands: `diary on`, `diary NewFileName`, `diary off`.

Warnings in Scripts

While writing M/MLX-files, including the function files M-file and MLX(Live)-file, editors automatically generate warning signs that are in many instances helpful hints to improve efficiency and locate missing or overlooked arguments. They do not prevent scripts from being executed. These warnings are indicated by the underlined wavy lines and hyphens ▬ and by a rectangular box ▢ on the right side of the M-file Editor window. They are by default orange in color. Similarly, the MLX (Live)-file editor indicates warnings by underlined wavy lines, as well as hyphens ▬ and a triangular warning sign ⚠ on the right side bar of the editor window.

Note that their color can be adjusted via the preferences settings. There are a few common warnings that are detected automatically by the M/MLX-file editors. They are as follows:

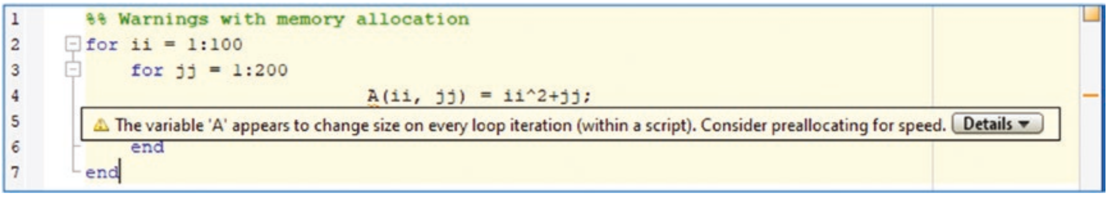
- To suppress the display of output in the Command Window detected by M-file editor only:



```
%% Warnings
A = 2
B = [A+1, 3]
C = A + B
```

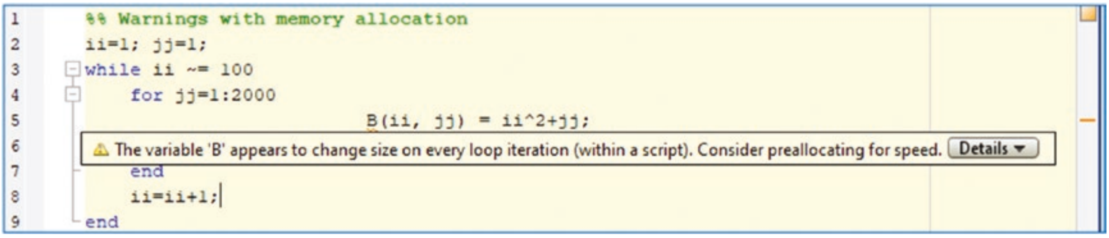
⚠ Terminate statement with semicolon to suppress output (within a script). Details ▾ Fix

- To suggest memory allocation (e.g., the variables A and B are underlined with an orange colored wavy line) when a variable size changes/increases in the loop iteration within `for ... end` and `while ... end`).



```
%% Warnings with memory allocation
for ii = 1:100
    for jj = 1:200
                    A(ii, jj) = ii^2+jj;
    end
end
```

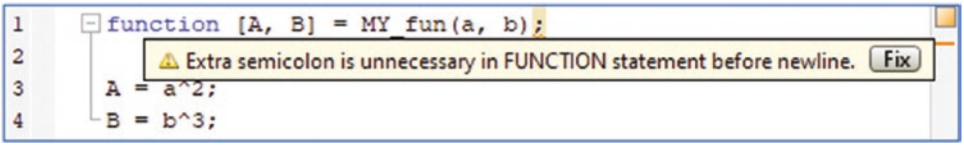
⚠ The variable 'A' appears to change size on every loop iteration (within a script). Consider preallocating for speed. Details ▾



```
%% Warnings with memory allocation
ii=1; jj=1;
while ii ~= 100
    for jj=1:2000
                    B(ii, jj) = ii^2+jj;
    end
    ii=ii+1;
end
```

⚠ The variable 'B' appears to change size on every loop iteration (within a script). Consider preallocating for speed. Details ▾

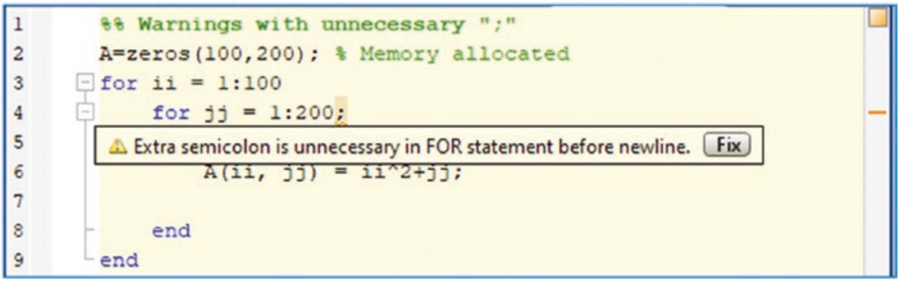
- To remove unnecessary semicolons. For example, at the end of line 1, the ; is unnecessary:



```
1  function [A, B] = MY_fun(a, b);
2
3  A = a^2;
4  B = b^3;
```

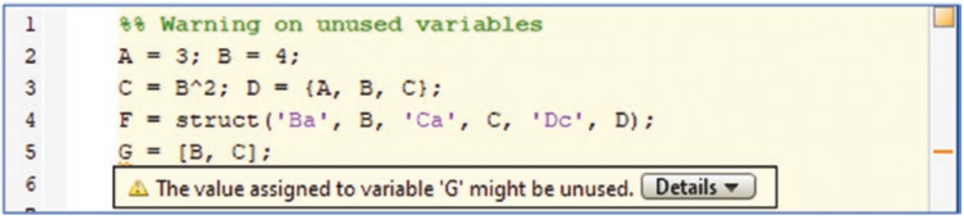
⚠ Extra semicolon is unnecessary in FUNCTION statement before newline. Fix

- To remove unnecessary semicolons in the for ... end loop's index declaration (on line 4):



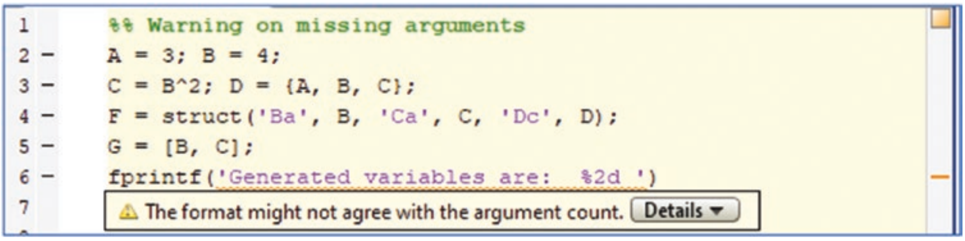
```
1      %% Warnings with unnecessary ";"
2      A=zeros(100,200); % Memory allocated
3      for ii = 1:100
4          for jj = 1:200;
5       ⚠ Extra semicolon is unnecessary in FOR statement before newline.  Fix
6              A(ii, jj) = ii^2+jj;
7
8          end
9      end
```

- To indicate unused but assigned variable names (G on line 5) within M-files:



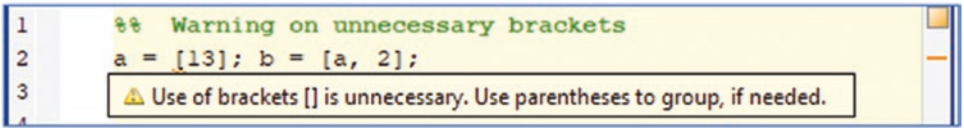
```
1      %% Warning on unused variables
2      A = 3; B = 4;
3      C = B^2; D = {A, B, C};
4      F = struct('Ba', B, 'Ca', C, 'Dc', D);
5      G = [B, C];
6      ⚠ The value assigned to variable 'G' might be unused.  Details ▾
```

- To indicate missing arguments when formatting fprintf( ):



```
1        %% Warning on missing arguments
2  -     A = 3; B = 4;
3  -     C = B^2; D = {A, B, C};
4  -     F = struct('Ba', B, 'Ca', C, 'Dc', D);
5  -     G = [B, C];
6  -     fprintf('Generated variables are:  %2d ')
7       ⚠ The format might not agree with the argument count.  Details ▾
```

- To indicate unnecessary brackets (a = [13] on line 2):

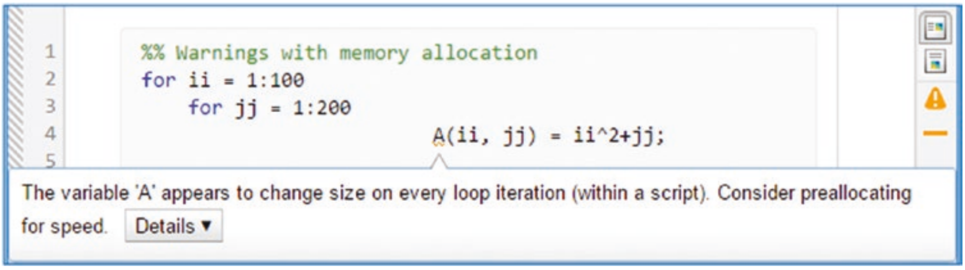


```
1      %%  Warning on unnecessary brackets
2      a = [13]; b = [a, 2];
3      ⚠ Use of brackets [] is unnecessary. Use parentheses to group, if needed.
```

It must be noted that some of the warning signs detected by the M Editor will not be picked up by the MLX Editor (Live Editor). For example, the missed ; used to suppress the display of the output in the Command Window is not applicable to the MLX Editor (Live Editor).

On the other hand, other warnings, such as memory allocation related warnings, unnecessary semicolons, missing arguments when formatting `fprintf()`, and unnecessary brackets are detected and highlighted very explicitly with the MLX Editor, which uses the ⚠ on the right side of the editor window.

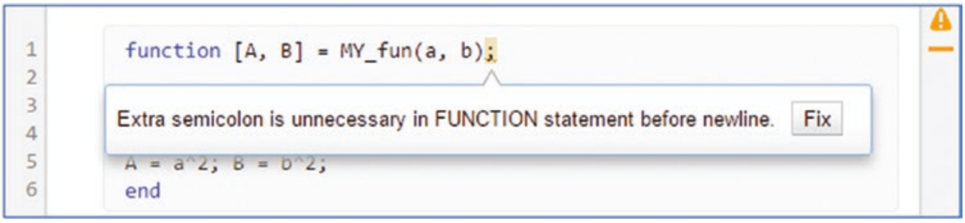
- To advise the memory allocation:



```
%% Warnings with memory allocation
for ii = 1:100
    for jj = 1:200
                A(ii, jj) = ii^2+jj;
```

The variable 'A' appears to change size on every loop iteration (within a script). Consider preallocating for speed.

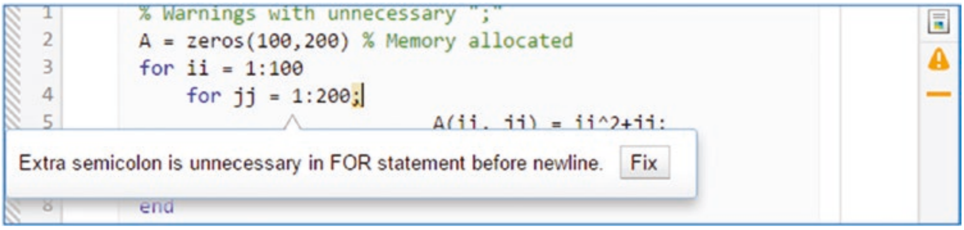
- To indicate unnecessary semicolons in a function statement:



```
function [A, B] = MY_fun(a, b);

A = a^2; B = b^2;
end
```

Extra semicolon is unnecessary in FUNCTION statement before newline.

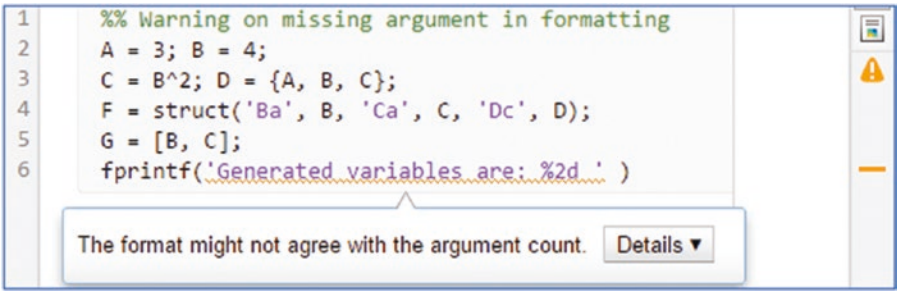
- To indicate unnecessary semicolons when defining indexes for a `for ... end` loop:



```
% Warnings with unnecessary ";"
A = zeros(100,200) % Memory allocated
for ii = 1:100
    for jj = 1:200;
                A(ii, jj) = ii^2+jj;

end
```

Extra semicolon is unnecessary in FOR statement before newline.

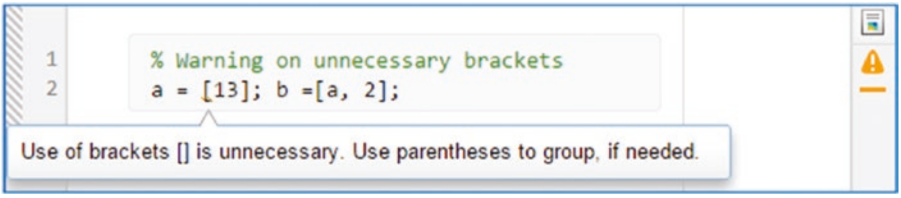
- To indicate a missing argument in `fprintf()`:



```
%% Warning on missing argument in formatting
A = 3; B = 4;
C = B^2; D = {A, B, C};
F = struct('Ba', B, 'Ca', C, 'Dc', D);
G = [B, C];
fprintf('Generated variables are: %2d ' )
```

The format might not agree with the argument count.

- To indicate unnecessary brackets:



```
% Warning on unnecessary brackets
a = [13]; b =[a, 2];
```

Use of brackets [] is unnecessary. Use parentheses to group, if needed.

Errors in Scripts

In MATLAB, scripts can contain code that performs various computations and analyses and defines functions. Let’s look at a few simple examples of writing scripts in the M-file and MLX-file editors and see how to locate and fix common errors that occur while writing scripts.

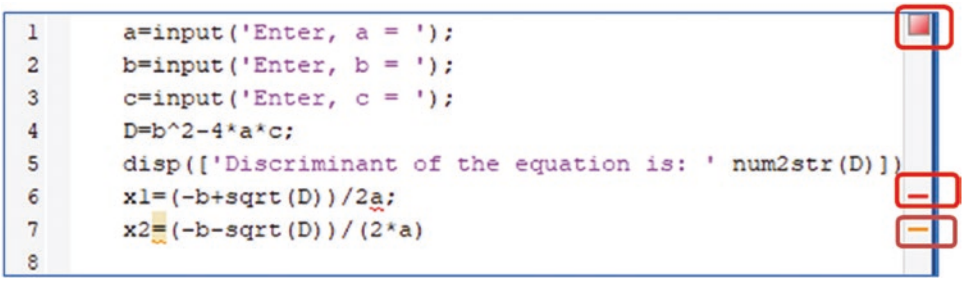
Example 1

Solve the quadratic equation represented in this general form: $ax^2 + bx + c = 0$. First, open an M-file editor and type in the following commands.

```
a=input('Enter, a = ');
b=input('Enter, b = ');
c=input('Enter, c = ');
D=b^2-4*a*c;
disp(['Discriminant of the equation is: ' num2str(D)])
```

Once the file is saved with the file extension *.m, it can be executed. When it is executed, the code prompts three input (**input()**) entries from the user and computes the discriminant of the quadratic equation with the user entries for *a, b, and c* and displays it in the Command Window. It should be noted that the **disp()** command on line 5 is optional. It's used to display the computation result in the Command Window with some comments. The **disp()** command does not make any changes to the output.

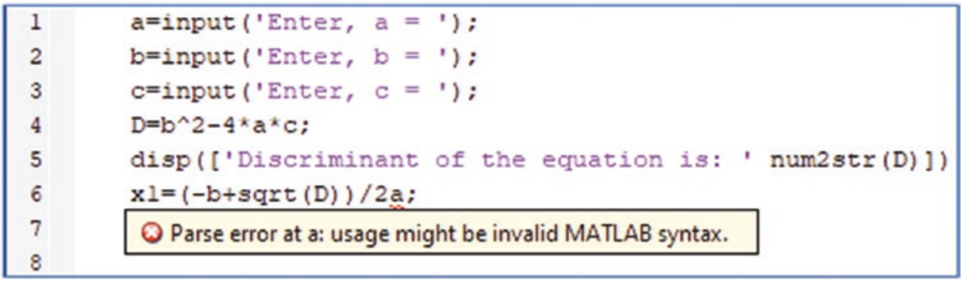
There are two more steps left in this code—computing the two roots of the quadratic equation. The remaining steps can be inserted after line 5. If illegal operations/errors occur while writing the script, the M/MLX-file editor will automatically detect them and underline them with red waves:



```
1    a=input('Enter, a = ');
2    b=input('Enter, b = ');
3    c=input('Enter, c = ');
4    D=b^2-4*a*c;
5    disp(['Discriminant of the equation is: ' num2str(D)])
6    x1=(-b+sqrt(D))/2a;
7    x2=(-b-sqrt(D))/(2*a)
8
```

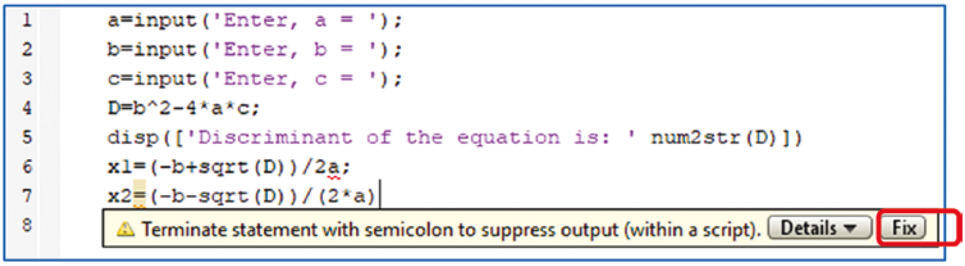
There is one error on line 6 (the red colored wavy line under a), where the multiplication * sign is missing, and one warning shown with an orange colored wavy line under the = sign on line 7 where the semicolon is recommended to suppress the display of the output from this line.

Error and warning messages like these are shown on the right edge of the editor's scrollbar. If there is a red colored wavy line showing an error, that script (M/MLX-file) cannot be executed. If there are any warning signs with orange wavy underlines , that script can be executed without a problem. A few different kinds of typos and illegal operations are detected automatically by the M/MLX editors, but they cannot be fixed automatically. Thus, you have to understand and work out such issues.



```
1    a=input('Enter, a = ');
2    b=input('Enter, b = ');
3    c=input('Enter, c = ');
4    D=b^2-4*a*c;
5    disp(['Discriminant of the equation is: ' num2str(D)])
6    x1=(-b+sqrt(D))/2a;
7    Parse error at a: usage might be invalid MATLAB syntax.
8
```

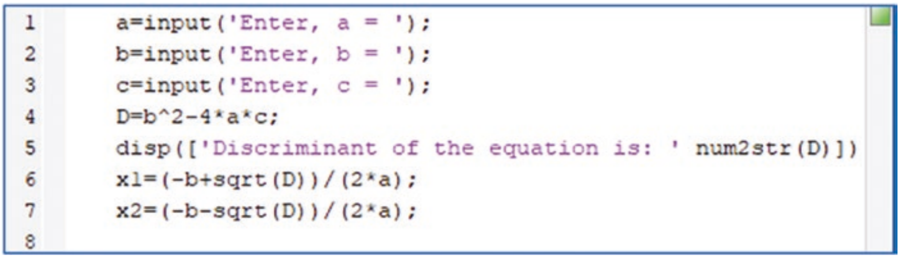
The warning message on line 7 can be fixed either by adding a semicolon where the cursor is or clicking the fix button.



```
1      a=input('Enter, a = ');
2      b=input('Enter, b = ');
3      c=input('Enter, c = ');
4      D=b^2-4*a*c;
5      disp(['Discriminant of the equation is: ' num2str(D)])
6      x1=(-b+sqrt(D))/2a;
7      x2=(-b-sqrt(D))/(2*a)
8
```

⚠ Terminate statement with semicolon to suppress output (within a script).

Finally, here is the fixed script. The green button in the right corner indicates that the syntax is correct and the script is ready for execution.

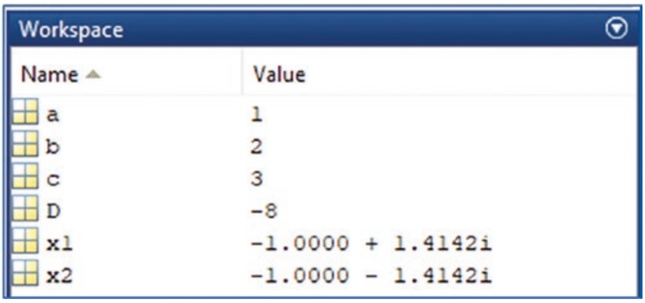


```
1      a=input('Enter, a = ');
2      b=input('Enter, b = ');
3      c=input('Enter, c = ');
4      D=b^2-4*a*c;
5      disp(['Discriminant of the equation is: ' num2str(D)])
6      x1=(-b+sqrt(D))/(2*a);
7      x2=(-b-sqrt(D))/(2*a);
8
```

After saving this script as `Eqn.m`, you can execute it by clicking the ▶ Run GUI button from the Editor's main menu, by pressing Ctrl+Enter from the keyboard, or by calling the script by its name (`>> run('Eqn')`) from the Command Window directly. After executing it, it prompts the input entries in the Command Window. You then would enter the 1, 2, 3 values for a, b, and c, respectively.

```
Enter, a = 1
Enter, b = 2
Enter, c = 3
Discriminant of the equation is: -8
```

Then the whole computation is completed and this is what is obtained in the Command Window. There are also some other variables saved in the workspace that are shown as well.



Workspace

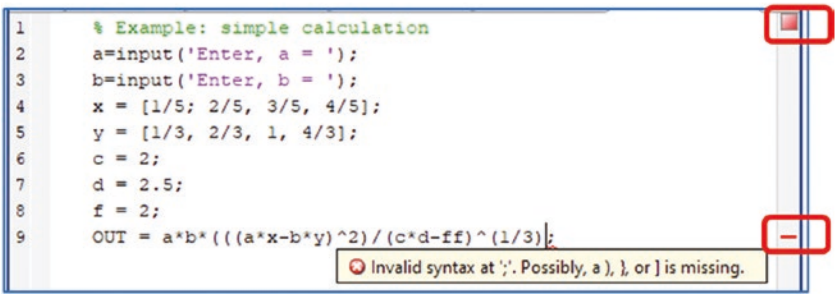
Name	Value
a	1
b	2
c	3
D	-8
x1	-1.0000 + 1.4142i
x2	-1.0000 - 1.4142i

All of the entries and processed/computed outputs from the scripts and M/MLX-files are saved automatically in the workspace. Except for when the function files are executed, not all results are saved in the workspace apart from the specified output variables in the function file. This issue is addressed in the function files section later in this chapter.

Example 2

Compute the following expression by writing an M-file (script): $ab\sqrt[3]{\frac{(ax-by)^2}{cd-f}}$ where $x=\left[\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5}\right];y=\left[\frac{1}{3},\frac{2}{3},1,\frac{4}{3}\right],c=2,d=2.5,f=2$ and the values *a*, *b* according to the user entries (scalars). According to the given values of the variables, *x* and *y* are row arrays and other variables—*a, b, c, d,* and *f*—are scalars.

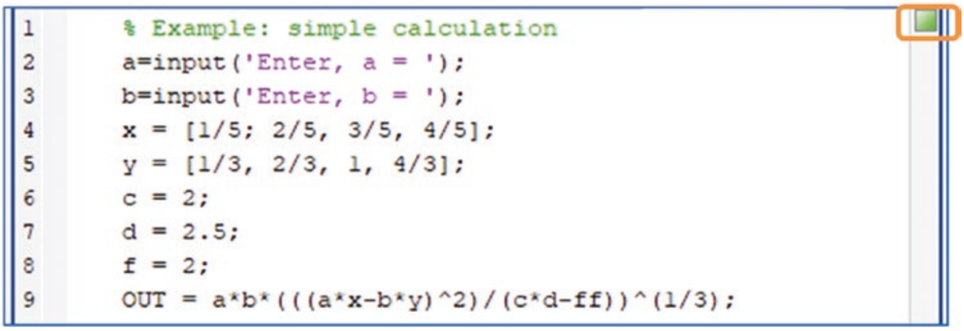
Like with the quadratic equation example, the input prompts are included in the script.



```
% Example: simple calculation
a=input('Enter, a = ');
b=input('Enter, b = ');
x = [1/5; 2/5, 3/5, 4/5];
y = [1/3, 2/3, 1, 4/3];
c = 2;
d = 2.5;
f = 2;
OUT = a*b*(((a*x-b*y)^2)/(c*d-ff)^(1/3);
```

Invalid syntax at ';'. Possibly, a), }, or] is missing.

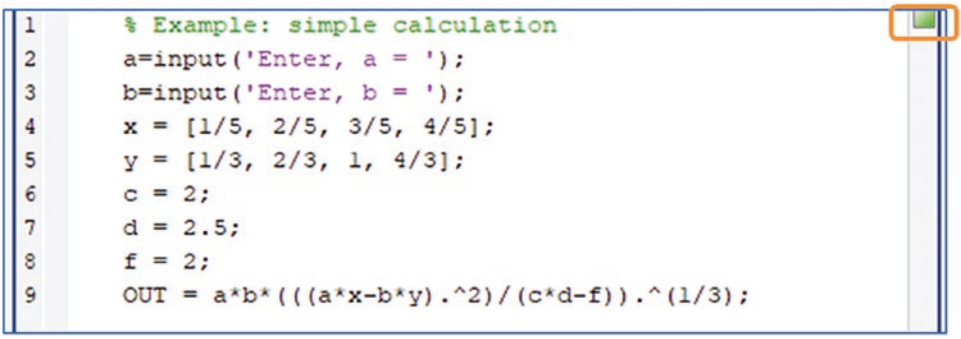
This is the created short script to compute the given assignment. By taking a quick look at this script, one can see that there is one error detected by the MATLAB editor. It's invalid syntax related to missing a parenthesis at the end of line 9. The missing parenthesis is before the first ^ power raise sign. Here is the fixed code with the green button in the upper-right corner.



```
1   % Example: simple calculation
2   a=input('Enter, a = ');
3   b=input('Enter, b = ');
4   x = [1/5; 2/5, 3/5, 4/5];
5   y = [1/3, 2/3, 1, 4/3];
6   c = 2;
7   d = 2.5;
8   f = 2;
9   OUT = a*b*(((a*x-b*y)^2)/(c*d-ff))^(1/3);
```

Now everything appears to be correct according to the editor syntax. However, there are still several errors:

- On line 4 while defining the elements of x, *a* semicolon is used as an element separator, but it must be a comma or a space.
- The f variable is defined on line 8, but in the expression on line 9, ff is used instead.
- The computation expression on line 9 is performed with the variables a, b, c, d, and f, which are scalars, and the variables x and y, which are row arrays. That's not correct. That line has to contain elementwise operations over the row array variables x and y. After fixing these errors, the script will be in the following form.

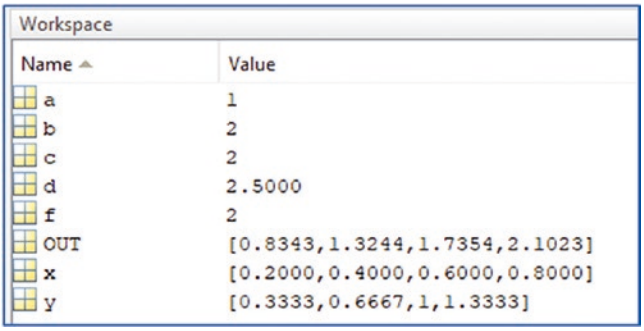


```
1   % Example: simple calculation
2   a=input('Enter, a = ');
3   b=input('Enter, b = ');
4   x = [1/5, 2/5, 3/5, 4/5];
5   y = [1/3, 2/3, 1, 4/3];
6   c = 2;
7   d = 2.5;
8   f = 2;
9   OUT = a*b*(((a*x-b*y).^2)/(c*d-f)).^(1/3);
```

Now the script can be executed by pressing Ctrl+Enter from the keyboard without saving it, or by clicking after saving it. Here are the results (the input entries from the Command Window):

```
Enter, a = 1
Enter, b = 2
```

And here are the variables in the workspace:



Workspace

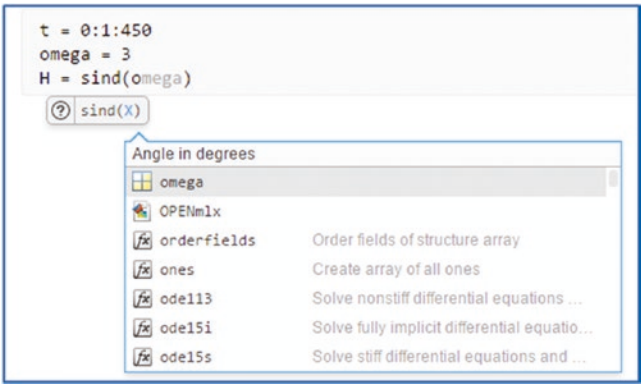
Name	Value
a	1
b	2
c	2
d	2.5000
f	2
OUT	[0.8343,1.3244,1.7354,2.1023]
x	[0.2000,0.4000,0.6000,0.8000]
y	[0.3333,0.6667,1,1.3333]

Example 3

Compute the mathematical expression's values and plot them using the MLX-file editor and creating an MLX-file. Given: $H(t) = \sin(\omega t)$; $\omega = 3$; $t = [0^0, 450^0]$ with $\Delta t = 1^0$.

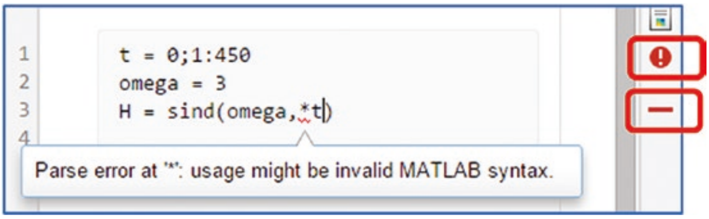
Note that the given argument values of t are in degrees, not in radians. In MATLAB, there are two functions—**sin()** and **sind()**—that compute sine function values in radians and degrees, respectively. Therefore, in such cases, there are two approaches—always use the right MATLAB function or alternatively employ the conversion function from degrees to radians (called **deg2rad()**), or vice versa, using **rad2deg()**.

Now the MLX-file (script) is created in a live editor by considering input argument t:



```
t = 0:1:450
omega = 3
H = sind(omega)
```

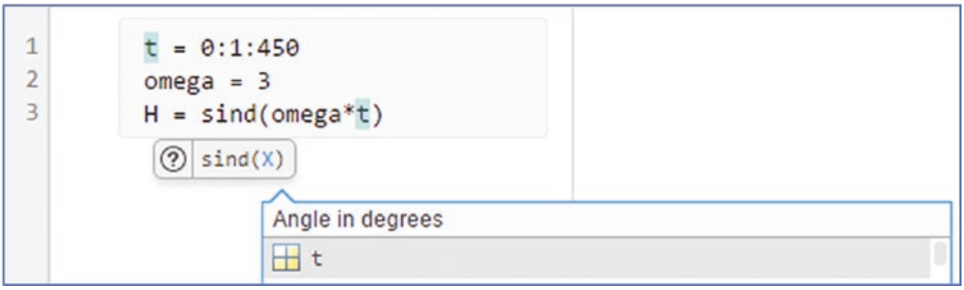
sind(X)
Angle in degrees
omega
OPENmlx
orderfields Order fields of structure array
ones Create array of all ones
ode113 Solve nonstiff differential equations ...
ode15i Solve fully implicit differential equatio...
ode15s Solve stiff differential equations and ...



```
t = 0;1:450
omega = 3
H = sind(omega,*t)
```

Parse error at "*": usage might be invalid MATLAB syntax.

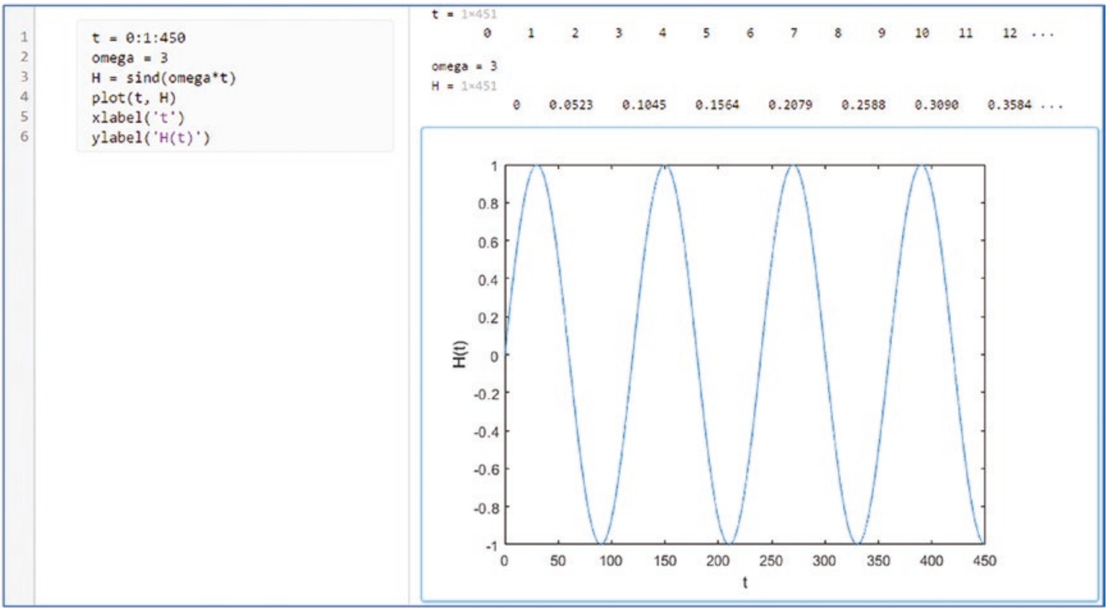
Note that the error is shown with a wavy underline, and ❗ and ▬ appear in red on the right side of the MLX editor. After fixing the error (the comma before the *), the corrected code will be in the following form.



```
t = 0:1:450
omega = 3
H = sind(omega*t)
```

Angle in degrees

There are several essential differences between the M-file editor and MLX-file editor. One of them is that the MLX editor can detect automatically defined variables after typing the first letter of the variable name. An additional hint shows the argument type (e.g., ω and `t` in angle in degrees). Another feature of the MLX-file editor is that it does not show warning messages if the semicolon is not placed at the end of commands assigning variable values. For example, `t = 0:1:450`, which we saw while working with the M-file editor. The MLX-file editor displays the output, not in the Command Window, but in the MLX editor's right window. At the same time, all of the variables and computed expressions are saved in the workspace after executing the MLX-files, just like with M-files. Execution of the MLX-files is similar to M-files and can be done via pressing Ctrl+Enter from the keyboard or by clicking the ▶ GUI button. Here is the final script with the **plot()** command and its computed results.



```
t = 0:1:450
omega = 3
H = sind(omega*t)
plot(t, H)
xlabel('t')
ylabel('H(t)')
```

```
t = 1×451
       0      1      2      3      4      5      6      7      8      9     10     11     12 ...

omega = 3
H = 1×451
          0   0.0523   0.1045   0.1564   0.2079   0.2588   0.3090   0.3584 ...
```

Example 4

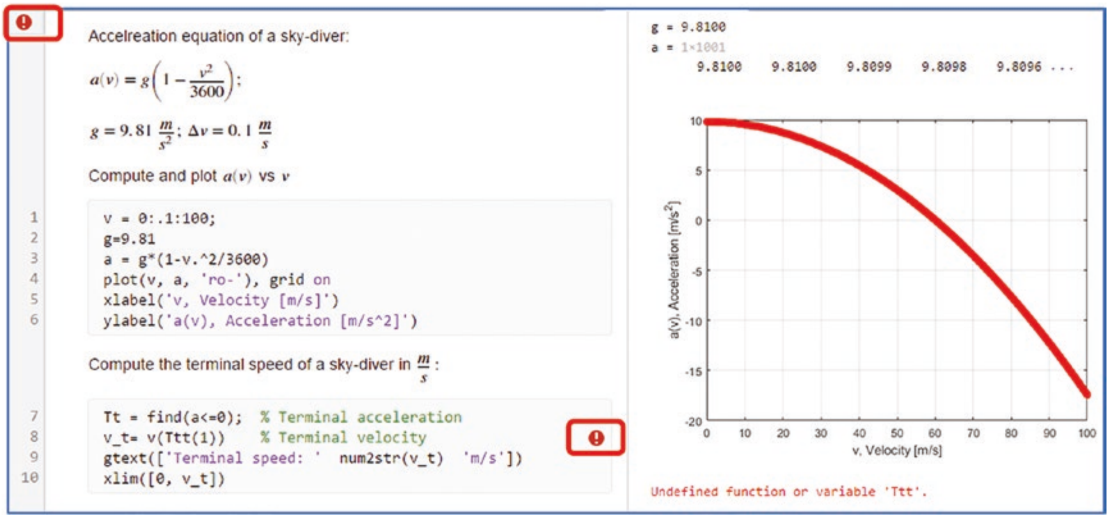
Write an MLX-file that analyzes and computes the following acceleration equation *a*(*v*) (acceleration as a function of velocity) of a skydiver:

$$a(v) = g\left(1 - \frac{v^2}{3600}\right)$$

$$g = 9.81\frac{m}{s^2}$$

1. Plot *a* versus *v*.
2. Compute the terminal speed of a skydiver when $a(v) \cong 0\frac{m}{s^2}$.
3. Take the vertical velocity *v* to be an independent variable with the step size of $\Delta v = 0.1\frac{m}{s}$ within $\left[0,\ 100\frac{m}{s}\right]$.
4. Analyze the code using the profile viewer.
5. Display the terminal speed value on the plot.

Here is the initial code with one error (a mistyped variable name—*Ttt* instead of *Tt*). This is indicated on line 8 indicated by the exclamation ❗ sign. These types of errors (mistyped variable names) can be detected in the live editor only after you execute the script.



Accelreation equation of a sky-diver:

$a(v) = g\left(1 - \frac{v^2}{3600}\right);$

$g = 9.81\ \frac{m}{s^2};\ \Delta v = 0.1\ \frac{m}{s}$

Compute and plot $a(v)$ vs v

```
1   v = 0:.1:100;
2   g=9.81
3   a = g*(1-v.^2/3600)
4   plot(v, a, 'ro-'), grid on
5   xlabel('v, Velocity [m/s]')
6   ylabel('a(v), Acceleration [m/s^2]')
```

Compute the terminal speed of a sky-diver in $\frac{m}{s}$:

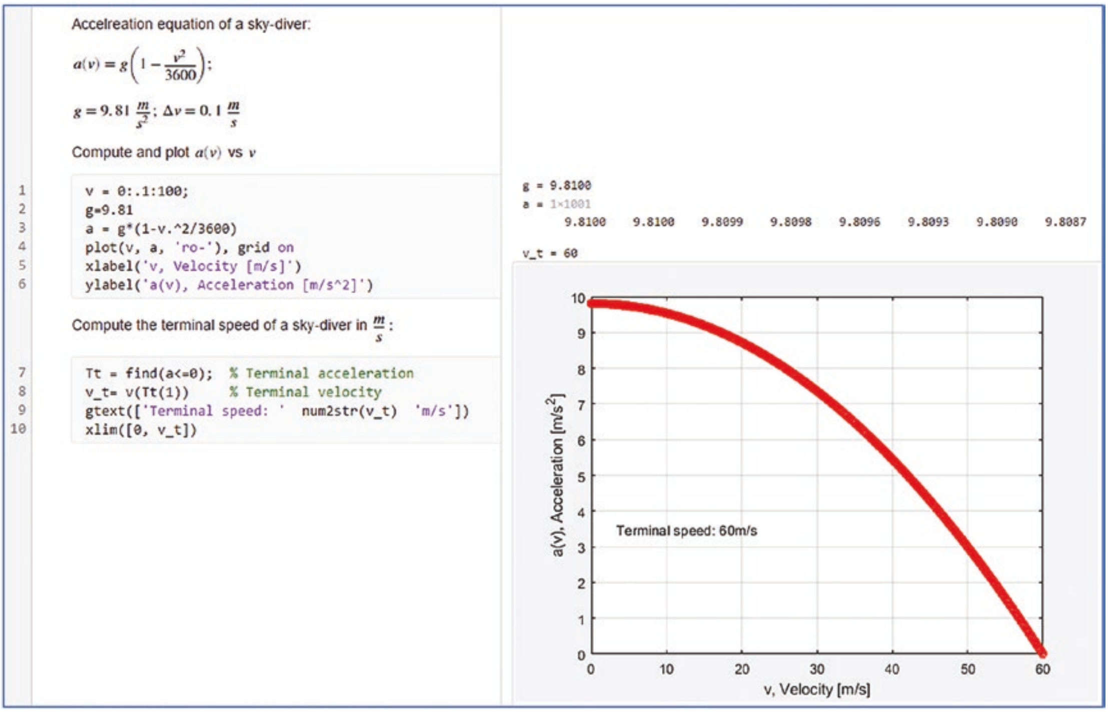
```
7   Tt = find(a<=0);  % Terminal acceleration
8   v_t= v(Ttt(1))    % Terminal velocity
9   gtext(['Terminal speed: '  num2str(v_t)  'm/s'])
10  xlim([0, v_t])
```

```
g = 9.8100
a = 1×1001
    9.8100    9.8100    9.8099    9.8098    9.8096 ...
```

```
Undefined function or variable 'Ttt'.
```

Note that the error with the mistyped variable name (*Ttt* instead of *Tt*) showed up after executing the MLX editor. The results of calculations from the MLX editor, unlike with the M-file editor, do not show up in the Command Window. Therefore, it is a good idea to execute the live script frequently in order to catch any flaws and errors while typing and editing the scripts in the MLX editor.

Here are the corrected and finalized solution scripts of this exercise in the MLX editor:



Accelreation equation of a sky-diver:

$a(v) = g\left(1 - \frac{v^2}{3600}\right);$

$g = 9.81\ \frac{m}{s^2};\ \Delta v = 0.1\ \frac{m}{s}$

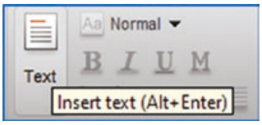
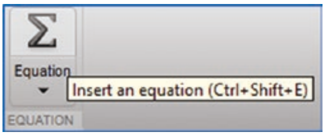
Compute and plot $a(v)$ vs v

```
v = 0:.1:100;
g=9.81
a = g*(1-v.^2/3600)
plot(v, a, 'ro-'), grid on
xlabel('v, Velocity [m/s]')
ylabel('a(v), Acceleration [m/s^2]')
```

Compute the terminal speed of a sky-diver in $\frac{m}{s}$:

```
Tt = find(a<=0);  % Terminal acceleration
v_t= v(Tt(1))     % Terminal velocity
gtext(['Terminal speed: '  num2str(v_t)  'm/s'])
xlim([0, v_t])
```

```
g = 9.8100
a = 1×1001
    9.8100    9.8100    9.8099    9.8098    9.8096    9.8093    9.8090    9.8087

v_t = 60
```

The outputs are displayed in the same window on the right side, including the plot figures. In this exercise, there are a few lines of text (non-executable) and comments. The equations are added between executable commands using the  and  buttons from the equation editor GUI tools of the MLX editor. These tools are not available in M-file editor.

Example 5

It is possible to call another M/MLX-file within an M-file. Let's look at an example that writes M- and MLX-files in order to compute the following expressions. You'll also obtain their computation results by calling/executing another M/MLX-file. This demonstrates how you link/connect several scripts (M/MLX-files). In other words, you'll execute several scripts, obtain their simulation results, and then use them in a single script.

1) Compute $F(\alpha) = e^{\sin(\alpha)}$, $H(\beta) = e^{\cos(\beta)}$, $S = 1 - (sin^2\alpha + cos^2\,\beta)$, for $\alpha = -\,2\pi \ldots 2\pi$, $\beta = -\,360^0 \ldots 360^0$ by writing an MLX-file.

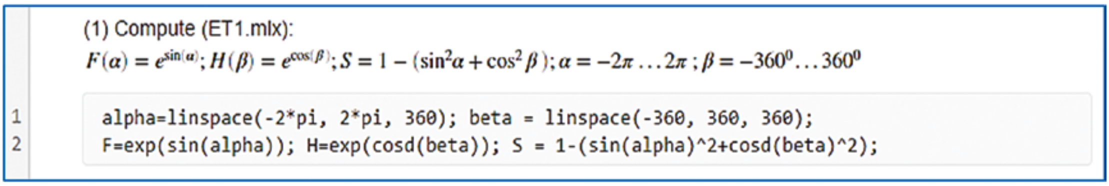
2) Compute $\mathrm{g}(a, b, t) = \cos(at) - \sin(\mathrm{b}t)$, for $a = 3$, $b = 2$, $-13 \le t \le -3$ with $\Delta t = \pi/50$ by writing an M-file.

3) Compute $f(t) = \cos(20t) - \sin(10t)$ for $t \in [-\pi, \ \ \pi]$ with $\Delta t = \pi/50$ and plot the computation results of t versus $f(t)$ by writing an M-file.

4) Compute $T(s) = \dfrac{13e^{-2s}}{s^2 + \delta\omega_n s + \omega_n^2}$ for $\delta = [0, \ \ 0.5, \ \ 1, \ \ 1.5, 2]$ and $\omega_n = 3\left[\dfrac{rad}{sec}\right]; s = [0{,}25], \Delta s = 5*10^{-2}$ by writing an MLX-file.

5) Write an M-file that executes all of the M- and MLX-files from the previous tasks and saves all the computation results as follows. In a single array called `A1` (for Step 1), a table array called `Btab2` (for Step 2), a cell array called `Ccell3` (for Step 3), and a structure array called `Dstruct4` (for Step 4).

Here are the solution scripts in the M/MLX-files of these five tasks.

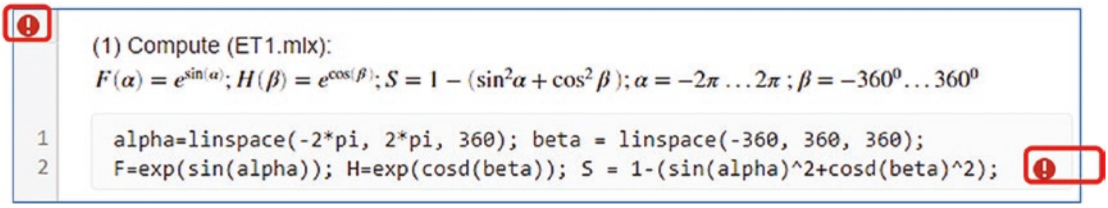


(1) Compute (ET1.mlx):

$F(\alpha) = e^{\sin(\alpha)}; H(\beta) = e^{\cos(\beta)}; S = 1 - (\sin^2\alpha + \cos^2\beta); \alpha = -2\pi \ldots 2\pi\,; \beta = -360^0 \ldots 360^0$

```
1   alpha=linspace(-2*pi, 2*pi, 360); beta = linspace(-360, 360, 360);
2   F=exp(sin(alpha)); H=exp(cosd(beta)); S = 1-(sin(alpha)^2+cosd(beta)^2);
```

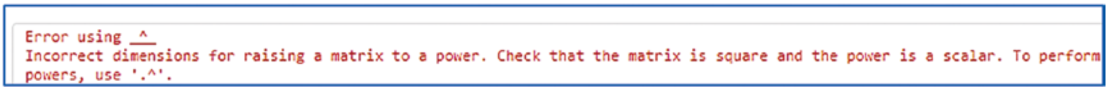
This script looks error-free and the MLX editor does not show any problematic issues with it. However, there are two errors (in array power operations) on line 2 in computing `S`, which will be detected by the editor only after you execute the script.



(1) Compute (ET1.mlx):

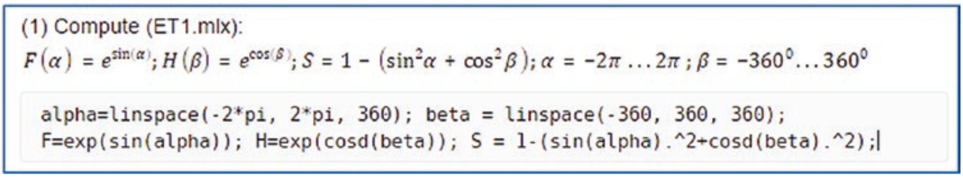
$F(\alpha) = e^{\sin(\alpha)}; H(\beta) = e^{\cos(\beta)}; S = 1 - (\sin^2\alpha + \cos^2\beta); \alpha = -2\pi \ldots 2\pi; \beta = -360^0 \ldots 360^0$

```
alpha=linspace(-2*pi, 2*pi, 360); beta = linspace(-360, 360, 360);
F=exp(sin(alpha)); H=exp(cosd(beta)); S = 1-(sin(alpha)^2+cosd(beta)^2);
```



```
Error using  ^
Incorrect dimensions for raising a matrix to a power. Check that the matrix is square and the power is a scalar. To perform
powers, use '.^'.
```

Here is the corrected script.

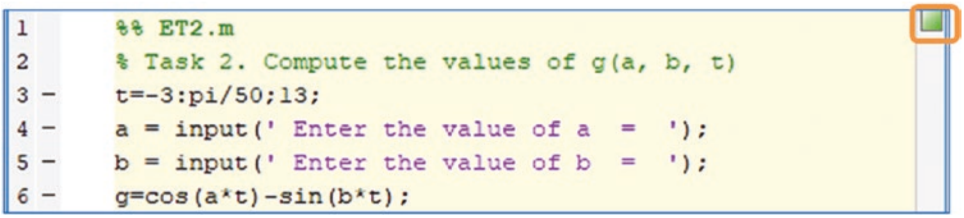


(1) Compute (ET1.mlx):

$F(\alpha) = e^{\sin(\alpha)}; H(\beta) = e^{\cos(\beta)}; S = 1 - (\sin^2\alpha + \cos^2\beta); \alpha = -2\pi \ldots 2\pi; \beta = -360^0 \ldots 360^0$

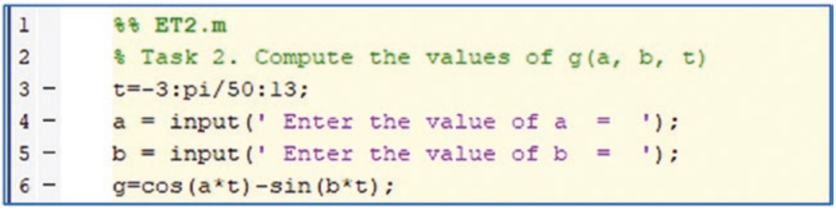
```
alpha=linspace(-2*pi, 2*pi, 360); beta = linspace(-360, 360, 360);
F=exp(sin(alpha)); H=exp(cosd(beta)); S = 1-(sin(alpha).^2+cosd(beta).^2);
```

Here is the initial version of the script that solves task 2:



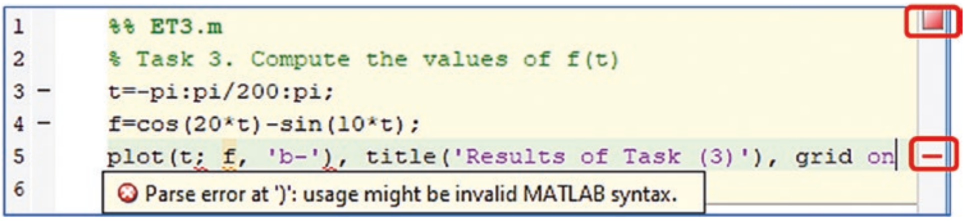
```
1     %% ET2.m
2     % Task 2. Compute the values of g(a, b, t)
3 -   t=-3:pi/50;13;
4 -   a = input(' Enter the value of a  =  ');
5 -   b = input(' Enter the value of b  =  ');
6 -   g=cos(a*t)-sin(b*t);
```

A quick glance at this code shows that it is correctly typed and ready to execute. That can be verified with the green box 🟩 on the right side of the editor window. Even with the execution of the script, no error will show up in the Command Window. However, the results of g show that many anticipated data points are missing in the results. The error is on line 3. A semicolon is typed instead of the colon `:` operator to create an array of `t`. Here is the corrected code.



```
1     %% ET2.m
2     % Task 2. Compute the values of g(a, b, t)
3 -   t=-3:pi/50:13;
4 -   a = input(' Enter the value of a  =  ');
5 -   b = input(' Enter the value of b  =  ');
6 -   g=cos(a*t)-sin(b*t);
```

Here is the initial version of the answer script for task 3:



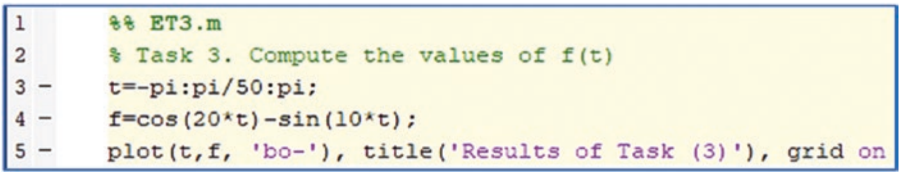
```
1      %% ET3.m
2      % Task 3. Compute the values of f(t)
3 -    t=-pi:pi/200:pi;
4 -    f=cos(20*t)-sin(10*t);
5      plot(t; f, 'b-'), title('Results of Task (3)'), grid on
6
```

Parse error at ')': usage might be invalid MATLAB syntax.

In this script, the editor detects two errors and one warning sign on line 5. The script can be executed up to line 5, at which point the following error message is displayed in the Command Window.

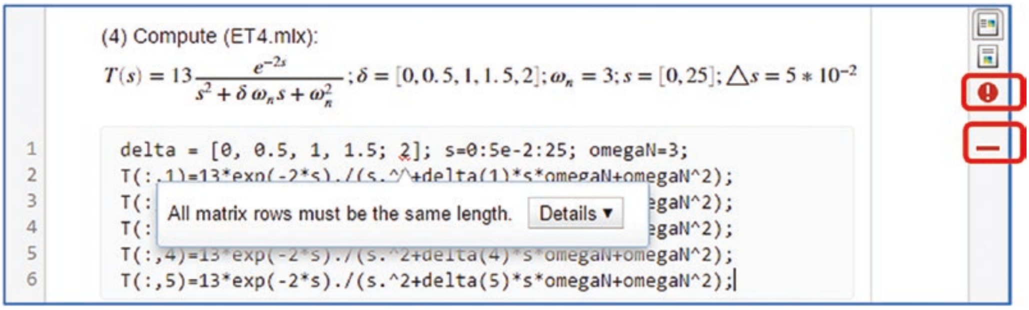
```
Error: File: ET3.m Line: 5 Column: 7
Invalid expression. When calling a function or indexing a variable, use
parentheses. Otherwise, check for mismatched delimiters.
```

Actually, there was one error, a misplaced semicolon instead of a comma in the `plot()` command on line 5: `plot( t; f,'b-')`. The two most common uses of semicolons in MATLAB are to terminate the expression and as an end of row element of an array. Therefore, in this example, the semicolon inside `plot()` is misplaced. Here is the corrected script.



```
1      %% ET3.m
2      % Task 3. Compute the values of f(t)
3 -    t=-pi:pi/50:pi;
4 -    f=cos(20*t)-sin(10*t);
5 -    plot(t,f, 'bo-'), title('Results of Task (3)'), grid on
```

Here is the initial solution script of Task 4:



(4) Compute (ET4.mlx):

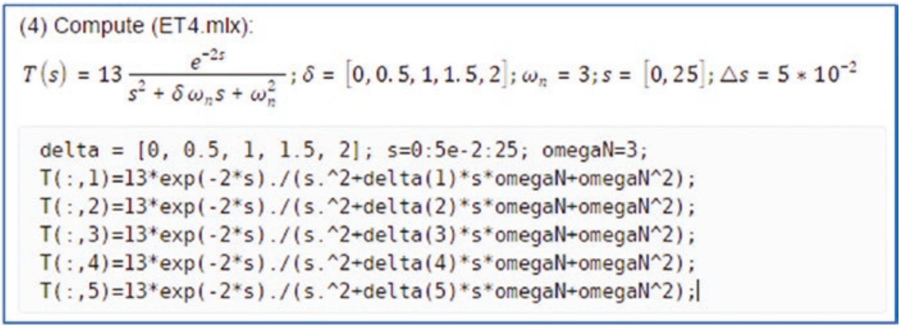
$$T(s) = 13\frac{e^{-2s}}{s^2+\delta\,\omega_n s+\omega_n^2};\delta=[0,0.5,1,1.5,2];\omega_n=3;s=[0,25];\triangle s=5*10^{-2}$$

```
delta = [0, 0.5, 1, 1.5; 2]; s=0:5e-2:25; omegaN=3;
T(:,1)=13*exp(-2*s)./(s.^^+delta(1)*s*omegaN+omegaN^2);
T(:                                              egaN^2);
T(:                                              egaN^2);
T(:,4)=13*exp(-2*s)./(s.^2+delta(4)*s*omegaN+omegaN^2);
T(:,5)=13*exp(-2*s)./(s.^2+delta(5)*s*omegaN+omegaN^2);
```

All matrix rows must be the same length.

It automatically detected an error—the mistyped semicolon instead of a comma—as an element separator of the array. With this error, the script cannot be executed.

Here is the corrected version of the script.

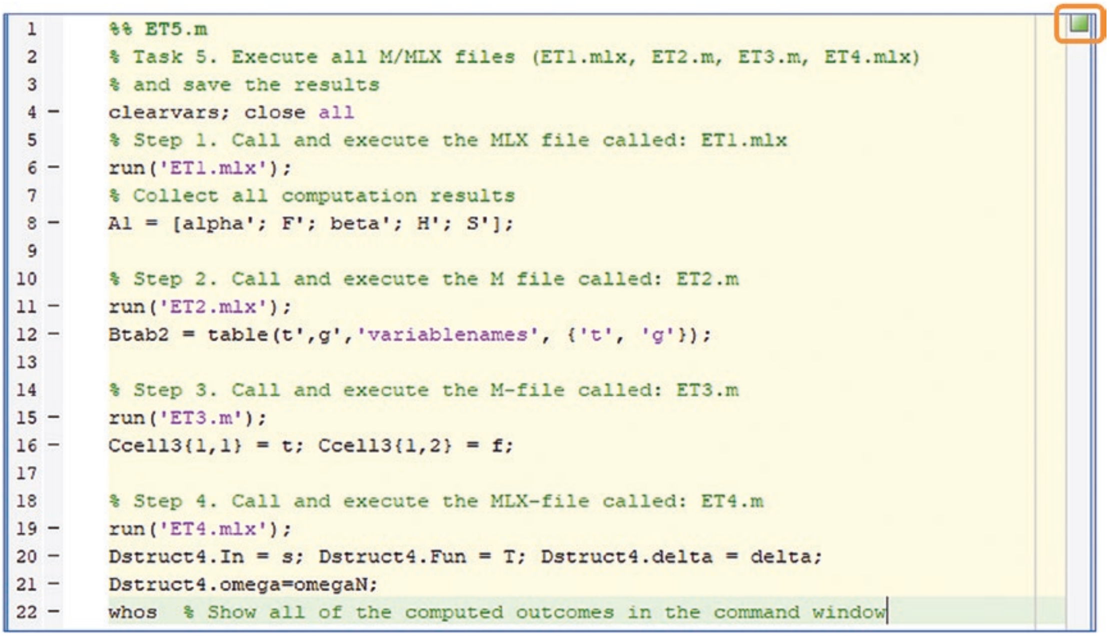


(4) Compute (ET4.mlx):

$$T(s) = 13\frac{e^{-2s}}{s^2 + \delta\,\omega_n s + \omega_n^2}; \delta = [0, 0.5, 1, 1.5, 2]; \omega_n = 3; s = [0, 25]; \Delta s = 5 * 10^{-2}$$

```
delta = [0, 0.5, 1, 1.5, 2]; s=0:5e-2:25; omegaN=3;
T(:,1)=13*exp(-2*s)./(s.^2+delta(1)*s*omegaN+omegaN^2);
T(:,2)=13*exp(-2*s)./(s.^2+delta(2)*s*omegaN+omegaN^2);
T(:,3)=13*exp(-2*s)./(s.^2+delta(3)*s*omegaN+omegaN^2);
T(:,4)=13*exp(-2*s)./(s.^2+delta(4)*s*omegaN+omegaN^2);
T(:,5)=13*exp(-2*s)./(s.^2+delta(5)*s*omegaN+omegaN^2);
```

Here is the script of Task 5. It calls the scripts in Tasks 1 to 4.



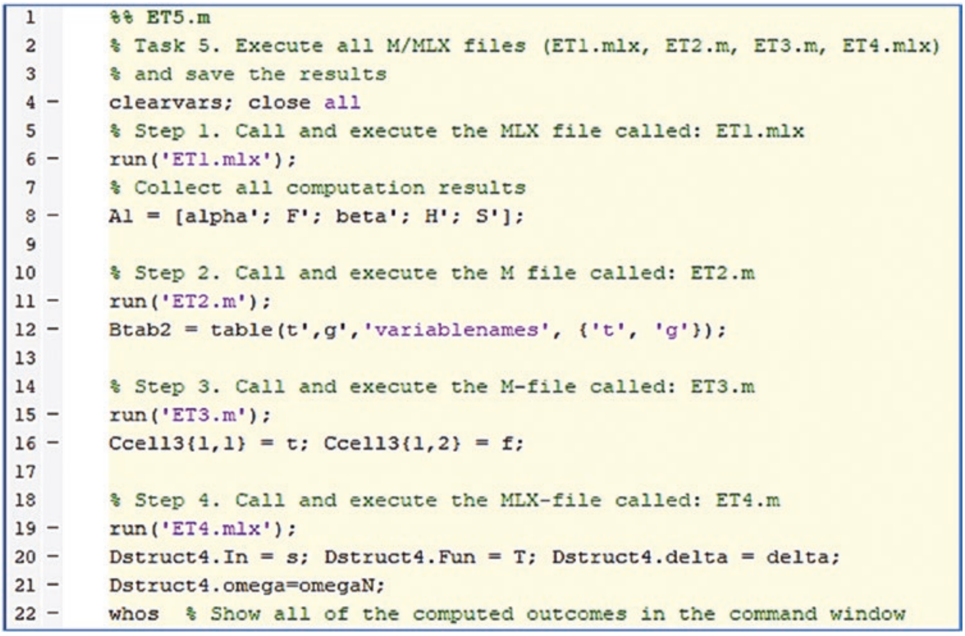
```
1       %% ET5.m
2       % Task 5. Execute all M/MLX files (ET1.mlx, ET2.m, ET3.m, ET4.mlx)
3       % and save the results
4  -    clearvars; close all
5       % Step 1. Call and execute the MLX file called: ET1.mlx
6  -    run('ET1.mlx');
7       % Collect all computation results
8  -    A1 = [alpha'; F'; beta'; H'; S'];
9
10      % Step 2. Call and execute the M file called: ET2.m
11 -    run('ET2.mlx');
12 -    Btab2 = table(t',g','variablenames', {'t', 'g'});
13
14      % Step 3. Call and execute the M-file called: ET3.m
15 -    run('ET3.m');
16 -    Ccell3{1,1} = t; Ccell3{1,2} = f;
17
18      % Step 4. Call and execute the MLX-file called: ET4.m
19 -    run('ET4.mlx');
20 -    Dstruct4.In = s; Dstruct4.Fun = T; Dstruct4.delta = delta;
21 -    Dstruct4.omega=omegaN;
22 -    whos  % Show all of the computed outcomes in the command window
```

The script editor does not show any problem or issues. However, when it is executed, the following error message pops up in the Command Window:

```
Error using run (line 87)
RUN cannot execute the file 'ET2.mlx'. RUN requires a valid MATLAB script
Error in ET5 (line 11)
run('ET2.mlx');
```

This error message indicates that there is a problem recognizing the file. That means the filename ET2.mlx is not correct (the file extension is wrong); it has to be ET2.m instead.

Here is the corrected script of Task 5.



```
%% ET5.m
% Task 5. Execute all M/MLX files (ET1.mlx, ET2.m, ET3.m, ET4.mlx)
% and save the results
clearvars; close all
% Step 1. Call and execute the MLX file called: ET1.mlx
run('ET1.mlx');
% Collect all computation results
A1 = [alpha'; F'; beta'; H'; S'];

% Step 2. Call and execute the M file called: ET2.m
run('ET2.m');
Btab2 = table(t',g','variablenames', {'t', 'g'});

% Step 3. Call and execute the M-file called: ET3.m
run('ET3.m');
Ccell3{1,1} = t; Ccell3{1,2} = f;

% Step 4. Call and execute the MLX-file called: ET4.m
run('ET4.mlx');
Dstruct4.In = s; Dstruct4.Fun = T; Dstruct4.delta = delta;
Dstruct4.omega=omegaN;
whos  % Show all of the computed outcomes in the command window
```

By executing the last file (ET5.m), the other four scripts are also called and executed consecutively. Here are the computed outcomes from all scripts in the Command Window:

```
Enter the value of a  =  2
 Enter the value of b  =  3
  Name                 Size             Bytes  Class     Attributes

  A1                1800x1              14400  double
  Btab2              255x2               5162  table
  Ccell3               1x2               1840  cell
  Dstruct4             1x1              24800  struct
  F                    1x360             2880  double
  H                    1x360             2880  double
  S                    1x360             2880  double
  T                  501x5              20040  double
  a                    1x1                  8  double
  alpha             1x360             2880  double
  b                 1x1                  8  double
  beta              1x360             2880  double
  delta             1x5                 40  double
  f                 1x101              808  double
  g                 1x255             2040  double
  omegaN            1x1                  8  double
  s                 1x501             4008  double
  t                 1x101              808  double
```

Also, there is a plot figure (not shown here) from Task 3 (`ET3.m`).

Note that there are many other common errors made while writing scripts that will be highlighted throughout the book. Moreover, there are a few common mistakes made while creating the function files that all will be highlighted in the section dedicated to function files.

Via plain examples, you have seen how the M and MLX (Live) editors can be employed while writing scripts to detect common errors automatically. There are some other errors that are not detected by the M/MLX editors automatically that can be found only after executing the scripts. These include various operations (arithmetic, matrix, and array) that are performed with mismatched sizes of variables or improperly spelled MATLAB commands.

Note Finding and fixing the errors you make while writing code are good exercises for learning how to write great programs.

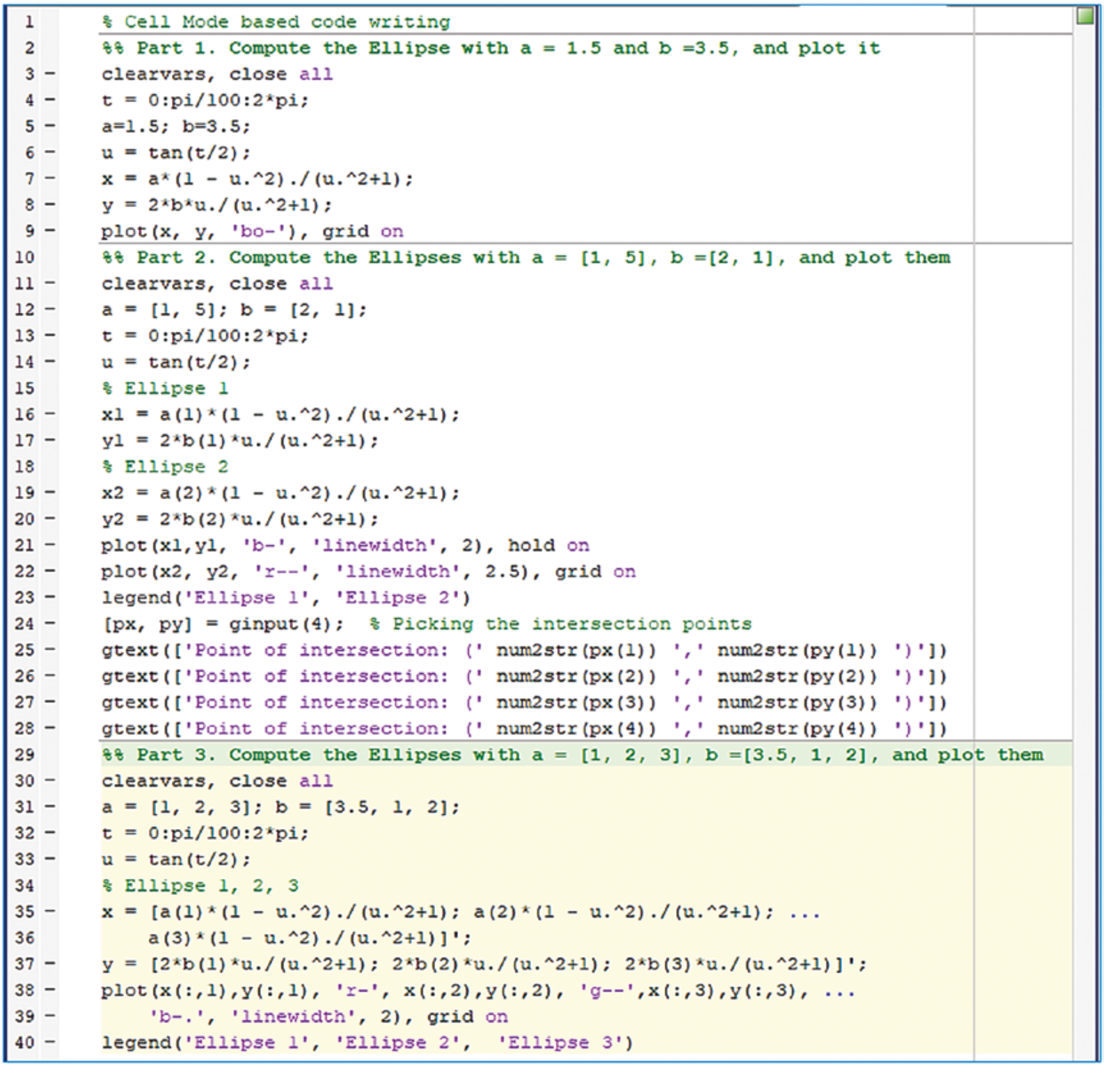
Cell Mode

In the M/MLX editors, there is a very handy tool for writing well-structured codes/scripts, called the cell mode option. Working in cell mode is simple and easy and can be accomplished by typing %%. Leaving one space creates a new cell by default. Writing long scripts separated in cells for every separate operation helps you execute your code cell by cell and detect where the bugs/flaws are. It also helps to visually separate the code into distinct blocks that can be highlighted one at a time. This helps with readability. Let's consider the following example.

Given that the equation for an ellipse centered at the origin of the Cartesian coordinates (x, y) is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (A)$, where *a* and *b* are constants that determine the shape of the ellipse, the variables *x*, *y* are defined by $x(u)=\frac{a(1-u^2)}{u^2+1}, y(u)=\frac{2bu}{u^2+1}$, and $u=tg\left(\frac{t}{2}\right)$ where $0 \le t \le 2\pi$

1) Compute the ellipse given in the equation in (A) for *a = 1.5* and *b=3.5* and plot it.
2) Plot and display the points of intersection of the two ellipses described by $x^2+\frac{y^2}{4}=1$ and $\frac{x^2}{25}+y^2=1$.
3) Compute three ellipses defined in (A) as a three-column array for these cases: *a = [1, 2, 3], b=[3.5, 1, 2].*

Here is the solution script created in three cell modes representing answers for each subsection of the exercise. Each cell contains the answer script for each part of the given exercise. Cell 1 (Part 1), composed of lines 2–9, computes the ellipse in (A) with respect to the values of *a* and *b*. Cell 2 (Part 2), composed of lines 10–27, computes two ellipses and plots their values, and subsequently displays the intersecting points of the two ellipses. Cell 3 (Part 3) computes three ellipses with respect to the values of *a* and *b* and plots the computed ellipses.



```
% Cell Mode based code writing
%% Part 1. Compute the Ellipse with a = 1.5 and b =3.5, and plot it
clearvars, close all
t = 0:pi/100:2*pi;
a=1.5; b=3.5;
u = tan(t/2);
x = a*(1 - u.^2)./(u.^2+1);
y = 2*b*u./(u.^2+1);
plot(x, y, 'bo-'), grid on
%% Part 2. Compute the Ellipses with a = [1, 5], b =[2, 1], and plot them
clearvars, close all
a = [1, 5]; b = [2, 1];
t = 0:pi/100:2*pi;
u = tan(t/2);
% Ellipse 1
x1 = a(1)*(1 - u.^2)./(u.^2+1);
y1 = 2*b(1)*u./(u.^2+1);
% Ellipse 2
x2 = a(2)*(1 - u.^2)./(u.^2+1);
y2 = 2*b(2)*u./(u.^2+1);
plot(x1,y1, 'b-', 'linewidth', 2), hold on
plot(x2, y2, 'r--', 'linewidth', 2.5), grid on
legend('Ellipse 1', 'Ellipse 2')
[px, py] = ginput(4);  % Picking the intersection points
gtext(['Point of intersection: (' num2str(px(1)) ',' num2str(py(1)) ')'])
gtext(['Point of intersection: (' num2str(px(2)) ',' num2str(py(2)) ')'])
gtext(['Point of intersection: (' num2str(px(3)) ',' num2str(py(3)) ')'])
gtext(['Point of intersection: (' num2str(px(4)) ',' num2str(py(4)) ')'])
%% Part 3. Compute the Ellipses with a = [1, 2, 3], b =[3.5, 1, 2], and plot them
clearvars, close all
a = [1, 2, 3]; b = [3.5, 1, 2];
t = 0:pi/100:2*pi;
u = tan(t/2);
% Ellipse 1, 2, 3
x = [a(1)*(1 - u.^2)./(u.^2+1); a(2)*(1 - u.^2)./(u.^2+1); ...
    a(3)*(1 - u.^2)./(u.^2+1)]';
y = [2*b(1)*u./(u.^2+1); 2*b(2)*u./(u.^2+1); 2*b(3)*u./(u.^2+1)]';
plot(x(:,1),y(:,1), 'r-', x(:,2),y(:,2), 'g--',x(:,3),y(:,3), ...
    'b-.', 'linewidth', 2), grid on
legend('Ellipse 1', 'Ellipse 2',  'Ellipse 3')
```

After writing the code in cell mode, you can execute each cell separately by using Ctrl+Enter from the keyboard when the cursor is within the cell that is meant to be executed, or by using Run Section and Advance or Run and Advance from the M-file editor. The advantage to using cell mode is that you can execute and test each cell separately. If there are any errors, you can fix them without interfering with other cells. You can save considerable time and effort when debugging and correcting long scripts.

Note When writing long, extended scripts and debugging them, it is more efficient to use cell mode.

As demonstrated, the M/MLX-file editors have many useful tools. For instance, they automatically detect common typographical errors. For instance, they detect and warn users about the following:

- Mistyped MATLAB built-in function/command names
- Unclosed mathematical expressions with brackets, such as `()`, `{}`, and `[]`
- Misused/improperly used or missed mathematical operators, such as `+-*/\`, ...`( )`, `{}`, and `[]`
- Invalid syntax
- Unused variable names
- Suppress display of (lengthy) outputs
- Identify which entries are numeric data, which entries are characters or text messages, and which entries are comments
- Warning messages that let you know how to improve computation efficiency of a script

Besides, the M/MLX-file editor has the following tools:

- Debugging modes, such as set/clear breakpoints, set/modify conditional breakpoints, enable/disable breakpoints, stop if errors/warnings...
- Working in a cell mode
- Text editing options under text
- Report writing with save and publish options

Debugging Mode

The **dbstop** function is used to temporarily halt the execution of a script and provide the user an opportunity to examine the local workspace. While debugging a script, you can set breakpoints at specific lines in the editor window by clicking on that line in the left gutter. There are over a dozen uses of the **dbstop** function:

```
(1)  dbstop in FILE at LINENO
(2)  dbstop in FILE at LINENO@
(3)  dbstop in FILE at LINENO@N
(4)  dbstop in FILE at SUBFUN
(5)  dbstop in FILE
(6)  dbstop in FILE at LINENO if EXPRESSION
(7)  dbstop in FILE at LINENO@ if EXPRESSION
(8)  dbstop in FILE at LINENO@N if EXPRESSION
(9)  dbstop in FILE at SUBFUN if EXPRESSION
(10) dbstop in FILE if EXPRESSION
(11) dbstop if error
(12) dbstop if caught error
(13) dbstop if warning
(14) dbstop if naninf  or  dbstop if infnan
(15) dbstop if error IDENTIFIER
(16) dbstop if caught error IDENTIFIER
(17) dbstop if warning IDENTIFIER
```

The name `FILE` is the filename in which you want to perform the debugging operations. It has to be specified as a character vector of string scalar. The `FILE` can also include a full or partial path to the file directory. `LINENO` is the line number within `FILE` (the script in which debugging operation is being performed) and `N` is an integer specifying the Nth anonymous function on the line. `SUBFUN` is the name of a sub-function within `FILE`. `EXPRESSION` is an executable conditional expression, specified as a character vector or string scalar. `IDENTIFIER` is a MATLAB message identifier (see help for `ERROR` for a description of message identifiers). The `AT` and `IN` keywords are optional.

M-Lint Code Check

The mlin() command is used to check the MATLAB code files for possible problems. When you run an M-Lint code check from the M-file editor window, by choosing Tools ➤ M-Lint ➤ Show M-Lint Report, it will prompt a full report (on the M-file) of warning messages. For instance, which variable is unused, which variable's size changes on every loop iteration, where there are opportunities for improvement, which one of the employed built-in functions of MATLAB is deprecated, and so forth. The general command syntax of mlint() is >>mlint('filename'). Starting with MATLAB 2019a, mlint() is not recommended and the checkcode() command is recommended instead.

Code Profiling

Code profiling measures where a program spends time and where the problems/errors are. By identifying the performance of the program, it can be improved. Code profiling can be called from the M-file editor window by clicking on Tools --> Open Profiler. The Code Profiling (Profile) window pops up. The profile report contains the whole benchmarking report of our script (M-file), including extensive information on each command and where the CPU (central-processing-unit) spends its time. By studying the profile report of your script, you can learn how to improve simulation time efficiency. The profile mode is off by default; it can be switched on by using the >> profile on command. The profile summary report of any M-file or Simulink model can be generated and viewed with general syntax, for example:

```
profile on
% Script - M-file starts here:
clc; close all; clearvars
M=magic(55); plot(M);
Minv=inv(M); Mans=M*Minv; MANS=M.*Minv;
% Script - M-file ends here:
profile viewer
```

Or

```
>> profile on; My_function; profile viewer
```

Or for Simulink models:

```
>> profile on; sim('My_Model'); profile viewer
```

Note that you can insert any M-filename (e.g., `My_function.m`) or function filename (e.g., `My_function.m`) or Simulink model name (e.g., `My_Model.mdl`) in between the two commands:

```
profile on; profile viewer
```

Dependency Report

The dependency report identifies all functions, scripts, and external programs (applications) that are called/used in your program. This report finds the information about dependent files and tools you employed in your script from the current folder. This is of great help before sharing your work with other users, who may not have all the M-files, function files, and toolboxes that you have. The dependency report shows dependencies among MATLAB files in a directory and can be called using the following steps:

1. Select Desktop --> Current Directory and navigate to the directory where the appropriate MATLAB files are.
2. Right-click on the Current Folder tab. From the right-mouse button options, click on Reports and then select Dependency Report.
3. The dependency report of all MATLAB files will be created automatically.

P-Codes

In MATLAB and Simulink, there is another important file type called P-code, with a file extension of `*.p`. It is very handy and recommended to create in some specific instances. The main reason to create a P-code file from your M-files is to prevent valuable files from unwanted editing and to speed up the simulation time of scripts.

Here are good reasons to create P-codes:

- To speed up the simulation process in MATLAB.
- To keep valuable M-files secure. However, P-codes should not be considered as a substitute for secure encryption.
- To hide proprietary algorithms.
- When you call an M-file function, MATLAB parses the M-code and stores the instructions as P-code in cache memory. The P-code remains in memory until it is cleared using the `clear` command or until MATLAB quits.
- It's platform independent pseudo-code for a virtual MATLAB machine.
- Since P-files are in binary format, their source code is hidden.

How to create P-codes:

– To create a P-code from a given M-file (e.g., `My_fileM.m` in this example), you type the following in the Command Window:

```
>> pcode My_fileM.m
```

After running the command, a P-code of the M-file `My_fileM.m` will be created under the same filename, but with the extension of `*.p` instead of `*.m`.

– To execute the P-code called `My_fileM.p`, type the following in the Command Window:

```
>> My_fileM.p
```

– Or type this in the Command Window:

```
>> run My_fileM
```

This executes the created P-code even if you have the primary M-file called `My_fileM.m` in the current directory.

Some Remarks About Scripts/M/MLX-Files

- In general, MATLAB commands can be executed from scripts or from the Command Window. The question of which method is best depends on which tasks/computations need to be performed. If there are simple computations composed of one-step processes and no repetitions are required, there is no need to create M/MLX-files or scripts. On the other hand, when large computations with different operations, including loop iterations and conditional statements, are required, writing M/MLX-files is the best option.
- The error and warning messages will not only help you locate errors in your scripts and models, but also will provide hints about the causes of the errors and how to eliminate them. In fact, warning messages make your programs more robust and inform you about the ignored data in computed/plotted outputs.
- To write scripts and M-files, the M/MLX-file editors must be employed since they include a number of very helpful tools. For instance, they automatically detect unused variables, display of data sets, and unclosed loops. They also enable you to work in cell and debug modes, help you detect errors, suggest memory allocation options, and much more.

Display and Printout Operators: display, sprintf, and fprintf

There are several commands and operators (built-in functions) that display computation results in the Command Window or export them into external files compatible with MATLAB. They are `disp()`, `display()`, `sprint()`, and `fprintf()`. Of these commands, `disp()` and `display()` are straightforward ways to display comments, strings, or numerical values in the Command Window without any additional formatting tools or characters. They are not robust enough to display comments/strings and numerical values in various formats. They cannot write data into external files. On the other hand, `sprint()` and `fprintf()` can substitute all functions of `disp()` and `display()`. They can be used to print various data types in the Command Window, by using formatting operators and characters. Moreover, they can print textual and numerical data into external files. Let's look at several examples to demonstrate how to employ these display and print commands.

Example 1

Given $f(t) = sin(t), t = \left[0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi\right]$, display computation results of the function *f*(*t*) and its argument *t* with short explanatory comments in the Command Window. This is straightforward with `disp()` and `display()`.

```
t=[0, pi/4, pi/2, 3*pi/4, pi];
disp(['Sine @', num2str(t(1)),' is equal to: ', num2str(sin(t(1))) ])
disp(['Sine @', num2str(t(2)),' is equal to: ', num2str(sin(t(2))) ])
disp(['Sine @', num2str(t(3)),' is equal to: ', num2str(sin(t(3))) ])
disp(['Sine @', num2str(t(4)),' is equal to: ', num2str(sin(t(4))) ])
disp(['Sine @', num2str(t(5)),' is equal to: ', num2str(sin(t(5))) ])
```

These commands display the following in the Command Window:

```
Sine @ 0 is equal to: 0
Sine @ 0.7854 is equal to: 0.70711
Sine @ 1.5708 is equal to: 1
Sine @ 2.3562 is equal to: 0.70711
Sine @ 3.1416 is equal to: 1.2246e-016
```

The results are correct and readable, but the demonstrated procedure is tedious. Note that there is a difference in outputs from `disp()` and `display()`. The `disp()` command displays outputs only without a variable name. On the other hand, `display()` displays the variable name and its value as a simple calculation, without a semicolon at the end. For instance:

```
>> D1=sind(90);
>> display(D1)
D1 =
     1
>> disp(D1)
     1
```

```
>> D1
D1 =
     1
>> D1=sind(90)
D1 =
     1
```

Example 2

The clock command is a built-in command used to see current year/date/time according to a user's computer clock in a row matrix format. Let's display the current time in a more explicit way with some explanation. How do you do it with the disp() and display() commands?

```
>> format short G
>> TT=clock

TT =

  Columns 1 through 5

         2013           11            9            4            5

  Column 6

       37.261
>> display('This year is: '); disp(TT(1))
This year is:
        2013

>> display('This month is: '); disp(TT(2));
This month is:
    11
>> display('Day of this month: '); disp(TT(3));
Day of this month:
     9
```

```
>> disp('Current time is '); display(['hour: ', num2str(TT(4))]);
display(['minutes: ', num2str(TT(5))])
Current time is
hour: 4
minutes: 5
```

The output results are legible and explicit, but the commands are too long and inefficient from a programming point of view.

fprintf()

For the previous two examples, the `fprintf()` command can be employed much more simply. General syntax of this command is the following:

```
fprintf(format, A, ...)
```

For Example 1, displaying the sine function values, `fprintf()` provides a much simpler solution:

```
fprintf('Sine @ %1.5f is equal to: %2.5f\n', t, sin(t));
```

Here is the output of the `fprintf()` command.

```
Sine @ 0.00000 is equal to: 0.78540
Sine @ 1.57080 is equal to: 2.35619
Sine @ 3.14159 is equal to: 0.00000
Sine @ 0.70711 is equal to: 1.00000
Sine @ 0.70711 is equal to: 0.00000
```

The formatting specifiers used in this example (`%1.5f`, `%2.5f`, and `\n`) define field width and precision with floating point number format (`%1.5f`, `%2.5f`) and newline (`\n`). More details of formatting specifiers are provided later in this section.

This is a much simpler and effortless method; it contains only one line of code.

For Example 2, displaying the time and date, you can use `fprintf()` again as follows:

```
>> TT=clock;
fprintf('Year:%g; Month: %g; Day: %g; Hour: %g; Min passed: %g\n',...
TT(1), TT(2), TT(3), TT(4), TT(5))
```

```
Year: 2013; Month: 11; Day: 9; Hour: 4; Min passed: 5
```

For such cases, sprintf() could also be an option. The general syntax of the sprintf() command is:

```
str = sprintf(format, A, ...)
[str, errmsg] = sprintf(format, A, ...)
```

Note that in order to display output (str string messages) from sprintf(), you need to use the disp() or display() command again. sprintf() is less flexible than fprintf(). Note that sprintf() always returns a string. To demonstrate how to employ sprintf() more explicitly and improve the script created to solve a quadratic equation, let's use the sprintf() command to display the computation results in the Command Window with some additional information. Let's compute roots of a quadratic equation and display the information in the Command Window. Here is the solution script (called QuadEq1.m).

```
% QuadEq1.m
% Solve quadratic equations based on coefficients of: a, b, & c
disp('Solve: ax^2+bx+c=0')
a=input('Enter value of a: ');
b=input('Enter value of b: ');
c=input('Enter value of c: ');
D=b^2-4*a*c;
[S, Errm]=sprintf('Discriminant of the equation is: %g', D); disp(S)
% Roots
x1=(-b+sqrt(D))/(2*a); x2=(-b-sqrt(D))/(2*a);
% Display roots in the command window
[xr1, Errm1]=sprintf('Root1 of the equation is x1= %g', x1);
disp(xr1); display(x1)
[xr2, Errm2]=sprintf('Root2 of the equation is x2= %g', x2);
disp(xr2); display(x2)
```

Let's test the M-file (QuadEq1.m) by executing it and providing the following entries (a=11, b=11, and c=13) in the Command Window:

```
Solve: ax^2+bx+c=0
Enter value of a: 11
Enter value of b: 12
Enter value of c: 13
```

```
Discriminant of the equation is: -428
Root1 of the equation is x1= -0.545455

x1 =

  -0.5455 + 0.9404i

Root2 of the equation is x2= -0.545455

x2 =

  -0.5455 - 0.9404i
```

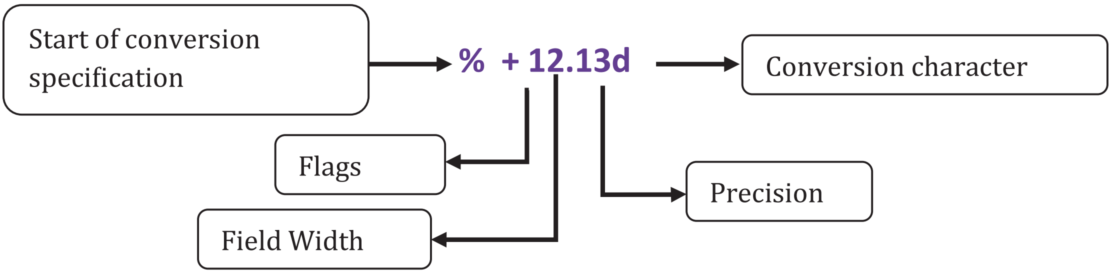
Note that within sprintf(), the numerical data is defined by the % sign followed by a formatting sign/letter g, which is called a conversion specifier. Here is a command syntax of declaring data formats with sprint() and fprintf().

```
[S, Errm]=sprintf('Discriminant of the equation is: %g', D);
```

Conversion specifications begin with the % character and contain these elements:

- Flags (optional)
- Width and precision fields (optional)
- A subtype specifier (optional)
- Conversion character (required)

These elements are used in the following order:



Flags are to control the alignment of the output, for instance, the - sign for left justification of the output, the + sign for right justification of the output, the space character for a space before the value, and '0' to put 0 (zero) before the output. The Field Width is defined with a nonnegative integer. It specifies the number of digits or characters in the output and the number (in precision) specifies the number of digits after the decimal point of the output. For example, `%12.0` produces no decimal digits after decimal digit number, whereas `%12.13` produces 13 decimal digits after the decimal sign. Table 2-1 lists the conversion characters and Table 2-2 lists the escape characters used to specify non-printing characters.[1]

Table 2-1. *Conversion Characters*

Specifier	Description
`%c`	Single character
`%d`	Decimal notation (signed)
`%e`	Exponential notation (using a lowercase e as in 3.1415e+00) for floating point
`%E`	Exponential notation (using an uppercase E as in 3.1415E+00) for floating point
`%f`	Fixed-point notation for floating point
`%g`	The more compact of `%e` or `%f`, as defined in [*]. Insignificant zeros do not print for floating point
`%G`	Same as `%g`, but using an uppercase G for floating point
`%i`	Base 10 values for integer signed
`%o`	Base 8 octal notation (unsigned)
`%s`	String of characters
`%u`	Base 10 Integer (unsigned)
`%x`	Base 16 hexadecimal notation (using lowercase letters a–f)
`%X`	Base 16 hexadecimal notation (using uppercase letters A–F)

[1] ANSI specification X3.159-1989: “Programming Language C,” ANSI, 1430 Broadway, New York, NY 10018.

Table 2-2. *Characters for Escape Formatting*

Symbol	Description
`\b`	Backspace
`\f`	Form feed
`\n`	New line
`\r`	Carriage return
`\t`	Horizontal tab
`\\`	Backslash
`'   or   ' (quote)`	Single quotation mark
`%`	Percent character

One of the most commonly used escape formatting commands is `\n`. It is used to write the following data in a new line. Note that the previous examples (Examples 1 and 2) used `\n` in the `fprintf()` command.

Let's consider the previous quadratic equation example to display and write its real and complex roots. Note that this exercise is designed on purpose to demonstrate how to employ `sprintf()` and `fprintf()`.

There are several ways to display complex numbers (e.g., complex roots of the quadratic equation) in the form of $\Re + \Im * i$ explicitly with additional comments and then write/export them into an external file. You can employ numerical conversions along with comments or employ the string conversion character, `s`.

Let's start with the first way. It requires you to add several formatting items from Tables 2-1 and 2-2 in order to display complex numbers correctly. The existing M-file (`QuadEq1.m`) will be updated with the following, which uses the `num2str()` conversion function to obtain the complex roots converted into strings. There is no need to separate the real and imaginary components of the complex roots of the equation.

```
 [xr1, Errm1]=sprintf('Root1 of the equation is x1= %10s', num2str(x1));
disp(xr1); display(x1)
[xr2, Errm2]=sprintf('Root2 of the equation is x2= %10s', num2str(x2));
disp(xr2); display(x2)
% Or an alternative way:
disp(sprintf('Root2 of the equation is x1= %10s', num2str(x1)))
disp(sprintf('Root2 of the equation is x2= %10s', num2str(x2)))
```

After executing the updated script (QuadEq1.m), you get three input prompts to enter values for *a, b,* and *c* consecutively. After you enter the entries (a =11, b=12, and c=13), you get the following results of the given equation's complex roots.

```
Solve: ax^2+bx+c=0
Enter value of a: 11
Enter value of b: 12
Enter value of c: 13
Discriminant of the equation is: -428
Root1 of the equation is x1= -0.54545+0.94037i
x1 =

     -0.54545 +    0.94037i
Root2 of the equation is x2= -0.54545-0.94037i
x2 =

     -0.54545 -    0.94037i
```

Note that in this script, fprintf() can be used instead of sprintf() in a similar manner. That would make the code simpler. Thus, let's employ fprintf() to display the complex roots of the quadratic equation and write the computation results of the quadratic equation into an external file.

The general syntax for writing data into an external file with fprintf() is as follows:

```
FileID=fopen('filename.extension', 'permission')
fprintf(FileID, 'format', data);
...
fclose(FileID)
```

Note that 'permission' can be 'w' for writing; 'r' for reading; 'r+' for reading and writing; or 'a' for appending permissions. MATLAB supports several common data file types—*.txt, *.dat, *.xls, *.csv, *.jpeg, etc. The type of data is defined in 'format'. The 'format' in fprintf() is defined just like in sprint(). Using the general syntax of writing/exporting data (existing data is discriminant and the roots of the quadratic equation are saved in the workspace) into an external file, the following short script (DataWrite.m) writes the computed data (saved data in the workspace) into a file called DataWrite.txt.

```
% DataWrite.m - Write computed data into an external file
fid1=fopen('Results_QE.txt', 'w');
fprintf(fid1,'Discriminant of the equation is: %g\n', D)
fprintf(fid1,'Root1 of the equation is x1= %10s\n', num2str(x1))
fprintf(fid1,'Root2 of the equation is x2= %10s\n', num2str(x2))
fclose(fid1); open('Results_QE.txt')
```

Note that this script first creates a new external file called `Results_QE.txt` with permission to write in it. This external file (`Results_QE.txt`) has a file ID of `fid1` (note that a file ID can be named freely), through which it can be called up to write in the assigned data, `D`, and roots `x1` and `x2`.

In the `fprintf()` command, the `\n` escape character is used to write the following data in a new line. Finally, the data writing procedure ends with a file close command `fclose(fid1).` This is a standard way of exporting data of any size and of any format into an external file. This procedure of data export is covered more extensively via different examples in other chapters of the book. After executing the script, the following `*.txt` file content is displayed in the M-file editor window.

```
Discriminant of the equation is: -428
Root1 of the equation is x1= -0.54545+0.94037i
Root2 of the equation is x2= -0.54545-0.94037i
```

There are many format tools that display data and write it to external files. The demonstrated script can be improved further with data entries and by displaying/writing the calculated data. These will be discussed further with control statements in the following section.

The if, else, elseif, end and switch, case, end Control Statements

The control statements **if ... else if... else ... end** are also called conditional statements or Boolean operators. They are employed to logically select which block of the code is to be executed while running the whole script according to the given information. They are crucial programming operators. These operators are employed in building and branching structures of programs. The general structure of these control statements follows:

```
if < logical statement A is true>
          DO X Operations
elseif < logical statement B is true>
          DO Y operations
...
else
         DO W operations
end
```

This indicates that if logical statement A is true, then X operations will be executed. If logical statement B is true, Y operations will be executed. If none of the logical statements is true, W operations will be executed.

Conditional operations can be expressed in two ways—either on operational forms or on M/MLX-file forms. The conditional and logic statements in Table 2-3 are applicable to all logical operations and conditional statements used in M/MLX-files/ scripts.

Table 2-3. *Control Logical Operators and Their Descriptions*

Operation	M/MLX-File	Description
A<B	LT(A,B)	Less than
A>B	GT(A,B)	Greater than
A<=B	LE(A,B)	Less than or equal to
A>=B	GE(A,B)	Greater than or equal to
A~=B*	NE(A,B)	Not equal to
A==B	EQ(A,B)	Equality
A&B	AND(A,B)	Logical AND
A|B	OR(A,B)	Logical OR

**The ~ tilde sign followed by = means "Not equal to".*

Note The < vs. lt(), > vs. gt(), <= vs. le(), >= vs. ne(), ~= vs. ne(), == vs. eq(), & vs. and(), and | vs. or() operators can be used interchangeably and lead to no difference in code performance.

In various examples, we employ these operators interchangeably. Let's look at several examples that demonstrate how to employ the control statements given in Table 2-3.

Example 1

The clock command displays the current year/date/time of the user's computer in row matrix form. Let's display the data in a readable format with some additional textual information. Here is a possible solution script (TimeDisp.m) of the given problem with the clock command.

```
% TimeDisp.m  Time Display
display(clock);
NOW=fix(clock);% Round to nearest integers
fprintf('This year is: %g \n', NOW(1));
if NOW(2)==1
    display(['Date is January ' num2str(NOW(2))]);
elseif NOW(2)==2
    display(['Date is February ' num2str(NOW(2))]);
elseif NOW(2)==3
    display(['Date is March ' num2str(NOW(2))]);
elseif NOW(2)==4
    display(['Date is April ' num2str(NOW(2))]);
elseif NOW(2)==5
    display(['Date is May ' num2str(NOW(2))]);
elseif eq(NOW(2),6)
    display(['Date is June ' num2str(NOW(2))]);
elseif eq(NOW(2),7)            % No difference between eq() and ==
    display(['Date is July ' num2str(NOW(2))]);
elseif eq(NOW(2),8)
    display(['Date is August ' num2str(NOW(2))]);
elseif eq(NOW(2),9)
    display(['Date is September ' num2str(NOW(2))]);
elseif eq(NOW(2),10)
    display(['Date is October ' num2str(NOW(2))]);
elseif eq(NOW(2),11)
    display(['Date is November ' num2str(NOW(2))]);
```

```
else
    display(['Date is December ' num2str(NOW(2))]);
end
if NOW(4)>=12
fprintf('Current time:  %g PM - %g min - %g sec\n', ...
        NOW(4), NOW(5), NOW(6))
else
fprintf('Current time:  %g AM - %g min - %g sec\n', ...
        NOW(4), NOW(5), NOW(6))
end
```

By running the script, the following outputs are displayed in the Command Window:

```
ans =

  Columns 1 through 5

         2018            11             1             4            51

  Column 6

       34.932

This year is: 2018
Date is November 1
Current time:  4 AM - 51 min - 34 sec
```

Let's now look at the previous example with a quadratic equation and correct the display of computed roots with `disp`, `sprintf`, `fprintf`, `if`, `elseif`, and `else` in order to demonstrate how to employ these programming operators.

Example 2

Here is one possible solution (`Quad_Eq2.m`) of computing and displaying the roots of quadratic equations with additional comments.

```
% Quad_Eq2.m
% Solve quadratic equations based on coefficients of: a, b, & c
a=input('Enter value of a: ');
b=input('Enter value of b: ');
```

```
c=input('Enter value of c: ');
fprintf('Solve: (%3g)x^2+(%3g)x+(%3g)=0\n', a,b,c)
D=b^2-4*a*c;
% Roots
x1=(-b+sqrt(D))/(2*a); x2=(-b-sqrt(D))/(2*a);
if lt(D,0)
disp('This equation does not have real value roots!');
Dm=sprintf('Because discriminant is negative. D = %g', D); disp(Dm)
fid1=fopen('Results_QE.txt', 'w');
fprintf(fid1, 'This equation does not have real value \n');
fprintf(fid1, 'roots!\n');
fprintf(fid1,'Because discriminant is negative. D = %g\n', D);
fprintf(fid1,'Complex Root1: x1= %10s\n', num2str(x1));
fprintf(fid1,'Complex Root2; x2= %10s\n', num2str(x2));
fclose(fid1); open('Results_QE.txt');
elseif eq(D,0)
disp('This equation has one unique root! ');
disp('Because discriminant is zero: D = 0 ');
fid1=fopen('Results_QE.txt', 'w');
fprintf('This equation has one unique root! \n');
fprintf(fid1,'Because discriminant is zero: D = 0 \n');
fprintf(fid1,'Unique Root: x = %g \n', x1);
fclose(fid1); open('Results_QE.txt');
else
    disp('This equation has two roots! ');
    Dm =sprintf('Because discriminant is: D = %g ', D); disp(Dm);
    fid1=fopen('Results_QE.txt', 'w');
    fprintf(fid1,'This equation has two roots \n');
    fprintf(fid1,'Because discriminant is: D = %g \n', D);
    fprintf(fid1,'Real Root1 is: x1= %g \n', x1);
    fprintf(fid1,'Real Root2 is: x2= %g \n', x2);
    fclose(fid1); open('Results_QE.txt');
end
```

This script employs a few combinations of the `if, elseif, else, end` conditional statements to provide legible output. Also, verbal versions (`gt`, `lt`, and `eq`) of the ≤, ≥ , < , > , = logical operators are employed. Yet the robustness of the script is still insufficient. For instance, the user can enter wrong entries by typing the value of *c* for *b* or *a* and vice versa, or users can skip entering any of the coefficients. As a result, the script would produce the wrong output. To attain full robustness of the script for different case scenarios and user entries, including mistakes entered by the users, the `while ... end` loop control statement will be employed. We discuss it in the following section.

Example 3

Warn drivers about their speed based on where they are driving (such as in a school zone, residential/business district, unpaved town road, two- and multi-lane highways, and interstates). The speed limits from North Dakota, USA are used in this example.

```
% Drive_Safe.m gives a warning sign what is the speed limit and
% how to behave in specific roads, highways, expressways, etc.
% E.g. CS is the only input variable
% Speed Limit [1] School area: 20 [mph]
% Speed Limit [2] Residential and business area: 25 [mph]
% Speed Limit [3] Town gravel roads: 55 [mph]
% Speed Limit [4] Two-lane highways: 65 [mph]
% Speed Limit [5] Multi-lane highways: 70 [mph]
% Speed Limit [6] Divided Interstate: 75 [mph]

CS = input('Enter your car"s current speed in [mph]:    ');
disp('Choose WHERE you are driving:  ')
disp('[1] School area;  [2] Residential and Business Area; ')
disp('[3] Town gravel roads; [4] Two-lane Highways; ')
disp('[5] Multi-lane Highways; [6] Divided Interstate Roads; ')
DZone = input('Enter your drive zone, e.g. 1, 2, 3, ... 6:  ');
if isempty(DZone) || DZone >7 || DZone<0
    warndlg('Not clear where you are driving!')
elseif CS ==0
    warndlg('Your car is not moving')
```

```
elseif CS<0
    warndlg('Your car is moving in a rear direction that is DANGEROUS! ')
else
    if DZone ==1 && CS > 0 && CS<10
       fprintf('Your speed = %g [mph] is too slow \n ', CS)
      fprintf('even if it is during the student arrival/departure time \n')
       warndlg('Speed UP to make your car speed around 15 mph!')
    elseif  DZone ==1 && CS > 10 && CS<20
        fprintf('Your speed = %g [mph] is OK in a school area \n', CS)
        fprintf('during the student arrival/departure time \n')
        warndlg('Keep your car speed around 15...20 mph!')
    elseif DZone ==1 && CS >20
    fprintf('Your speed = %g [mph] is TOO fast for a school area \n', CS)
    warndlg('DANGER! Slow DOWN up to 20 mph!')
    elseif DZone ==2 && CS > 15 && CS <20
fprintf('Your speed = %g [mph] is too slow for a residential area!\n', CS)
warndlg('Speed up! Speed limit for residential/business areas is 25 mph')
    elseif DZone == 2 && CS > 20 && CS < 28
    fprintf('Your speed = %g [mph] is adequate for a residential area!',
CS)
    warndlg(' Do not speed over 25 mph!')
    elseif DZone == 2 && CS > 28
    fprintf('Your speed = %g [mph] is higher for a residential area!\n',
CS)
    warndlg(' DANGER!  Slow DOWN and do not speed over 25 mph!')
    elseif DZone == 3 && CS < 48  && CS > 25
    fprintf('Your speed = %g [mph] is slower for gravel town roads!\n', CS)
    warndlg(' Speed up to 55 mph!')
    elseif DZone == 3 && CS > 48  && CS < 58
fprintf('Your speed = %g[mph] is within limits for gravel town roads!\
n',CS)
        warndlg(' Keep your speed around 55 mph!')
    elseif DZone == 3 && CS > 58
        fprintf('Your car speed = %g [mph] is beyond \n', CS)
        fprintf('the limits for a gravel town roads! \n')
        warndlg(' DANGER!  Slow DOWN up to 55 mph!')
    elseif DZone == 4 && CS > 55  && CS < 60
        fprintf('Your speed = %g [mph] is slower for two-lane HW!\n', CS)
        warndlg(' Speed up to 65 mph!')
    elseif DZone == 4 && CS > 60  && CS < 68
fprintf('Your speed = %g[mph] is within the limits for two-lane HW! \n', CS)
        warndlg(' Keep your speed around 65 mph!')
    elseif DZone == 4 && CS > 68
        fprintf('Your speed = %g [mph] is beyond  \n', CS)
        fprintf('the limits for two-lane HW! \n')
        warndlg(' DANGER!  Slow DOWN up to 65 mph!')
    elseif DZone == 5 && CS > 55  && CS <65
        fprintf('Your speed = %g [mph] is slower than \n' , CS)
        frpintf('the speed limits for multi-lane lane HW! \n')
        warndlg(' Speed up to 70 mph!')
    elseif DZone == 5 && CS > 65  && CS <75
        fprintf('Your speed = %g [mph] is within \n', CS)
        fprintf('the speed limits for multi-lane lane HW! \n')
        warndlg(' Keep your speed around 70 mph!')
    elseif DZone == 5 && CS > 75
        fprintf('Your speed = %g [mph] is beyond \n', CS)
        frintf('the speed limits for multi-lane lane HW! \n')
        warndlg(' DANGER! Slow DOWN up to 70 mph!')
    elseif DZone == 6 && CS > 60  && CS <70
        fprintf('Your speed = %g [mph] is slower than \n', CS)
        fprintf('the speed limits for Divided Interstate in ND! \n')
        warndlg(' Speed up to 70 mph!')
    elseif DZone == 6 && CS > 70  && CS <78
        fprintf('Your speed = %g [mph] is within \n', CS)
        fprintf('the speed limits for Divided Interstate in ND! \n')
        warndlg(' Keep your speed around 75 mph!')
    else
        fprintf('Your speed = %g [mph] is beyond  \n', CS)
        fprintf('the speed limits for Divided Interstate in ND! \n')
        warndlg(' DANGER!  Slow DOWN up to 75 mph!')
    end
end
```

In this answer script, the conditional statements and Boolean operators `if ... elseif ... else ... end` are used twice. The first `if... elseif ...elseif... else ...` identifies whether the user has specified where he is driving (`DZone`) and what his speed (`CS`) is. The second `if... elseif ... ... else ... end` starts only if the first `if...elseif ...` conditions are met. In other words, only when the user has entered her driving speed and zone.

If the user information is valid for the pre-defined six driving zones, then under `elseif CS ==0`, it verifies that the given car is moving. If it is moving, `elseif CS<0` indicates in which direction (backward for negative speed values). The last `else` evaluates the positive values of speed.

Beyond this first `else`, all the positive values (forward movement of a car) and a second loop of `if ... elseif ... ... else ... end` are executed. They define the speeding level and provide warning signals to the driver with respect to where (zones and roads) he is driving and what his speed is.

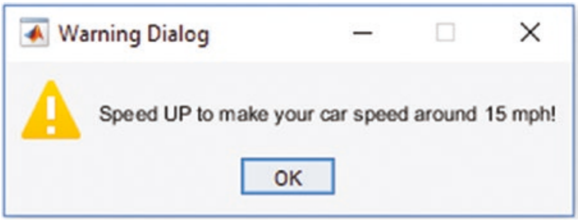
Let's test this script with different input speed values and the different driving zones

Case 1

The given car is moving at 9 mph in a school zone.

```
Enter your car"s current speed in [mph]:    9
Choose WHERE you are driving:
[1] School area;  [2] for Residential and Business Area;
[3] Town gravel roads; [4] Two-lane Highways;
[5] Multi-lane Highways; [6] Divided Interstate Roads;
Enter your drive zone, e.g. 1, 2, 3, ... 6:  1
Your car speed = 9 [mph] is too slow
 even if it is during the student arrival/departure time
```

The following warning dialog box is displayed.



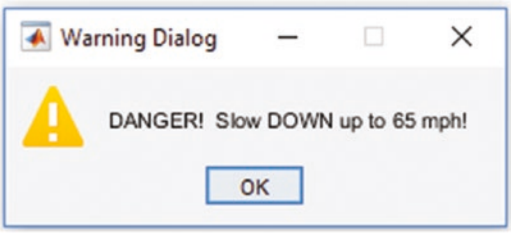
Speed UP to make your car speed around 15 mph!

Case 2

The given car is moving at 72 mph on a two-lane highway.

```
Enter your car"s current speed in [mph]:    72
Choose WHERE you are driving:
[1] School area;  [2] for Residential and Business Area;
[3] Town gravel roads; [4] Two-lane Highways;
[5] Multi-lane Highways; [6] Divided Interstate Roads;
Enter your drive zone, e.g. 1, 2, 3, ... 6:  4
Your car speed = 72 [mph] is beyond
the limits for two-lane HW!
```

The following warning dialog box is displayed.



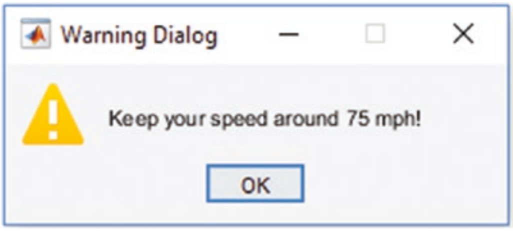
DANGER! Slow DOWN up to 65 mph!

Case 3

The given car is moving at 77 mph on a divided interstate highway.

```
Enter your car"s current speed in [mph]:    77
Choose WHERE you are driving:
[1] School area;  [2] for Residential and Business Area;
[3] Town gravel roads; [4] Two-lane Highways;
[5] Multi-lane Highways; [6] Divided Interstate Roads;
Enter your drive zone, e.g. 1, 2, 3, ... 6:  6
Your car speed = 77 [mph] is within
the speed limits for Divided Interstate in ND!
```

The following warning dialog box is displayed.



Keep your speed around 75 mph!

Another set of control statements are `switch`, `case`, `otherwise`, and `end`. These statements have one big advantage over `if`, `else`, `elseif`, and `end`. They can handle many cases and simplify the code. They have the following general syntax structure:

```
switch expression of scalar or string (variable)
     case variable value1
     A statements
     case variable value2
     B statement
     case variable value3
     C statement
...
     otherwise
     X, ... W statements
end
```

Let's look at a simple example to understand how these conditional statements—`switch`, `case`, `otherwise`, and `end`—operate.

Example 4

Determine whether an integer the user entered (from 1 to 50) is odd or even or a prime number. The following script (called `Num50Type.m`) defines the type of integer from 1 to 50.

```
% Num50Type.m - Defines entered integer type
clear all; clearvars; clc
N=input('Enter ANY integer up to 50:  ');
if N<=50
```

```
fprintf('Your entry is: %d \n', N)
switch N
case {2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32, 34 36 38 40 43 44 46 48 50}
 fprintf('you have entered %g which is EVEN number \n', N)
case {9,15,21,25,27,33,35,39,45,49}
fprintf('You have entered %g which is ODD number but not prime\n', N)
otherwise %N=[3,5,7,11,13,17,19,23,29,31,37,41,43,47]
 fprintf('You have entered %g which is PRIME number \n ', N)
end
else
disp('This code works with integers up to 50 to identify if they are even,
odd or prime!')
end
```

If you were to enter 23 as the integer, you would get the following output in the Command Window.

```
Enter ANY integer up to 50:  23
You have entered 23 which is PRIME number
```

This script correctly defines all entered numbers up to and including 50. However, you can improve its robustness by adding conditional statements and `while` loops to verify whether the integer is within the range of 1 to 50.

The robustness of scripts using control branching statements `if, elseif, else, end` can be improved by employing them within loop statements, such as `while ... end]` and `for ... end`. This is discussed via examples in the following section.

Loop Control Statements: while, for, continue, break, and end

There are two types of control statements—`while...end` and `for...end`. The first loop control statement is `while...end` and its general syntax is the following:

```
while expression is NOT true
perform A, B, C,..., W operations
end
```

Unlike `if .. elseif ... else... end` statements, a `while...end` loop keeps executing the operations until the expression behind the `while` operator becomes valid. Once the expression becomes true/valid, the execution process halts.

Example 1

Compute the sum of all odd numbers from 1 to 100 using the `while ... end` loop control (called `Sum100.m`).

```
% Sum100.m sums of all odd numbers with while
Sum=0; N=1;
while ne(N, 100)
if ne(mod(N,2),0) % Verifies all odd numbers are added.
    Sum=Sum + N;
else              % Verifies if the number is even, NO sum.

end
    N=N+1;
end
disp(['Sum of all odd numbers 1...100 is:  ', num2str(Sum)]);
```

This loop-based script (`Sum100.m`) will keep running until the iteration process reaches the final value of 100. The `if ... else ... end` conditional statements verify whether the number is odd or even with the help of `mod()`. It is used to compute the remainder of `N/2`. If the number is odd, the sum will take place; otherwise, it will be ignored.

Note In many summation types of exercises that use `while ... end` and `for ... end` iteration loops, users have difficulty understanding the computation algorithm and writing code to perform the summation process. The summation algorithm is not intuitive for many beginners in programming. The difficulty comes from the point that they overlook the necessity of assigning `0` (value) to the sum variable in the initial step of the summation process. That means that adding any number to `0` gives the same number back. Subsequently, within a `for ... end` or `while ... end` loop, this summation variable will get a new value in each iteration. For example, in the previous example:

Step 1. `Sum(1) = 0`

Step 2. `Sum(2) = Sum(1)+N(1) = 0+1=1`; when `N(1)= 1` and `ne(mod(N(1),2), 0)` is valid

Step 3. `Sum(3)=Sum(2)=1`; when `N(2)= N(1)+1=2` and because `N(2)/2 = 1` and remaining `0` ← `ne(mod(N,2), 0)` is not valid

Step 4. `Sum(4)=Sum(3)+N(3)=1+3 =4`; when `N(3)=N(2)+1=2+1=3` and `ne(mod(N(3),2), 0)` is valid

Step 5. `Sum(5)=Sum(4)=4`; when `N(4)=N(3)+1=3+1=4`; because `N(4)/2 = 2` and remaining `0` ← `ne(mod(N(4),2), 0)` is not valid

Step 6. `Sum(6)= Sum(5)+N(5)=4+5=9`; when `N(5)=N(4)+1=4+1=5` and `ne(mod(N(5),2), 0)` is valid

And so on. The iteration process continues until `N` equals 100.

The final output of the script in the Command Window is:

```
Sum of all odd numbers 1...100 is:  2500
```

Example 2

In order to better understand how to employ a control loop statement `while ... end`, let's take the example of solving a quadratic equation: $ax^2 + bx + c = 0$

Problem: Users can enter the values of *a, b,* and *c* in the wrong order or can mistype their values or enter wrong numbers.

To make the program more robust and resolve any wrong entries, the conditional operators `if ... elseif ... else ...end` are employed along with the `while ... end` loop iteration operators.

Here is one possible solution to the problem. The script (called `Quad_Eqn.m`) is very robust. It accepts different entries and computes all possible roots, real and complex, and then displays them correctly in the Command Window. It resolves many possible wrong entries, such as wrong values, wrong data sizes, or mixed up values of *a, b, and c*.

Note that a first `while` loop statement verifies the size of the entry row array containing the values of *a, b, and c* and a second one with conditional operator `if ... else ... end` verifies the order of the *a, b, and c* entries.

```
% Quad_Eqn.m solves quadratic equations based on the user entries for:
% a, b, c
clearvars; clc
SIZE_chk = 0;
while SIZE_chk ~=1
    disp('Solution of: ax^2+bx+c=0')
    abc=input('Enter values as [a, b, c]:   ');
    SIZE_abc=numel(abc);
    if SIZE_abc ==3
        SIZE_chk=1;
        a=abc(1);
        b=abc(2);
        c=abc(3);
        CorrectEntry=0;
        while CorrectEntry~=1
            if SIZE_abc == 3
                CorrectEntry=1; CorrectABC=0;
                % Check the order of a, b, c
                while CorrectABC~=1
                    disp('Is this the equation you"d like to find roots?')
                    disp('   ')
                    Eqn=sprintf(' %g*x^2+(%g)*x + (%g)=0', a, b, c);
                    disp('   ')
                    disp(Eqn);
            CorrectABC=input('If yes, enter 1, otherwise any integer!  ');
                    disp('   ')
                    if CorrectABC~=1
                        abc=input('Enter values as [a, b, c]:   ');
                        a=abc(1); b=abc(2); c=abc(3);
                    end
                end
                D=b^2-4*a*c;
                % Roots
                x1=(-b+sqrt(D))/(2*a); x2=(-b-sqrt(D))/(2*a);
                if lt(D,0)
                    disp('The equation has no real roots!');
                    Dm=sprintf('Because discriminant <0. D = %g', D);
                    disp(Dm)
                    % Display the results
                    fprintf(' %g*x^2+(%g)*x + (%g)=0\n', a, b, c);
                    fprintf('It does not have real roots!\n');
                    fprintf('Because its discriminant <0. D=%g\n',D);
fprintf('Complex Roots are: x1=%10s; x2=%10s\n', num2str(x1),num2str(x2));
                elseif eq(D,0)
                    disp('This equation has one unique root! ');
                    disp('Because discriminant is zero. D=0 ');
                    fprintf('%g*x^2+(%g)*x + (%g)=0\n', a, b, c);
                    fprintf('It has a unique root! \n');
                    fprintf('Because discriminant is "0". D=0 \n');
                    fprintf('Unique Root: x = %g \n', x1);
                else
                    disp('This equation has two real roots! ');
     [Dm, Errm2]=sprintf('Because discriminant >0. D = %g ', D); disp(Dm);
                    fprintf(' %g*x^2+(%g)*x + (%g)=0\n', a, b, c);
                    fprintf('It has two real roots \n');
                    fprintf('Because discriminant >0. D = %g\n', D);
     fprintf('Roots of the equation are x1= %g; x2= %g \n', x1, x2);
                end
            else
                CorrectEntry=0;
                disp(' a, b, c cannot have more than 1 element (value)');
                disp('Re-enter values of a, b, c');
                disp('    ')
                abc=input('Enter values as [a, b, c]:   ');
                a=abc(a); b=abc(2); c=abc(3);
                sa=numel(1); sb=numel(b); sc=numel(c);
            end
```

```
        end
    else
fprintf('Your entry is wrong \n')
fprintf('Enter the correct entries for [a, b, c] as a row vector in [ ]: ')
    end
end
```

The script is tested for robustness with intentionally wrong entries for *a, b,* and *c,* as stated in Cases 1 and 2.

```
Solution of: ax^2+bx+c=0
Enter values as [a, b, c]:   [12 1 1 1 ]
Your entry is wrong
Enter the correct entries for [a, b, c] as a row vector in [ ]:  Solution
of: ax^2+bx+c=0
Enter values as [a, b, c]:   [12 1 1 1 ]
Your entry is wrong
Enter the correct entries for [a, b, c] as a row vector in [ ]:  Solution
of: ax^2+bx+c=0
Enter values as [a, b, c]:   [1 2 1 1 ]
Your entry is wrong
Enter the correct entries for [a, b, c] as a row vector in [ ]:  Solution
of: ax^2+bx+c=0
Enter values as [a, b, c]:   [1 2 1  ]
Is this the equation you"d like to find roots?

 1*x^2+(2)*x + (1)=0
If yes, enter 1, otherwise any integer!  0

Enter values as [a, b, c]:   [1 1 2]
Is this the equation you"d like to find roots?

 1*x^2+(1)*x + (2)=0
If yes, enter 1, otherwise any integer!  1

The equation has no real roots!
Because discriminant <0. D = -7
 1*x^2+(1)*x + (2)=0
```

```
It does not have real roots!
Because its discriminant <0. D=-7
Complex Roots are: x1=-0.5+1.3229i;   x2=-0.5-1.3229i
```

From this simple example, it is clear that `while ... end` loop operations have more power than `if ... elseif ... else ... end` operators to enhance the robustness of scripts.

A second loop control statement is `for ... end` and it has the following general syntax:

```
for i=startloop:stepsize:endloop
     Iteration 1
     Iteration 2
...
     Iteration N
end
```

The `for ... end` loop control statement works in the following fashion. It starts executing every statement of `1, 2, ..., N` starting from `startloop` for every step, with a step equal to `stepsize`, until the loop iteration reaches the value of `endloop`, which corresponds to `N` number of iterations.

Note If the step size is not specified when you're assigning vector spaces or index ranges in `for ... end` loops, the default step size is 1.

Example 3

Compute the sum of all the odd numbers within [1, 20] using a `for ... end` loop (called `Sum20odd.m`).

```
% Sum20odd.m sums odd numbers within 0...20 with for loop
clearvars;
S=0;
for N=1:20;
```

```
if rem(N,2)==0 %Verifies if the number is even, then no sum.
        S=S;
else           %All odd numbers are added:
        S=S+N;
end
end
fprintf('Sum of all odd numbers 1 to %g is equal to %g \n', N, S)
```

The `if ... else ... end` conditional statements determine if the number is odd. If the number is odd, it is added to the summation variable (`S`). Otherwise, nothing will be added to the variable. The algorithm behind this script is the same one in Example 1 and this short script is an alternative solution to Example 1. The only difference here is that the number of odd numbers is 20.

Two alternative, simpler solutions of this example using `for ... end` and `while ... end` are as follows:

```
% Sum20odd_alt.m sums odd numbers within 0...20 with [for ... end] loop
clearvars;
S=0;
for N=1:2:20
    S=S+N;
end
fprintf('Sum of all odd numbers 1 to 20 is equal to %g \n', S)
% Sum of odd numbers within 0...20 with [while ... end] loop
clearvars;
S=0; jj=1;
while le(jj,20)
    S=S+jj; jj=jj+2;
end
fprintf('Sum of all odd numbers 1 to 20 is equal to %g \n', S)
```

The computation algorithm of this script for `for ... end` is:

1. `S(1) = 0`
2. `S(2)=S(1)+N(1)=0+1=1` when `N(1)=1`

3. `S(3)=S(2)+N(2)=1+3=4` when `N(2)=3` because the step size is equal to 2 (i.e., `N(2)=N(1)+2=1+2=3`)
4. `S(4)=S(3)+N(3)=4+5=9` when `N(3)=5` because `N(3)=N(2)+2=3+2=5`
5. And so on... The iteration loop runs until `N=20`. In fact, in this case, `N` cannot be 20 because the step size is 2. So the maximum value of N is 19.

Finally, the output of these scripts is identical:

```
Sum of all odd numbers 1 to 20 is equal to 100
```

As demonstrated, `for ... end` can be easily substituted by the `while ... end` loop and vice versa; their efficiencies are similar. However, in some specific examples (e.g., in Example 2), the robustness gained by using the `while ... end` loop cannot be attained using `for ... end`. Theoretically in some problems (in Example 2), `while ... end` can be substituted by `for ... end` when the number of iterations is unknown.

Note In many loop iteration-based computation problems, `for ... end` can be easily substituted with the `while ... end` loop and vice versa. Their computation efficiencies are similar. However, there are many specific cases where `while ... end` operations are employed to attain the high robustness to the script, and `for ... end` cannot provide as much robustness and flexibility. This is illustrated in Example 2 in this section.

It is recommended to use `ii` or `jj` in loop iterations for indices in order to not confuse them with imaginary numbers, which use `i` and `j` as reserved in MATLAB by default. If these letters (`i` and `j`) are employed for indices, then `sqrt(-1)` can be used for imaginary numbers or you can clear these variables from the workspace.

Example 4

Evaluate square and cubic powers of even numbers with the loop control statement for, end.

Here is one possible solution script (called SqCube10.m). Note in this example, startloop=2, stepsize=2, and endloop=10:

```
% SqCube10.m computes square and cube of 2...10
%%  [for  ... end]
for k=2:2:10
    Square = k^2;
    Cube   = k^3;
    fprintf('Square of %g is %g \n', k, Square);
    fprintf('Cube of %g is %g \n'  , k, Cube);
end
%% Alternative way: [while ... end]
jj =2;
while le(jj, 10)
     Square = jj^2;
    Cube   = jj^3;
    fprintf('Square of %g is %g \n', jj, Square);
    fprintf('Cube of %g is %g \n'  , jj, Cube);
    jj =jj+2;
end
```

When the script is executed, the following will be displayed in the Command Window.

```
Square of 2 is 4
Cube of 2 is 8
Square of 4 is 16
Cube of 4 is 64
Square of 6 is 36
Cube of 6 is 216
Square of 8 is 64
Cube of 8 is 512
Square of 10 is 100
Cube of 10 is 1000
```

Example 5

Compute the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}\ldots$ by using `for ... end` and `while ... end`.

Here are four possible solution scripts of the given exercise gathered within one script called `SumSeries.m`.

```
% SumSeries.m computes a sum of Series:
%% Version A. [for ... end]
clearvars
sign=1; sum_ser=0;
n=input('Enter number of series to compute:  ');
for N=1:n
    sum_ser=sum_ser+sign/N; sign=-sign;
end
fprintf('Sum of %g series is equal to %2.5f \n', n, sum_ser)
%% Version B. [while ... end]
clearvars
S=0;
N=input('Enter number if series to compute:  ');
ii=1;
while ii~=N+1
if mod(ii,2)==0
        S=S-1/ii;
else
        S=S+1/ii;
end
    ii=ii+1;
end
fprintf('Sum of %g series is equal to %2.5f \n', ii, S)
%% Version C. [for ... end]
clearvars
Sum=1;
n = input('Enter number of series to compute:  ');
for k=2:n
    if eq(mod(k,2), 0)
        Sum=Sum-1/k;
    else
        Sum=Sum+1/k;
    end
end
fprintf('Sum of %g series is equal to %2.5f  \n', k, Sum)
```

All of these solution methods with `for ... end` and `while ... end` result in identical solutions. It must be noted that the MATLAB function `mod(N, 2)` defines whether the number is even or odd. It works in this script to separate even and odd components of `ii` and `k`.

In all of these examples, the final values from the iteration processes are saved. What about saving all the values from all iteration steps?

```
%% Version A. [for ... end]
clearvars
Sum=1;
n = input('Enter number of series to compute:  ');
k=2:1:n;
for jj=1:numel(k)
    if eq(mod(k(jj),2), 0)
        Sum(jj+1)=Sum(jj)-1/k(jj);
    else
        Sum(jj+1)=Sum(jj)+1/k(jj);
    end
    fprintf('Sum of %g series is equal to %2.5f  \n', k(jj), Sum(jj+1))
end
disp('         ')
fprintf('Sum of %g series is equal to %2.5f \n', k(end), Sum(end))
```

```
%% Version B. [for ... end]
clearvars
sign=1; sum_ser=0;
n=input('Enter number if series to compute:  ');
for N=1:n
    sum_ser=sum_ser+sign/N; sign=-sign;
    S(N)=sum_ser;
    fprintf('Sum of %g series is equal to %2.5f \n', n, sum_ser)
```

```
end
%% Version C. [while ... end]
clearvars
S=0;
N=input('Enter number if series to compute:  ');
ii=1;
while ii~=N+1
if mod(ii,2)==0
        S(ii+1)=S(ii)-1/ii;
        else
        S(ii+1)=S(ii)+1/ii;
        end
    fprintf('Sum of %g series is equal to %2.5f \n', ii, S(ii+1))
    ii=ii+1;
end
%% Version D. [while ... end]
clearvars
S=0;
N=input('Enter number if series to compute:  ');
ii=1;
while ii~=N+1
if mod(ii,2)==0
        S=S-1/ii;
        Ss(ii)=S;
else
        S=S+1/ii;
        Ss(ii)=S;
end
    fprintf('Sum of %g series is equal to %2.5f \n', ii, Ss(ii))
    ii=ii+1;
end
```

One of the easiest and most straightforward approaches to preserving all values from all iterations is to initiate a new series of variables, e.g. Sum(jj), S(N), and Ss(ii). When saving all values of calculated outputs within loops, it is salient to interpret Sum(jj+1), Sum(jj), S(ii+1), and S(ii) correctly. For example, Sum(jj) takes the value of Sum=1 and Sum(1+1) = Sum(1)-1/k(1) equals Sum(2) = 1-1/2 = ½. In the next step (Step 2), Sum(3)=Sum(2) +1/k(2) equals Sum(3) =1/2+1/3 =5/6, and so forth. In Version B, S(N) preserves every value of S from every iteration within 1:n.

Example 6

Compute the expression $f(t) = e^{sin(3t)}$ $t = 0 : 0.001 : 6.28$; by employing the for ... end and while ...end loop iteration operators and preserve all of the values from all iterations. From every iteration, one value of $f(t)$ is saved, corresponding to each value of t.

```
%% Ex6_FOR_WHILE.m
clearvars
% Ver 1
ii =1;
for t=0:.001:6.28
    f(ii)=exp(sin(t));
    ii=ii+1;
end
% Ver 2
t=0:.001:6.28;
for k=1:numel(t)
    f(k)=exp(sin(t(k)));
end
%% Ver 3
jj=0;
while jj~=numel(t)
    f(jj+1)=exp(sin(t(jj+1)));
    jj=jj+1;
end
% Ver 4
m=1;
while m~=numel(t)+1
    f(m)=exp(sin(t(m)));
    m=m+1;
end
```

Note It is salient to use `f(ii)`, `f(k)`, `f(jj+1)`, or `f(k)` while computing *f*(*t*) within `for ... end` and `while ... end` loops. This preserves all the values of *f*(*t*) from the iteration process with respect to the values of *t*. If only `f` is used instead of `f(ii)`, `f(k)`, `f(jj+1)`, or `f(k)`, then only the last value of *f*(*t*) will be saved.

One of the most common mistakes that users make when working with loop operators (`for ... end` and `while ... end`), is that they overlook the index `(ii)`, `(k)`, `(jj)`, or `(m)` after the main variable. For example, `f(ii)`, `f(k)`, `f(jj+1)`, and `f(k)` in the previous example, in order to collect all values from the entire iteration process.

Another common mistake that users make while working with loop operators is that they assign the wrong index values to `(ii)` and `(jj)`. For example, they assign negative values, or start them with `0`, or use non-integer values or mismatched sizes. In the previous example Ver 3, we overlook assigning the value `0` to `jj` before `while` or assign `jj=0` and at the same time assign `f(jj)`. Because `f(0)` is meaningless, a variable cannot have an index of `0`. The indexes can be 1, 2,3, ... 10^{12}, ...10^{25} + 1, ... but not ..., -3,-2, -1, 0 or 0.12, 2.1, 3.35, 5.5, 7/8, 100/899, etc.

There is a good alternative approach to collecting every value from every computation within a loop, which is to assign a new variable with an index. For example, this can be attained via the following:

```
%% Ex6_FOR_WHILE.m
clearvars
% Ver 1
ii =1;
for t=0:.001:6.28
    f=exp(sin(t));
    F1(ii)=f;
    ii=ii+1;
end
```

```
% Ver 2
t=0:.001:6.28;
for ii=1:numel(t)
    f=exp(sin(t(ii)));
    F2(ii)=f;
end
%% Ver 3
jj=0;
while jj~=numel(t)
    f=exp(sin(t(jj+1)));
    F3(jj+1)=f;
    jj=jj+1;
end
% Ver 4
jj=1;
while jj~=numel(t)+1
    f=exp(sin(t(jj)));
    F4(jj)=f;
    jj=jj+1;
end
```

Note that in this example, the best and most efficient computation approach is vectorization, as shown.

```
t=0:.001:6.28
f=exp(sin(t));
```

Note For the efficiency of computation processes and to improve your code, it is recommended you avoid using the `for ... end` and `while ...end` loop control statements whenever feasible. Instead of these loop iterations, it is better to employ the vectorized code approach shown in Example 6.

Example 7

Let's use a famous "wheat and chessboard" story in this example. The problem is defined as follows: grains are placed in each chessboard square in the order of one grain in the first square, two grains in the second, four grains in the third, eight grains in the fourth, 16 gains in the fifth, 32 grains in the sixth, and so on, doubling the number of grains upon each subsequent square of the chessboard.

The sum of all grains to be placed on the chessboard is 1+2+4+8+16+32 ... and so forth. This can also be expressed as sum of powers of 2 $2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + \dots 2^{63}$ and is equal to $2^{64} - 1$ = 18,446,744,073,709,551,615. This is indeed a huge number of grains. In simple calculations, if one grain weighs 0.025 grams, the whole amount would weigh over 461 billion tons. That would be a mountain of wheat grains bigger than the Mount Everest. Let's compute the number of grains using the `for ... end` and `while ... end` loops.

```
% ChessInventorReward.m
% Example: Grains of Wheat on the Chess Board
%% Version A. [while ... end] loop
clearvars
S=1; Ncell=64; jj=1;
while jj~=Ncell
        S=S+2^jj; jj=jj+1;
end
display(['Number of cells: ', num2str(Ncell)])
display('& total sum of grains is: '), disp(uint64(S))
% Now test our results with a simple solution: 2^64-1;
Error=S-2^64 %#ok
%% Version B. [for ... end] loop
clearvars
S=0; Ncell=64;
for ii=0:Ncell-1
        S=S+2^ii;
end
display(['Number of cells: ', num2str(Ncell)])
display('& total sum of grains is: '), disp(uint64(S))
% Now test our results with a simple solution: 2^64-1;
Error=S-2^64 %#ok
```

Both of the solution scripts output the same number of grains = 18,446,744,073,709, 551,615 with 0 error. Note that 18,446,744,073,709,551,615 = $2^{64} - 1$ is the maximum length of any integer to be displayed correctly in MATLAB's 64-bit version, which is installed on a 64-bit computer.

Example 8

Prove that $\sum_{k=1}^{\infty}\frac{1}{k^2}=\frac{\pi^2}{6}$. Compute this summation by using `for ... end` and `while ... end` loops.

```
% SumPi6.m
%% Example: sum(1/k^2)=pi^2/6
% Version A. [while...end] loop
clearvars
S=0; jj=1; k=input('Enter number of iterations to compute:  ');
while le(jj, k)
    S=S+1/jj^2; jj=jj+1;
end
display(['Number of cells: ', num2str(Ncell)])
display('& total sum of grains is: '), disp(uint64(S))
Error = (pi^2/6)-S; display(Error)
% Example: sum(1/k^2)=pi^2/6
% Version B. [for ... end] loop
clearvars -except k
S=0;
for jj=1:k
    S=S+1/jj^2;
end
display(['Number of cells: ', num2str(Ncell)])
display('& total sum of grains is: '), disp(uint64(S))
Error = (pi^2/6)-S; display(Error)
```

Memory Allocation

When all values from all iterations are saved, it is recommended to employ memory allocation methods in order to improve the computation efficiency. Let's analyze how the computation efficiency is enhanced by the memory allocation approaches. With the memory allocation technique, the specified memory is allocated to the computed variable with a user-specified size, and during the loop computation, all the variable's values are recorded in the pre-specified memory. This speeds up the computation process considerably. Using the memory allocation technique, the exact size of the variable is created before the loop iteration, via the standard matrix/array generators (e.g., `ones()` or `zeros()`).

The general syntax/pseudo-code of the memory allocation is as follows:

```
% [FOR ... END]
M = zeros(1, n);
for ii = 1:n
M(ii) = [computation];
end
% [WHILE ... END]
MM = zeros(1, k); jj=1;
while jj ~=k
 MM(jj) = [computation];
 jj=jj+1;
end
```

This syntax can be applied to loops within loops. Let's consider the memory allocation technique in the following examples.

Note It is important to employ memory allocation techniques while using `for ... end` and `while ... end` loops with considerable computations. This will decrease computation time and save computing resources.

Example 9

In the 17th Century, Leibnitz used the series expansion of *arctan*(x) to determine an approximation of π. By using the Maclaurin Series formula, we can write it as:

$$arctan\left(x\right)\approx 0+\left(1\right)x+\frac{0}{2}x^2-\frac{2}{3!}x^3+\frac{0}{4!}x^4+\frac{0}{5!}x^5\ldots=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}\ldots$$

Considering that $\left(1\right)=\frac{\pi}{4}$, you can substitute $x = 1$ into the expression, which will yield the following expression.

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\ldots$$

That can also be written for N terms in this form:

$$\frac{\pi}{4}\cong\sum_{k=0}^{N}\frac{\left(-1\right)^k}{2k+1}=S_N$$

The task is to write a script that computes the sum (S_N) of N terms and the approximated value of the $\frac{\pi}{4}$, and then plots the iteration process by computing the difference (error) $Error=\frac{\pi}{4}-S_N$ using `for ... end` and `while ... end` loops.

Here are the possible solution scripts collected under one script `LeibnitzSeries.m` file, which compute the Leibnitz Series for an approximation of $\frac{\pi}{4}$.

```
%% LeibnitzSeries.m
% Version A.
clearvars
N=input('Enter number of terms to approximate pi/4:  ');
S=0;                       % Initial value of summation
Err=zeros(1, N);   % Memory allocation
for k=0:N-1
    p=(-1)^k;
    S=S+p/(2*k+1);
    Err(k)=pi/4-S;  % Accumulates all of Errors from all iterations
end
```

```
plot(1:N, Err), grid on
title(['\pi/4 Approximation of ', num2str(N), ' terms'])
ylabel('Error'), xlabel('Number of terms')
%% Leibnitz series
% Version B
clearvars
n=input('Enter number of terms to approximate pi/4:  ');
S=0;  % Initial value of summation
Error=zeros(1, n);  % Memory allocation
k=0;
while k<=n-1
    p=(-1)^k;
    S=S+p/(2*k+1);
    Error(k+1)=pi/4-S; % Accumulates all of Errors from all iterations
    k=k+1;
end
plot(1:n, Error), grid on
title(['\pi/4 Approximation of ', num2str(n), ' terms'])
ylabel('Error'), xlabel('Number of terms')
```

If you execute the script with N=200 terms, you'll get the plot shown in Figure 2-2, which displays the results of the series approximation.

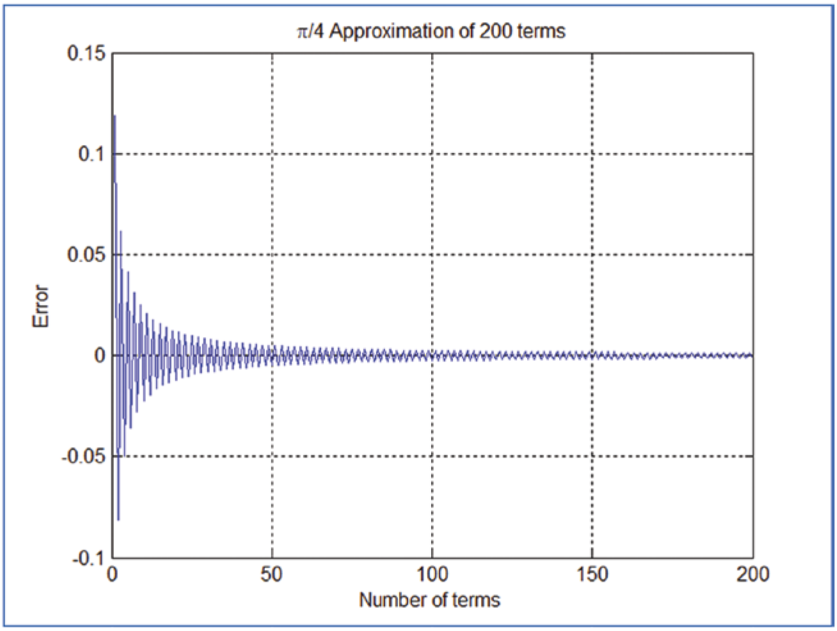


Figure 2-2. *$\frac{\pi}{4}$ approximation by 200 terms of the Leibnitz series*

It is also possible to have one loop within another. Let's look at several examples that generate loops within loops, using `for ... end` and `while ... end`.

Note It is also possible to make multiple nested `for ... end` loops.

Example 10

Create elements of a 5-by-7 matrix with pseudo-random integer numbers of 1 to 10. Here is the solution script `PseudoRandi.m`.

```
% PseudoRandi.m
for k=1:5
    for n=1:7
        AB(k,n)=randi(10,1);
    end
end
display(AB)
```

By executing the script, the following results are obtained.

```
AB =

     3     5     3     6     4     8     6
     2     1     5     6     7     2     9
     9     6     8     1     9     4     9
     4     9     9     4     7     3     3
     3     7     2     8     8    10     8
```

Note that every time the script is executed, different pseudo-random integers ranging from [1, 10] are generated. As demonstrated, for many problems, the `for, end` and `while, end` loop control statements can be employed interchangeably.

Example 11

Compute the sum of the following series by using the `for ... end` and `while ... end` loop iterations.

$f(b_n, x)= \sum_{n=1}^{\infty} \frac{b_n sin\left((2n-1)\pi x\right)}{2L}$ using `for, end` loop control statements. Take $n = 1 : 11$, $b_n = [-5...5]$, $x = [0...L]$, and $L = 10$. For x, take a step size of $\Delta x = \frac{L}{2000}$. Plot all the computation results.

Here's the solution script with a `for ... end` loop (called `Ex11_sine_series.m`).

```
%% Ver A. [FOR ... END]
clearvars, close all
n=11; b = -5:5;x=0:.01:10; f=0; L=10;
f = zeros(n, numel(x)); % Memory allocation
for k=1:n
    f=f+(b(k)*sin((2*k-1)*pi*x(:))/(2*L));
    Fun(k,:)=f;
    plot(x(:), Fun(k,:)), label{k}=(['Iteration: ' num2str(k)]);
    legend(label{:}), hold all,
end
grid on, title('sum of series'), xlabel('x'), ylabel('f(b_n, x)')
```

```
%% Ver B.
clearvars
n=11; b = -5:5;x=0:.01:10; L=10;
f = zeros(n, numel(x)); % Memory allocation
for k=1:n
    f(k, :)=(b(k)*sin((2*k-1)*pi*x(:))/(2*L));
end
Fun = sum(f(:,:));  % Obtain the final values (total sum of all).
```

Here's an alternative solution script with a `while ... end` loop (see Figure 2-3).

```
%% Ver C. [WHILE...END]
clearvars, close all
n=11; b = -5:5;x=0:.01:10; f=0; L=10; k=1;
Fun = zeros(n, numel(x)); % Memory allocation
while k<n+1
    f=f+(b(k)*sin((2*k-1)*pi*x(:))/(2*L));
    Fun(k,:)=f;
    plot(x(:), Fun(k,:)), label{k}=(['Iteration: ' num2str(k)]);
    legend(label{:}), hold all
    k=k+1;
end
grid on, title('sum of series'), xlabel('x'), ylabel('f(b_n, x)')
```

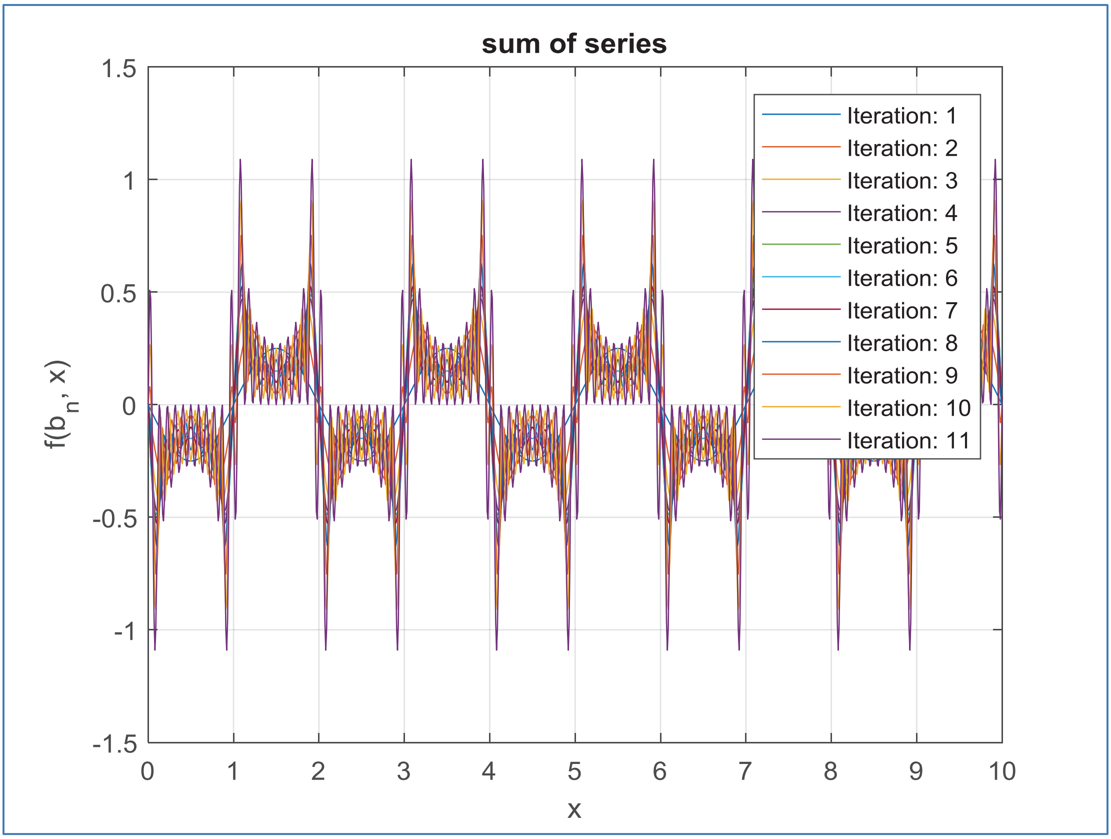


Figure 2-3. *Sum of series* $f(b_n, x) = \sum_{n=1}^{\infty} \frac{b_n sin((2n-1)\pi x)}{2L}$

Note that the commands related to plot building—such as `plot()`, `label{}`, and `legend()`—do not have an effect on the accuracy of the calculations, but they considerably slow down the computation process. Therefore, it is advisable to avoid plotting the computed data within loops if possible.

Example 12

Compute the values of the function $F(\mu,t) = \sin(2t) * \sqrt{\sin\left(\mu t \sqrt{t + t^2 + t^3}\right)}$ for $t = 0 \ldots \pi$ ($\Delta t = \pi/1000$) and $\mu = 1 \ldots 7$ by using `for ... end` and `while ... end` loop operators.

Here is the plain script (called `Ex12_FS.m`) with two loop operators.

```
% [FOR ... END]
mu=1:7; t=0:pi/1e3:pi;
F=zeros(numel(mu), numel(t)); % Memory allocation
for mu=1:7
    for ii=1:numel(t)
    F(mu, ii)=sin(2*t(ii))*sqrt(sin(mu*t(ii)*sqrt(t(ii)+t(ii)^2+t(ii)^3)));
    end
    plot(t, F(mu,:)), hold all
end
xlabel('t'), ylabel('F'), title('Sine waves'), grid on
%% [WHILE ... END]
mu=1:7; t=0:pi/1e3:pi;
F=zeros(numel(mu), numel(t)); % Memory allocation
MU = 1;
while ne(MU, 8)
    ii=1;
    while ii~=numel(t)+1
    F(MU, ii)=sin(2*t(ii))*sqrt(sin(MU*t(ii)*sqrt(t(ii)+t(ii)^2+t(ii)^3)));
    ii=ii+1;
    end
    plot(t, F(MU,:)), hold all
    MU=MU+1;
end
xlabel('t'), ylabel('F'), title('Sine waves'), grid on
```

Both iteration loops produce identical solutions and plot figures. While simulating this script, there will be complex outputs from the square root of negative values as well. Therefore, warning messages will be displayed in the Command Window indicating omitted/ignored imaginary components in the plot in Figure 2-4.

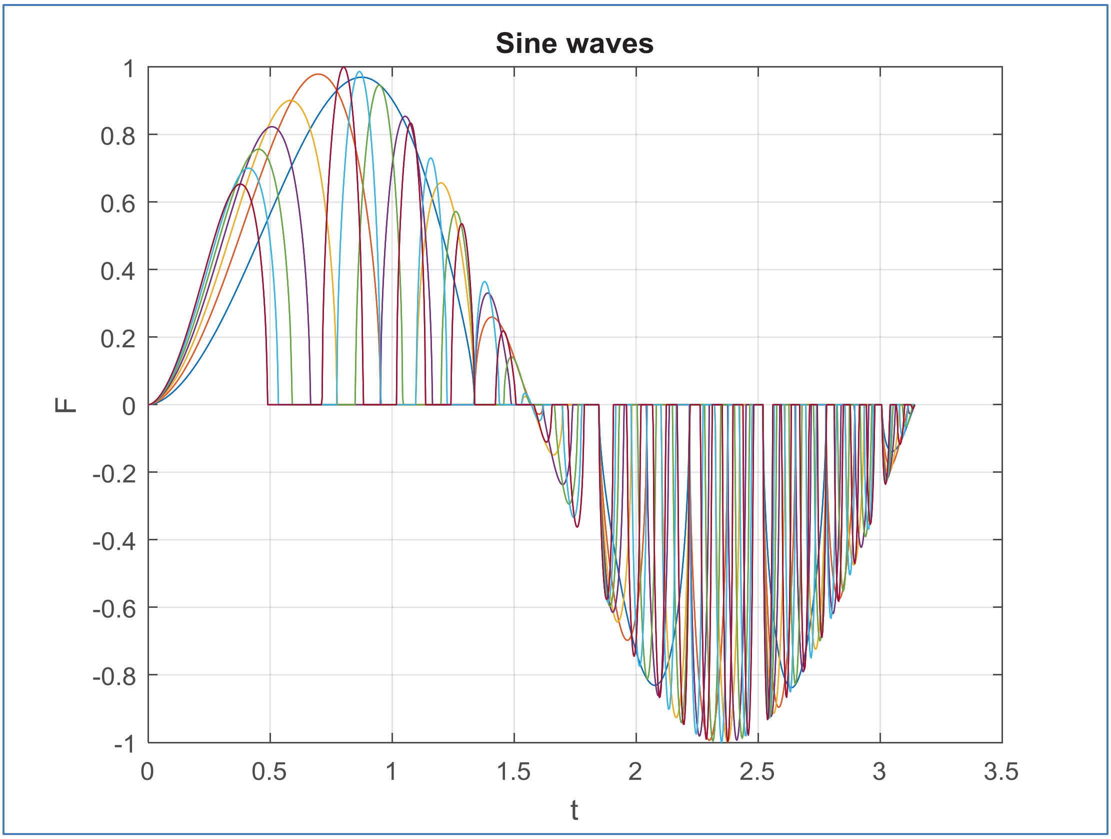


Figure 2-4. *Plot of* $F(\mu,t)=sin(2t)*\sqrt{sin\left(\mu t\sqrt{t+t^2+t^3}\right)}$ *for* $t = 0...\pi$ $(\Delta t = \pi/1000)$, $\mu = 1...7$

Example 13

Compute $f(t)=e^{\frac{-3t}{7}}$ function values for $t = 0...5$ with a time step of $\Delta t = 10^{-6}$ and $\Delta t = 10^{-3}$ and plot the computed function values against time values. The very small time step $\Delta t = 10^{-6}$ is taken to demonstrate the importance of memory allocation and loop iteration versus vectorized calculations, and $\Delta t = 10^{-3}$ is taken to show the importance of where to put the `plot()` command while using the loop iterations `for ... end` and `while ... end`.

There are four different ways (combined into one script called `Ex13_CF.m`) to solve the given exercise with the `for, end` and `while, end` loop control statements. All of these ways produce identical results, but their computation (elapsed) time differs substantially. Note that in this exercise, the `for, end` and `while, end` loop control statements are used for demonstration purposes only to explain how to employ loop statements and to improve their efficiency. For this type of exercises or any other vector computations, direct computation of vector values of *f*(*t*) is much more efficient than using loop statements.

```
% Ex13_CF.m computes Exp(-3t/7) Math function
% Solution Way 1
clc; clear all; close all

tic
t=0:1e-6:5;
m_num=length(t);
for m=1:m_num
    f(m)=exp(-3*t(m)/7);
end
Tfor=toc; fprintf('[1] FOR loop computation TIME: %g \n', Tfor)
plot(t, f, 'ro'), xlabel('t'), ylabel('f(t)'), title('t vs. f(t)')
% % Solution Way 2
clearvars, close all
tic
t=0:1e-3:5;
m_num=length(t);
for m=1:m_num
    f(m)=exp(-3*t(m)/7); plot(t(m), f(m), 'ro')
end
TforP=toc;
xlabel('t'), ylabel('f(t)'), title('t vs. f(t)')

fprintf('[2] FOR loop computation TIME with plot() in it: %g \n', TforP)
% Solution Way 3
clearvars, close all
tic
t=0:1e-3:5;
m_num=length(t); m=1;
```

```
while m<=m_num
    f(m)=exp(-3*t(m)/7); plot(t(m), f(m), 'ro')
    m=m+1;
end
Twhile=toc;
xlabel('t'), ylabel('f(t)'), title('t vs. f(t)')

fprintf('[3] WHILE loop computation TIME with plot() in it: %g \n', Twhile)
% Solution WAY 4. Improved by memory allocation.
clearvars, close all
tic
t=0:1e-6:5;
m_num=length(t); f=zeros(1,m_num-1);
for m=1:m_num
    f(m)=exp(-3*t(m)/7);
end
plot(t,f, 'bo'); TforIm=toc;
xlabel('t'), ylabel('f(t)'), title('t vs. f(t)')
fprintf('[4] Improved FOR loop TIME with memory allocation: %g\n', TforIm)
% Solution WAY 5. Vectorized method
clearvars, close all
t=0:1e-6:5;
tic
f=exp(-3*t/7);
Tvec=toc; plot(t, f, 'bo')
xlabel('t'), ylabel('f(t)'), title('t vs. f(t)')

fprintf('[5] Recommended VECTORIZED method time: %g\n', Tvec)
```

If you were to execute the `Ex13_CF.m` script on a personal computer (with processor: Intel COREi5, RAM: 4GB, OS: Windows 7, in MATLAB 7.9.0), the following computation time results would be obtained in the Command Window. Note that the plot figure is not shown here.

```
[1] FOR loop computation TIME: 0.532263
[2] FOR loop computation TIME with plot() in it: 9.43334
[3] WHILE loop computation TIME with plot() in it: 9.30924
```

```
[4] Improved FOR loop TIME with memory allocation: 0.205544
[5] Recommended VECTORIZED method time: 0.0335956
```

The simulation results demonstrate once more the influence of the `plot()` command on the computation time within the `for, end` and `while, end` control loops. In this plain example, it is clear how much CPU time (~17.5 times more time) is spent on plotting the computed data using methods 2 and 3 than when using methods 1 and 4, even though the step size $\Delta t = 10^{-3}$ is chosen to be 10^3 larger than the one ($\Delta t = 10^{-6}$) used with the three other ways.

Note If possible, avoid using the `plot()` command within `for...end` and `while ... end` loops in order to enhance the computation/simulation time efficiency.

Memory allocation is necessary whenever the use of the `for, end` and `while, end` loop control statements is unavoidable. By using memory allocation, you can improve the time efficiency of the loop control statements substantially. In the previous simple example, the computation time was improved more than 2.5 times by using the memory allocation technique (method 1 versus method 4).

Note that the time efficiency with a memory allocation technique can be observed if the number of iterations is considerably large, for instance, several thousand iterations or beyond. The vectorization approach is indeed superior. When method 4 with memory allocation is compared to the vectorized approach (method 5), the vectorization is over six times faster than the memory allocation method.

Note If the computation/simulation problem requires using `for...end` and `while ... end` loops, use the memory allocation technique in order to enhance the computation/simulation time efficiency.

In order to calculate elapsed (computation) time, stopwatch timers and `tic ... toc` operators are recommended to track computation time of a script. In fact, there are several alternative operators in MATLAB that count elapsed (computation) of a script or function file. They are `cputime` and `clock, etime`. These operators can be used as follows within M-files. This is the comparative analysis script (`Ex13_CT.m`) of these operators with the previous example to compute $f(t)=e^{\frac{-3t}{7}}$ function values for $t = 0 \ldots 5$ with a time step of $\Delta t = 0.0001$.

```
% Ex13_CT.m
%% [tic ... toc]
clc; clear all, close all
T1=tic;
t=0:1e-4:5; m_num=length(t);
for m=1:m_num
    f(m)=exp(-3*t(m)/7);
end
Tfor1=toc(T1);
fprintf('FOR loop time [tic...toc]: %g \n', Tfor1)
%% [cputime ... cputime-Time]
clear all, close all
TT=cputime;
t=0:1e-4:5; m_num=length(t);
for m=1:m_num
    f(m)=exp(-3*t(m)/7);
end
Tfor2=cputime-TT;
fprintf('FOR loop time with [cputime]: %g \n', Tfor2)
%% [clock ... etime()]
clear all, close all
TT=clock; t=0:1e-4:5; m_num=length(t);
for m=1:m_num
    f(m)=exp(-3*t(m)/7);
end
Tfor3=etime(clock, TT);
fprintf('FOR loop time with [clock...etime]: %g \n', Tfor3)
```

By executing this script, the following results are obtained in the Command Window:

```
FOR loop time [tic...toc]: 2.49076
FOR loop time with [cputime]: 2.46482
FOR loop time with [clock...etime]: 2.494
```

The results show that these operators work similarly and produce very close numerical results. Besides these operators, there is another operator called `timeit` that can also be employed to compute overall computation time spent for all simulation processes of function files. This operator was introduced in recent versions of the MATLAB package (R2014b).

The `break` command is a powerful and handy operator that's used mainly within `while, end` and `for, end` loop control statements in order to halt evaluation processes when certain conditions are met. This is another technique that improves iteration time efficiency. We look at the `break` command's implementation in the following example.

Example 14

Compute Fibonacci numbers. The Fibonacci sequence is defined by the following expressions: $f_1 = 1, f_2 = 1, f_3 = f_1 + f_2, \ldots, f_n = f_{n-2} + f_{n-1}$

Thus, if $f_1 = 1, f_2 = 1, f_3 = 3, f_4 = 3 + 1, f_5 = 4 + 3, \ldots$:

1) Show the first 13 elements of the Fibonacci sequence.

2) Compute the first 111 elements of the Fibonacci sequence or stop the iteration when the sequence reaches the value of 123456789.

Here is the solution script (called `Ex14_FN.m`). It computes the first 13 elements (series) with `for ... end` and `while ... end` and the first 111 elements (series) of the sequence or halts the computation if the sequence reaches 12345789 by employing the `break` operator.

```
% Ver 1. Computation of 13 elements of the Fibonacci Numbers: [for ... end]
clearvars
f(1) = 1; f(2)=1;
f = [f(1), f(2), zeros(1, 11)]; % Memory allocation

fprintf('Element: %.0f  FN: %.0f \n', 1, f(1));
fprintf('Element: %.0f  FN: %.0f \n', 2, f(2));
for jj=3:13
    f(jj)=f(jj-2)+f(jj-1);
    fprintf('Element: %.0f  FN: %.0f \n', jj, f(jj));
end
```

```
%% Ver 2. Computation of 13 elements of the Fibonacci Numbers: [while...end]
clearvars
f(1) = 1; f(2)=1; jj=3;
f = [f(1), f(2), zeros(1, 11)]; % Memory allocation

fprintf('Element: %.0f  FN: %.0f \n', 1, f(1));
fprintf('Element: %.0f  FN: %.0f \n', 2, f(2));
while jj<=13
    f(jj)=f(jj-2)+f(jj-1);
    fprintf('Element: %.0f  FN: %.0f \n', jj, f(jj));
    jj=jj+1;
end
%% [BREAK]. Iteration of the Fibonacci Numbers is controlled by [BREAK]
clearvars
F(1) = 1; F(2) = 1;
F = [F(1), F(2), zeros(1, 40)]; % Memory allocation

fprintf('Element: %.0f  FN: %.0f \n', 1, F(1));
fprintf('Element: %.0f  FN: %.0f \n', 2, F(2));
for ii=3:111
    F(ii)=F(ii-2)+F(ii-1);
    fprintf('Element: %.0f  FN: %.0f \n', ii, F(ii));
    if F(ii)>=123456789
    fprintf('Iteration is halted because \n ')
    fprintf('the last computed value is greater than 123456789 \n')
    break
    else
        continue
    end
end
```

This script produces the following output in the Command Window:

```
Element: 1  FN: 1
Element: 2  FN: 1
Element: 3  FN: 2
Element: 4  FN: 3
```

```
Element: 5  FN: 5
Element: 6  FN: 8
Element: 7  FN: 13
Element: 8  FN: 21
Element: 9  FN: 34
Element: 10  FN: 55
Element: 11  FN: 89
Element: 12  FN: 144
Element: 13  FN: 233
```

The results from Ver 2 are identical to the ones from Ver 1, so they are not shown here. Here are the results from the last section with the `break` operator.

```
Element: 1  FN: 1
Element: 2  FN: 1
Element: 3  FN: 2
Element: 4  FN: 3
Element: 5  FN: 5
Element: 6  FN: 8
Element: 7  FN: 13
Element: 8  FN: 21
Element: 9  FN: 34
Element: 10  FN: 55
Element: 11  FN: 89
Element: 12  FN: 144
Element: 13  FN: 233
...
Element: 25  FN: 75025
Element: 26  FN: 121393
Element: 27  FN: 196418
...
Element: 39  FN: 63245986
Element: 40  FN: 102334155
Element: 41  FN: 165580141
Iteration is halted because
 the last computed value is greater than 123456789
```

Note that in the last section with the break operator, the memory allocation operation F = [F(1), F(2), and zeros(1, 40)] creates [1, 1, 40 elements of "0"]. Thus, after 41 iterations, the last element (the 42nd element) of F remains 0.

Example 15

Compute the values of the sine function $h(\theta) = \ sin\ (\theta)$ for $\theta = 0\ldots2\pi$ with 1000 incremental steps and stop computation when the value of the function $h(\theta) \approx 0.9999$..

The sine function fluctuates between 0 ... 1; 1...0; 0...-1; -1...0 for $\theta = 0\ldots2\pi$; therefore, you should stop the computation loop when the function value reaches 0.9999. Here is the solution script (Ex15_SINE.m), which computes and halts the computation process with a break statement when the pre-set condition is met.

```
% Ex15_SINE.m computes sin(theta)
clearvars
theta=linspace(0, 2*pi, 1000); k=length(theta); h=ones(1,k-1);
for ii=1:k
    h(ii)=sin(theta(ii));
    data(ii,:)=[theta(ii);h(ii)];
if abs(h(ii))>=0.9999
    fprintf('Computation is halted after %g iterations\n', ii);
    fprintf('The function value is: %1.5f \n', h(ii))
fprintf('When theta is equal to % 1.5f degrees\n', theta(ii)*180/pi)
break
else
continue
end
end
plot(data(:,1), data(:,2), 'bd'), hold on
plot(data(ii,1),data(ii,2), 'p', 'markerfacecolor','c',  'markersize', 18)
grid on
```

When the script (Ex15_SINE.m) is executed, the following outputs are displayed in the Command Window.

```
Computation is halted after 249 iterations
The function value is: 0.99994
When theta is equal to  89.36937 degrees
```

The next plot figure is shown in Figure 2-5.

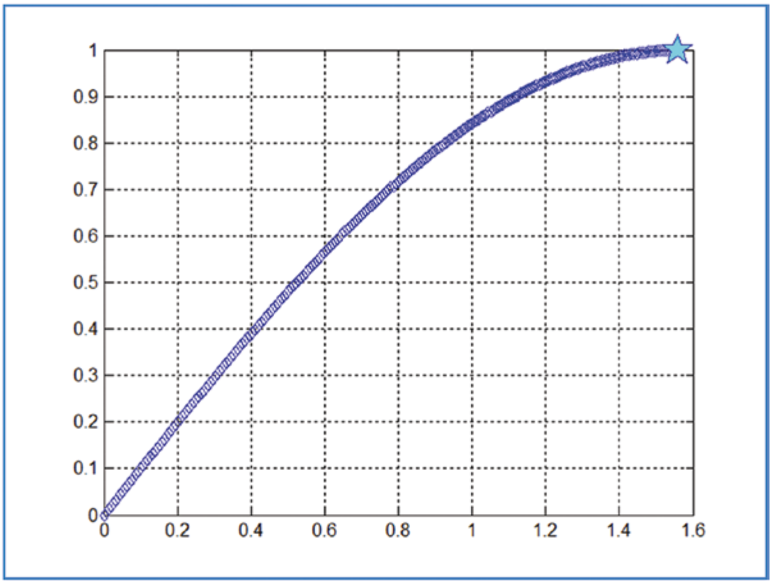


Figure 2-5. *Plot of $h(\theta) = \sin(\theta)$ for $\theta = 0\ldots\frac{\pi}{2}$ and computation halted at $h(\theta)=0.9999$*

This solution script can also be edited or rewritten with `while, end` in a very similar way. That is left for the reader to try for self-testing purposes.

Example 16

Write a script file with a `for ... end` loop and the conditional statements `if, elseif, else, end` to compute the values of the function. Then plot the computed values:

$$f(t)=\begin{cases} e^{-2t}+100t & t\le -13 \\ \cos(100t)-\sin(10t) & -3\ge t\ge -13 \\ 100t+e^{t} & -3\le t\le 0 \\ 1 & t>0 \end{cases}$$

For the values of $t \in [-4\pi,\ \ 3\pi]$. Take $\Delta t = 0.0001$, here is the solution script (called `Ex16_FNs.m`).

```
%% Ex16_FNs.m
clc, clearvars
t=-4*pi:.0001:3*pi;  % Whole t series
F=ones(size(t));     % Memory allocation
tic;
for ii = 1:numel(t)
    if t(ii)<=-2*pi
        F(ii)=exp(sin(2*t(ii)))+exp(cos(100*t(ii)));
    elseif t(ii)>-2*pi && t(ii)<=pi
        F(ii) = sin(2*t(ii))+cos(100*t(ii));
    elseif t(ii)>pi && t(ii)<=2*pi
        F(ii)=exp(sin(100*t(ii)))+exp(cos(2*t(ii)));
    else
        F(ii)=1;
    end
end
   Tma=toc;
   plot(t, F), grid on;
   plot(t, F), grid on, xlabel('\it t'), ylabel('\it F(t)'),
   title('Plot of function values w.r.t value ranges of t'), shg
   fprintf('Computation Time with memory allocation:  %2.6g \n', Tma)
```

Here are the outputs in the Command Window (called `Ex16_FNs.m`). Figure 2-6 shows the plot figure of the script.

```
Computation Time with memory allocation:  0.0286769
```

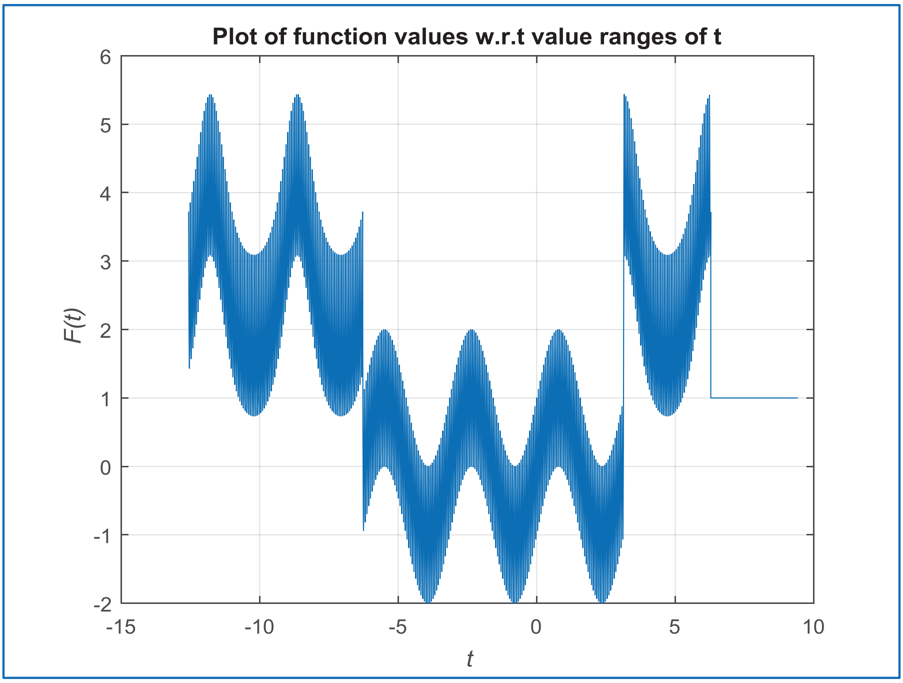


Figure 2-6. *Plot of a function* $(t)=\begin{cases} e^{-2t}+100t & t\leq -13 \\ \cos(100t)-\sin(10t) & -3\geq t\geq -13 \\ 100t+e^{t} & -3\leq t\leq 0 \\ 1 & t>0 \end{cases}$

One of the tricky steps in this example is separating out the four value ranges of `t` according to the problem statement and then computing the four separate function values. The loop operation `for ... end` before the conditional statements `if... elseif... elseif... elseif ... else ... end` performs the whole computation with respect to the indexes of `t`. Whereas the `if... elseif... elseif... elseif ... else ... end` will separate out the values of `t` by comparing every value of `t(ii)` according to the given conditions. In this case, the index `(ii)` is mandatory.

Symbol References in Programming

In MATLAB, there are dozens of important symbols used for various purposes in programs/ scripts. The symbol references are discussed in the following sections.

Asterisk

In general, the * symbol is used as a multiplication operator, but in other instances, it can be used as a wildcard for filename identification. For instance, if you need to delete all files with a file extension of `*.asv`, the following command performs the task:

```
>> delete *.asv
```

This way, you can delete all of the files with the file extension of `*.asv` located in the current directory of MATLAB. Another example is if you need to locate an M-file called `midpoint_rule.m`. You can type part of the filename as so:

```
>>  dir('midp*.m')
midpoint_rule.m
```

Also, you can use * to get information about variables that reside in the workspace by typing part of their names or delete them by typing part of their name:

```
>> whos
>> who
Your variables are:
H    N    NaH
>> clear N*
>> whos
  Name      Size            Bytes  Class     Attributes
  H         5x1              3525  struct
```

At (@) Sign

The `@` sign/operator is mostly used to construct a function handle. For example:

```
F=@(argument lists)expression
```

Moreover, the `@` operator is used to assign a new MATLAB class directory designator used to override MATLAB files. For example:

```
\@myclass\myfun.m
```

Note that MATLAB classes can be used to define object-oriented designs, which are not discussed in this book.

Colon

The `:` operator is used to generate a sequence of numbers or elements of arrays or assign indices. For example:

```
>>M=1:3:300;
>>N=1:200;
>>K=[N(1:10); M(1:10)];
>>Knew=K(1,:)
Knew =

     1     2     3     4     5     6     7     8     9    10
```

Note that when you use the colon `:` operator to generate a sequence of numbers by specifying the step size, the last element of the generated sequence may not match the specified last element. For example, A = 1:3:20 generates A = [1 4 7 10 13 16 19], whereas B = 1:3:21 generates B = [1 4 7 10 13 16 19].

Comma

The `,` operator separates variables or elements of arrays and indices, function input and output arguments, commands or statements. For example:

```
>> M, N,K
>> K(1,2)
>> G=@(t, x) sin(x*t)
>> for ii=1:2:20, Knewm(:)=K(1, ii)*2, end
```

Curly Brackets

The `{ }` are used to assign or construct cell arrays and write fractions, superscripts, and subscripts via the LaTEX interpreter in plot figure titles and axis labels. For instance:

```
>> KnM=[{Knew}, {K}, {N},{M}];
>> title('$sin(\frac{2\phi}{5})$','interpreter', 'latex');
>> xlabel('t_{at}');
>> ylabel('F^{dot}')
```

Dollar Sign

The $ is used to express fractions via the LaTEX interpreter in plot figure titles and axis labels. For instance:

```
>> ylabel('$\Omega(\frac{2\beta}{\alpha})$', 'interpreter', 'latex')
```

Dot

The . operator is used to add fields to an existing structure field and object method specifier. It is used for decimal digits and elementwise matrix operations. For example:

```
>> MH=H(1,1).bytes
>> P = [-3.1416, -1.5708, 0, 1.5708, 3.1416];
>> A = P.^2;
```

Dot-Dot

The .. two dots operator is used to refer to a sequence in the current directory. For instance, the following commands jump up one and two directories (if they already exist) from the current one:

```
>>cd ..\Fax
>>cd ..\..\Desktop
```

Dot-Dot-Dot (Ellipsis)

The **...** three dots (or ellipsis) operator is a line continuation operator. For example:

```
>>  KmN=[1,2; ...
3,2; ]

KmN =

     1     2
     3     2
```

```
>> fprintf('This is what you have entered: %f\n', ...
KmN)
This is what you have entered: 1.000000
This is what you have entered: 3.000000
This is what you have entered: 2.000000
This is what you have entered: 2.000000
```

Parentheses

The ( ) are used to index array elements or specify arguments passed to a called function. For example:

```
>> K(3, 4)
>> F=@(x, y)sqrt(x.^2+y.^2)
>> [m, n]=myFUN(x, y, k)
```

Percent

The % operator is one of the most commonly used reference symbols in MATLAB programming. It is used for five main purposes (of which four are for commenting purposes):

- To assign comments within M-files (%).
- To create cell modes with double percent signs (%%) and then leave a space.
- To assign conversion specifiers when used within `fprintf()` and `sprintf()`.
- To assign a block of comments when used with curly braces (%{). They are opened in one line and %} closed in another.
- They are (%#ok) used to remove a warning sign to display computation results in the Command Window.

Let's look at several examples of using the % sign.

```
% This script is used for ...
% Created by ...
%% New cell mode starts from here ...
fprintf('you have entered %g which is EVEN number \n', N)
sprintf('you have entered %g which is EVEN number \n', N)
X1=sprintf('Root2 of the equation is x1= %g*i', x1);
%{
 Start comments and remarks...
 This script is used for....
 Created by ...
 Define entered integer type
 End comments and remarks...
%}
K=randi(100,3);
M=K*2.2        %#ok
```

Note When using % with { } to insert comments in your script, the %{ open bracket must be on a separate line before the starting comments and the %} closing must be at the end of comments block on a separate line as well. No other entries should follow them on the same line.

Semicolon

The ; is mainly used to construct arrays, suppress output display in the Command Window, and separate commands entered on one line. For example:

```
>>K=[1 2; 3,4];
>> clear all; close all; clc
```

Single Quotes

Single quotes (`' '`) are one of the most commonly used reference symbols of MATLAB. They are used as character and string constructors, and with the inline function. For example:

```
>> a=input('Enter value of a: ');
>> Hurray = 'Today is a great DAY ! '
>> F_1 =inline('2*x^2-3*x+13')
```

Note that the single quote is also used as a matrix transpose operation:

```
>> A = [1, 2; 3, 4]
A =
     1     2
     3     4
>> A'
ans =
     1     3
     2     4
```

Slash and Backslash

The `/ \` characters are used to separate elements of a path or directory string. Note that these characters are also used for division (and matrix left division) operations and \ is used to write Greek letters in the LaTEX format. For example:

```
>> dir(['..\Circle13.m']) % Path or directory
>> title(' \alpha(\theta) vs. \Omega(\theta)')  % LaTEX interpretation
>> A = [2, 3, 4]; B=A/2;  % Division
```

Square Brackets

The square brackets `[ ]` are used to construct, declare, and concatenate arrays, and to declare and capture values returned by functions. They also indicate text and labels in plot figures. For example:

```
>> K=[1 2; 3,4];
>> B=[K, K*2; rand(2,2), eye(2)];
>> [xout yout] = myfunction(z, t);
>> gtext(['End point: ' num2str(yout(end)) ' [m]'], 'background', 'y');
```

Function Files

Function files are a remarkable strength of MATLAB for efficient code generation and computation. Once the function files are saved, they will have the symbol of , while other M-files have and MLX-files signs showing their file types. The way function files are executed and the way their simulation results are obtained are substantially different from M- and MLX-files.

First, let's address why function files are necessary. The main reason for using the function files is that they are faster, computationally more efficient, and consume less space in the workspace. Moreover, they can be called and executed within any M or MLX-file and from the Command Window. Function files are very powerful and flexible and they can be used for various purposes. Not only to evaluate mathematical functions or perform calculations, but also to evaluate, compare, and assess evaluated data inputs as arguments. The general syntax and structure of the function file can be expressed in the following way:

```
function [A, B, ..., W] = FCN_name(a, b, ..., w)
% Help. ....
....
end
```

Where `[A, B, ..., W]` are output variables and `(a, b, ..., w)` are input variables, also called input arguments. Note that function files have to start with the word `function` and the filename of the function file has to match (e.g., `FCN_name`). In other words, the previous function file has to be named `FCN_name.m`. Otherwise, you cannot execute it. MATLAB automatically recognizes the function file as soon as you type in the word `function` and prompts the function filename when you save it the first time. Moreover, you must end the function file with the keyword `end`. Naming function files is like naming any M-files or MLX-files; they have to start with a letter and must not contain any empty spaces or symbols, except for an underscore _.

Note One of the most common mistakes while recalling function files is misspelling the name of the file.

In more recent versions of MATLAB, function files can have a different filename and function name. (That is, the function file can be called `MY_fun` but its script may be written as `function [Out1, Out2, ...] = fun(In1, In2, ...)`.) This will not cause an error. However, it is still recommended to use the same filename and function name.

In more recent versions of MATLAB, the function file syntax (called sub-functions/member functions) can also be implemented/embedded within M-files and called/executed as part of the M-files.

Function files can be created for various purposes, for instance, to perform computations or display simulation results. Function files can be created/written in several ways:

- The first way. By using the M-file or MLX-file editors or using the Notepad text editor.
- The second way. By using the main menu New and selecting fx Function or Live Function from the drop-down box. The latter option (Live Function) is only available in MATLAB 2018b.
- The third way. By using the Symbolic Math[2] toolbox's symbolic object identifier, `syms`.

With the first way, you type in the function file and save a precisely matched function name with a file extension of `*.m` or `*.mlx`. It will be recognized by MATLAB automatically as a function file. In the second and third ways, the editors will automatically generate the standard syntax components of the function file. The file can be edited without having to type standard parts.

Let's look at several examples to demonstrate how to create function files and work with them.

[2] Symbolic Math is a registered trademark of The MathWorks Inc.

Example 1

Write a function file to compute $f\left(t\right)=\frac{3}{e^{sin(t)}}+\frac{5}{e^{cos(t)}}$ and plot the computed values. This can be implemented as shown in the following script called `myfunction.m`. It uses the first way of creating the function file.

```
function f=myfunction(t)
% HELP: this function file (myfunction.m) computes values of the expression
f(t) = 3./exp(sind(t))+5./exp(cosd(t)) for
% given values of t, saves the computed values of f(t) in the workspace,
and plots computed results.
% Note that [t] is input argument that has to be in degrees not in radians,
e.g.numerical array, e.g. >> t = -360:720360;
% To execute this file: >> t = -720:10:720;  f=myfunction(t);  % that
assigns output values to a variable f     % and shows the plot.
% To execute this file: >> t = 0:1:180;     myfunction(t);  % that assigns
output values to a variable ans % shows the plot.
% To execute this file: >> FF=myfunction(-180:2:360);  % that assigns
output values to a variable FF           % shows the plot.

f=3./exp(sind(t))+5./exp(cosd(t));
plot(t, f, 'ro-', 'linewidth', 1.5), grid on
title('Plot of myfunction.m file')
end
```

It is a good habit and practice to include comments and remarks in the function files that specify the functionality of the function file, specifics about the input arguments, helpful hints on how to use it with example input data, and so forth. When there are help comments in the function files, users can search for help about function files from the Command Window. They do so by recalling the function filename with the `help` command:

```
>> help myfunction
  HELP: this function file (myfunction.m) computes values of the expression
  f(t) = 3./exp(sind(t))+5./exp(cosd(t)) for
  given values of t, saves the computed values of f(t) in the workspace,
  and plots computed results.
  Note that [t] is input argument that has to be in degrees not in radians,
  e.g.numerical array, e.g. >> t = -360:720360;
  To execute this file: >> t = -720:10:720;  f=myfunction(t);  % that
  assigns output values to a variable f     % and shows the plot.
  To execute this file: >> t = 0:1:180;     myfunction(t);  % that assigns
  output values to a variable ans % shows the plot.
  To execute this file: >> FF=myfunction(-180:2:360);  % that assigns
  output values to a variable FF           % shows the plot.
```

Note that in order to call and use function files, they must reside in the current (active) MATLAB path directory. Or, you must add the path (directory) of the function file to the search directory of MATLAB. You can do that using the `addpath(' ')` command and specifying the directory inside `(' ')`. If the function file's directory is not in the current directory or is not added to the list of the MATLAB paths, it cannot be recalled and executed. Subsequently, you'll get error messages indicating an undefined function or variable.

You can simulate this function file (called `myfunction.m`) from the Command Window or within another M-file with the input variable of `t`:

```
>> % If the output variable is not specified, "ans" will be the default
variable name.
>> % That has all values of "f" of the function file.
>> myfunction(0:2:720);  % The input argument "t" values are defined
directly.
>> t=0:2:720; f=myfunction(t); % Input and output variables are defined and
given.
```

A second way to create a function file is by clicking the main menu New and selecting fx Function or Live Function. The following function file will be opened with a default name of `untitled`.

This file is opened with the fx Function button in the M-file editor window.

```
function [outputArg1,outputArg2] = untitled(inputArg1,inputArg2)
%UNTITLED6 Summary of this function goes here
%   Detailed explanation goes here
outputArg1 = inputArg1;
outputArg2 = inputArg2;
end
```

This one is opened with the button in the MLX-file editor window.

```
function z = Untitled(x, y)
z = x + y;
end
```

The following example explains how to employ the third way of creating function files.

Example 2

Given: $f(u_1,u_2)=\begin{cases} u_2-e^{2u_1} \\ 1.25u_1-e^{3u_2} \end{cases}.$

A third way of creating a function file is to employ Symbolic Math[3] toolbox's symbolic object identifier, syms. The MATLAB tool called matlabFunction creates an anonymous function file to compute $f(u_1, u_2)$ with input arguments of u_1, u_2.

```
syms u1 u2         % Define variables (input data names)
% Formulate the given function equations
f=[u2-exp(2*u1), 1.25*u1-exp(3*u2)];
% Define a function file name, e.g. myFUN
matlabFunction(f, 'file', 'myFUN');
```

When you execute these commands, a function file called myFUN.m is created in the current MATLAB directory.

```
function f = myFUN(u1, u2)
%MYFUN
% F = MYFUN(U1,U2)

% This function was generated by the Symbolic Math Toolbox version 5.9
%    24-Apr-2013 14:20:01

f = [u2-exp(u1.*2.0),u1.*(5.0./4.0)-exp(u2.*3.0)];
```

Note that once a function is created with matlabFunction, you should not re-use this function tool to re-create the already created function file. If you try to do so, error

[3] Symbolic Math is a registered trademark of The MathWorks Inc.

messages will pop up. There is another salient point in employing the `matlabFunction` tool—this tool cannot generate function files containing variables with indexes, so if there are variables with indexes, the `matlabFunction` tool prompts errors.

Certain operations, commands, computations, and execution of several function files can be performed within the function file without any input arguments or output. The general syntax of such function files without input arguments and output variables is very simple:

```
function functionNAME
% ...
```

Let's now look at the following example.

Example 3

Create a function file that clears the Command Window from all previously typed commands and displays benchmarking results of your computer. This `getREADY.m` function file performs the assigned tasks without any input arguments or any output data.

```
function getREADY
% HELP. getREADY.m
% This function file cleans up command history of MATLAB  and
% performs benchmarking calculations of several MATLAB functions (LU, FFT,
% ODE, etc.) installed computer against some of the computer configurations
of the date.

clc; close all; bench
end
```

Note There are two cases when function files can be executed with Run Run from the M/MLX-file editor window. If the function file has no input arguments or a varying number of input arguments (i.e., no input), or one, two, or more input arguments.

Example 4

Let's create a function file that computes the roots (x_1, x_2) and discriminant (D) of the quadratic equation $ax^2 + bx + c = 0$ with respect to the input arguments of *a, b, c,* which are scalar values. This function file (called QUAD.m) will produce three output variables according to the three input arguments.

```
function [x1, x2, D]=QUAD(a, b, c)
% QUAD.m
% Solves quadratic equations based on coefficients of: a, b, c
% Note that a, b, c need to be scalars.
fprintf('Solve: (%g)x^2+(%g)x+(%g)=0\n', a,b,c)
D=b^2-4*a*c;
% Roots
x1=(-b+sqrt(D))/(2*a); x2=(-b-sqrt(D))/(2*a);
if lt(D,0)
    fprintf('This equation does not have real value roots!\n');
    fprintf('Because discriminant is negative. D = %g\n', D);
    fprintf('Complex Root1: x1= %10s\n', num2str(x1));
    fprintf('Complex Root2: x2= %10s\n', num2str(x2));
elseif eq(D,0)
    disp('This equation has one unique root! ');
    disp('Because discriminant is zero. D=0 ');
    fprintf('This equation has one unique root! \n');
    fprintf('Because discriminant is zero. D=0 \n');
    fprintf('Unique Root: x = %g \n', x1);
else
    fprintf('This equation has two roots \n');
    fprintf('Because discriminant is: D = %g \n', D);
    fprintf('Root1 of the equation is: x1= %g \n', x1);
    fprintf('Root2 of the equation is: x2= %g \n', x2);
end
```

Let's test this function file with different input arguments and obtain the output variables from the Command Window.

```
>> a=1; b=2; c=3;
>> [x1, x2, D]=QUAD_01(a, b, c)     % Case 1. The values of a, b, c are
predefined
Solve: (1)x^2+(2)x+(3)=0
This equation does not have real value roots!
Because discriminant is negative. D = -8
Complex Root1: x1= -1+1.4142i
Complex Root2: x2= -1-1.4142i
x1 =
  -1.0000 + 1.4142i
x2 =
  -1.0000 - 1.4142i
D =
    -8
>> [x1, x2, D]=QUAD_01(1, 2, 3)   % Case 2. The values of a, b, c are
entered directly
Solve: (1)x^2+(2)x+(3)=0
This equation does not have real value roots!
Because discriminant is negative. D = -8
Complex Root1: x1= -1+1.4142i
Complex Root2: x2= -1-1.4142i
x1 =
  -1.0000 + 1.4142i
x2 =
  -1.0000 - 1.4142i
D =
    -8
>> [x1, x2, D]=QUAD_01(b, c, a)  % Case 3. The values of a, b, c are mixed up
Solve: (2)x^2+(3)x+(1)=0
This equation has two roots
Because discriminant is: D = 1
Root1 of the equation is: x1= -0.5
Root2 of the equation is: x2= -1
```

```
x1 =
   -0.5000
x2 =
    -1
D =
     1
>> QUAD_01(b, c, a);   % Case 4. The values of a, b, c are mixed up and
outputs are not specified
Solve: (2)x^2+(3)x+(1)=0
This equation has two roots
Because discriminant is: D = 1
Root1 of the equation is: x1= -0.5
Root2 of the equation is: x2= -1
>> a1=1; a2=5; a3=8;
>> [X, Y, Dis]=QUAD_01(a1, a2, a3) % Case 5. The output variable names are
changed
Solve: (1)x^2+(5)x+(8)=0
This equation does not have real value roots!
Because discriminant is negative. D = -7
Complex Root1: x1= -2.5+1.3229i
Complex Root2: x2= -2.5-1.3229i
X =
  -2.5000 + 1.3229i
Y =
  -2.5000 - 1.3229i
Dis =
    -7
```

From the five simulations, it is clear that while calling and executing the function files, the names of the input arguments and output variables are not essential. That means users can change the output variables and use different input arguments. Furthermore the function files take the input arguments according to their order (Case 2 vs. Case 3 and Case 4). The output variables can be omitted and the function file will still run, but there will be only one output corresponding to the first output variable specified in the function file's context.

Most Common Errors with Function Files

There are several common mistakes to avoid when working with function files:

- Misspelling function names when calling them
- Providing the wrong number of input arguments than is actually defined in the function file
- Calling the wrong number of output variables than is actually defined in the function file
- Incorrectly using input arguments, such as mismatched size, variable type, etc.
- Defining the input arguments in the function file and providing their values within the function file context. In other words, defining the input arguments twice without any preconditions of missing arguments and use of alternative (default) values.

Let’s test some of the most common errors that occur when working with the function files in the case of Example 1.

```
>> t=0:2:720;
>> % Error:  Misspelled function name with "M" instead of "m"
>> f=Myfunction(t);
Cannot find an exact (case-sensitive) match for 'Myfunction'

The closest match is: myfunction in
C:\ ...\Documents\MATLAB\myfunction.m
Did you mean:
>> f=myfunction(t);   % MATLAB's automatically detected similar  function
name

>>  t1 = [0:2:720]; t2= -720:2:0;
>> % Error: Two input arguments are inserted instead of one as defined in
the function file
>> f=myfunction(t1, t2);
Error using myfunction
Too many input arguments.
```

```
>>  t2= -720:2:0;   % Input argument name can be altered that is not an
ERROR!
>> % Error: Two output variables are requested instead of one as defined in
the function file
>> [f1, f2]=myfunction(t2);
Error using myfunction
Too many output arguments.
```

```
>> % Error: Wrong type of input arguments
>> t3 =['a', 'b', 'c', 'd', 'f', '-a'  '-b' '-c' '-d' '-f'];  % t3 is a
character array and not numerical array
>> f=myfunction(t3);
Undefined function 'sind' for input arguments of type 'char'.
Error in myfunction (line 4)
f=3./exp(sind(t))+5./exp(cosd(t));
```

Let's analyze the case whereby the input argument is defined within the function file and is called as the input argument while executing the function file. Let's make some changes to the function file (myfunction.m).

```
function f=myfunction(t)
% HELP: this function file (myfunction.m) computes values of the expression
f(t) = 3./exp(sind(t))+5./exp(cosd(t)) for
% given values of t, saves the computed values of f(t) in the workspace,
and plots computed results.
% Note that [t] is input argument that has to be in degrees not in radians,
e.g.numerical array, e.g. >> t = -360:720360;
% To execute this file: >> t = -720:10:720;  f=myfunction(t);  % that
assigns output values to a variable f     % and shows the plot.
% To execute this file: >> t = 0:1:180;     myfunction(t);  % that assigns
output values to a variable ans % shows the plot.
% To execute this file: >> FF=myfunction(-180:2:360);  % that assigns
output values to a variable FF
t = 0:720;  % Input variable t is assigned internally
f=3./exp(sind(t))+5./exp(cosd(t));
plot(t,f, 'ro-', 'linewidth', 1.5), grid on
title('Plot of myfunction.m file')
```

In this case, the function file takes the input argument values and ignores the input arguments for t when it is called/executed. For example:

```
>> t =0:1:180; f=myfunction(t)
```

The example outputs the computed values of f(t) for t = 0:720 by ignoring the specified input variable t =0:1:180.

Varying Number of Inputs and Outputs

One of the remarkable advantages of function files over M- or MLX-files is that they take a varying number of input arguments and output variables. This flexibility of the function files can be attained via several of MATLAB's built-in tools/commands/functions. To vary the number of input arguments, you use varargin and to vary the output variables, you use varargout. To make these functions more useful with respect to the given problem, there are two other MATLAB built-in tools/commands/functions—nargin and nargout—that determine how many input arguments and output variables are called while executing the function file. The varying inputs and outputs of the function files can be used in a few different combinations while declaring function input arguments and output variables:

```
function [Out1, Out2] = function_name(varargin)
```

```
function varargout =  function_name(Input1, Input2, Input3)
```

```
function varargout = function_name(varargin)
```

To find more detailed information on these tools, type the following in the Command Window:

```
>> help varargin
>> help varargout
>> help narargin
>> help narargout
>> doc varargin
>> doc varargout
```

Let's look at two different examples dealing with varying input arguments and output variables.

- Varying number of inputs and three output variables
- Varying number of inputs and output variables

Example 5

Create a function file (Quad_Var.m) that solves the quadratic equation ($ax^2 + bx + c = 0$) with a varying number of input variables ($a, b, and\ c$) and with three outputs (x_1, x_2, D).

```
function [x1, x2, D]=Quad_Var(varargin)
% Quad_Var.m computes roots of the quadratic equation with varying number
% of input arguments for a, b, c. Gives three output variables: x1, x2, D.
% There are four cases considered:
% Case 1. No Input arguments: a =1; b=2; c=3; values are taken as inputs.
% Case 2. One Input argument given: a, and b=2, c=3 are taken as inputs.
% Case 3. Two Input arguments given: a and b, c=3 is taken as a 3rd input
% Case 4. Three Input arguments given: a, b and c.
% E.g.
% Run: [x1, x2, D] = Quad_Var();        % No Input
% Run: [x1, x2, D] = Quad_Var(1);       % One Input: a=1;
% Run: [x1, x2, D] = Quad_Var(1, 2);    % Two Inputs: a=1; b=2;
% Run: [x1, x2, D] = Quad_Var(1, 2, 3); % Three Inputs: a=1; b=2; c=3;

if nargin==0          % Case 1. No input arguments
    a=1; b=2; c=3;
elseif nargin==1      % Case 2. One input argument only
    a=varargin{1}; b=2; c=3;
elseif nargin==2      % Case 3. Two input arguments
    a=varargin{1}; b=varargin{2}; c=3;
else                  % Case 4. Three input arguments
    a=varargin{1}; b=varargin{2}; c=varargin{3};
end
D=b^2-4*a*c;x1=(-b+sqrt(D))/(2*a); x2=(-b-sqrt(D))/(2*a);
end
```

In this function file called `Quad_Var.m`, the MATLAB's built-in command/tool **nargin** automatically counts how many inputs are given while calling the function file. **varargin{1}** represents the first input argument, **varargin{2}** represents the second input argument, and **varargin{3}** represents the third variable. In this order, an `n` number of function files can be created and called.

Let's test the function file (called `Quad_Var.m`) in four scenarios:

- No inputs: the values of *a, b*, and *c* are taken from the function file context *a=1, b=2*, and *c=3*.
- One input: *a* is given (e.g., *a = 1* or any scalar to be assigned to *a* and the two values (*b, c*) are taken from the context of the function file: *b =2* and *c=3*.
- Two input arguments: *a* and *b* are given (e.g., *a = 1, b = 2* or any scalars to be assigned to *a* and *b*) and one input *c* is taken from the context of the function file: *c = 3*.
- Three input arguments: *a, b*, and *c* are given (e.g., *a = 1, b = 2, c=3* or any scalars to be assigned to *a, b, c*).

```
>> [x1, x2, D]=Quad_Var         % Case 1. No Input
x1 =
  -1.0000 + 1.4142i
x2 =
  -1.0000 - 1.4142i
D =
    -8
>> [x1, x2, D]=Quad_Var(1)     % Case 2. One Input
x1 =
  -1.0000 + 1.4142i
x2 =
  -1.0000 - 1.4142i
D =
    -8
>> [x1, x2, D]=Quad_Var(1,2)   % Case 3. Two Inputs
x1 =
  -1.0000 + 1.4142i
```

```
x2 =
  -1.0000 - 1.4142i
D =
    -8
>> [x1, x2, D]=Quad_Var(1,2, 3) % Case 4. Three Inputs
x1 =
  -1.0000 + 1.4142i
x2 =
  -1.0000 - 1.4142i
D =
    -8
```

Let's look at the following example, which analyzes the varying number of outputs and specific (non-varying) input arguments (two input arguments).

Example 6

Write a function file to compute the Leibnitz series expansion of *arctan*(*x*). Find an approximation of *π* using the sum of the Maclaurin Series formula for *N* terms in the form of:

$$\frac{\pi}{4} \cong \sum_{k=0}^{N} \frac{(-1)^k}{2k+1} = S_N$$

Write a script that computes the sum (S_N) of *N* terms, which is the approximation of $\frac{\pi}{4}$, the difference (error) $Error = \frac{\pi}{4} - S_N$, and `Nhalt` when the iteration halted due to the user specified error tolerance (`E_tol`) using `for ... end` and `while ... end` loops.

```
function [varargout] = Leibnitz_VarOut(varargin)
% Leibnitz_VarOut.m computes and plot the Leibnitz series expansion of
%         arctan(1) to find an approximation of PI.
%         Input arguments:
%         N (number of terms, e.g. 10000)
%         E_tol (Error tolerances, e.g. 0.0001)
%         OUTPUT variables are:
%         Out1 = Error;
%         Out2 = SN (Sum of series);
%         Out3 = Nhalt (terms when simulation is halted).
```

```
if nargin==0
    N =1000; E_tol = 0.001;
elseif nargin<2
    E_tol = 0.001;
    N = varargin{1};
else
    N = varargin{1}; E_tol=varargin{2};
end

SN=0;
for ii=0:N
    SN = SN+((-1)^ii)/(2*ii+1);
    Error=abs(pi/4-SN);
    if abs(Error)<=E_tol
        Nhalt=ii;
        break
    else
        Nhalt=ii;
        continue
    end
end
if nargout == 0        % No Output specified: NO outputs
    disp('NO Outputs!')
elseif    nargout ==1  % One Output: Error
    varargout{1}=Error;
elseif nargout ==2     % Two Outputs: Error and Sum of series
    varargout{1}=Error; varargout{2}=SN;
else      % Three Outputs: Error, Sum of Series, Number of Iterations,
    varargout{1}=Error; varargout{2}=SN;varargout{3}= Nhalt;
end
```

If no input arguments are given, the output will be error and two input arguments. For example, the number of iterations and error tolerance will be 1000 and 0.001, respectively. If there is one input, that will be the number of iterations and the error tolerance will take the value of 0.001.

While executing this function file, if the output variable is not specified, then no outputs will be obtained and a short note (NO Outputs!) will be displayed in the Command Window. If one output variable is called (with any variable name), the last error (Error) value will be the output variable's value. If two output variables (names) are specified while calling/executing the function file, error (Error) and sum of series (S_N) will be output. If three output variables are called, Nhalt, S_N, and Error values will be obtained. Note that, from the specified conditions, if the given (N input) number of iterations cannot produce the specified input (E_tol) error tolerance, then Nhalt (the output variable) will be equal to N (the input argument) .

```
[EE, SM]=Leibnitz_VarOut()
EE =
   1.0000e-03
SM =
    0.7844
>> [EE, SM]=Leibnitz_VarOut(10000)
EE =
   1.0000e-03
SM =
    0.7844
>> Leibnitz_VarOut(1e5, 1e-4)    % No output variable specified and thus,
No output
NO Outputs!
>> E = Leibnitz_VarOut(1e5, 1e-4) % One output specified and thus, Error
displayed
E =
   1.0000e-04
>> [ Ee Ss]  = Leibnitz_VarOut(1e5, 1e-4) % Two outputs: Error and Sum of
series
Ee =
   1.0000e-04
Ss =
    0.7853
```

```
>> [ Ee Ss Nn ]  = Leibnitz_VarOut(1e5, 1e-4) % Three outputs: Error, Sum
of series, N Iterations
Ee =
   1.0000e-04
Ss =
    0.7853
Nn =
        2499
```

Let's look at another example that varies the number of input arguments and output variables.

Let's look at another example with some computations to demonstrate and analyze function files with varying numbers of input arguments and output variables.

Example 7

Compute the values of the function

$$f(t)=\begin{cases} e^{-2t_1}+100t_1 & t_1\le -13 \\ \cos(100t_2)-\sin(10t_2) & -3\ge t_2\ge -13 \\ 100t_3+e^{t_3} & -3\le t_3\le 0 \\ 1 & t_4>0 \end{cases}$$

The function $f(t)$ will have four input arguments (t_1, t_2, t_3, t_4), whose default set values are $t = [t_1, t_2, t_3, t_4] \in [-20,\ \ 6]$ and $t_1 = -20{:}\ 0.001{:}\ -13$; $t_2 = -12.999{:}\ 0.001{:}\ -3$; $t_3 = -2.999{:}\ 0.001{:}\ 3$; $t_4 = 3.001{:}\ 0.001{:}\ 6$.

Task 1

Simulate a system within the values of $t = [t_1, t_2, t_3, t_4],\ \ t \in [-20,\ \ 6]$ with $\Delta t = 0.001$.. Note that this function file may have

1) No input argument and as default values of the function file: $t_1 = -20{:}\ 0.001{:}\ -13$; $t_2 = -12.999{:}\ 0.001{:}\ -3$; $t_3 = -2.999{:}\ 0.001{:}\ 3$; $t_4 = 3.001{:}\ 0.001{:}\ 6$ will be taken.

2) One input argument that has to represent t as a vector space of data points within [-20, 6].

3) Two or three input arguments, which will be ignored and the default set values: $t_1 = -20: 0.001: -13$; $t_2 = -12.999: 0.001: -3$; $t_3 = -2.999: 0.001: 3$; $t_4 = 3.001: 0.001: 6$ will be taken instead.

4) Four input arguments that have to represent t_1, t_2, t_3, t_4 in consecutive order.

5) More than four input arguments, which means all entries need to be ignored and a warning message needs to be displayed. Instead of the entries, the default set values of the function file: $t_1 = -20: 0.001: -13$; $t_2 = -12.999: 0.001: -3$; $t_3 = -2.999: 0.001: 3$; $t_4 = 3.001: 0.001: 6$ will be taken.

Task 2

The function file (`Ex7Var.m`) has to produce any of the following underlined outputs with respect to the call commands of the function file:

1) No outputs and plot of the computed values of $f(t)$

2) One output: $f_1(t) = e^{-2t_1} + 100t_1$

3) Two outputs: $f_1(t) = e^{-2t_1} + 100t_1, f_2(t) = \cos(100t_2) - \sin(10t_2)$

4) Three outputs: $f_1(t) = e^{-2t_1} + 100t_1, f_2(t) = \cos(100t_2) - \sin(10t_2)$,
$f_3\ (t) = 100t_3 + e^{t_3}$

5) Four outputs: $f_1(t) = e^{-2t_1} + 100t_1, f_2(t) = \cos(100t_2) - \sin(10t_2)$,
$f_3(t) = 100t_3 + e^{t_3}, f_4(t) = 1$

6) Five outputs: $f_1(t) = e^{-2t_1} + 100t_1, f_2(t) = \cos(100t_2) - \sin(10t_2)$,
$f_3(t) = 100t_3 + e^{t_3}, f_4(t) = 1, t = t$

7) Six outputs: $f_1(t) = e^{-2t_1} + 100t_1, f_2(t) = \cos(100t_2) - \sin(10t_2)$,
$f_3(t) = 100t_3 + e^{t_3}, f_4(t) = 1, t = t, t_1$

8) Seven outputs: $f_1(t) = e^{-2t_1} + 100t_1, f_2(t) = \cos(100t_2) - \sin(10t_2)$,
$f_3(t) = 100t_3 + e^{t_3}, f_4(t) = 1, t = t, t_1, t_2$

9) Eight outputs: $f_1(t)=e^{-2t_1}+100t_1, f_2(t)=\cos(100t_2)-\sin(10t_2),$
$f_3(t)=100t_3+e^{t_3}, f_4(t)=1, t=t, t_1, t_2, t_3$

10) nine outputs: $f_1(t)=e^{-2t_1}+100t_1, f_2(t)=\cos(100t_2)-\sin(10t_2),$
$f_3(t)=100t_3+e^{t_3}, f_4(t)=1, t, t_1, t_2, t_3, t_4$

If more than nine outputs are requested, the function file will prompt a warning message and note stating that this function file can produce a maximum of nine output variables.

The final solution script is called `Ex7Var.m` and has incorporated all points in Tasks 1 and 2 concerning the varying number of input arguments and output variables.

```
function [f1, f2, f3, f4, t, t1, t2, t3, t4, varargout
] = Ex7Var(varargin)
% HELP: Ex7Var.m is a function file to compute a complex function whose
% computation function components differ w.r.t the values of t
% It may have no, one, two, three, or four input arguments for t
% [f1, f2, f3, f4, t] = Ex7Var(t1, t2, t3, t4)
% [f1, f2, f3, f4, TT, t1, t2, t3, t4]=Ex7Var(linspace(-20, 6, 10000));
if eq(nargin,0)  % No input argument
t1 = -20:.001:-13; t2=-12.999:.001:-3; t3=-2.999:.001:3; t4 = 3.001:1e-3:6;
t = [t1,t2, t3, t4];
f1=exp(sin(2*t1))+exp(cos(100*t1)); f2=cos(2*t2)+sin(100*t2);
f3=exp(sin(100*t3))+exp(cos(2*t3))+sin(100*t3)+cos(2*t3);
f4=ones(size(t4));
elseif eq(nargin,1)&&min(varargin{1})<=-13&&max(varargin{1})>=6 % One Input
    t = varargin{1};
    for ii=1:numel(t)
        if le(t(ii),-13)
            t1(ii)=t(ii);
        elseif gt(t(ii),-13) && le(t(ii),-3)
            t2(ii)=t(ii);
        elseif gt(t(ii),-3) && le(t(ii),3)
            t3(ii)=t(ii);
        else
            t4(ii)=t(ii);
        end
    end
                      f1=exp(sin(2*t1))+exp(cos(100*t1));
  t2=t2(find(t2~=0)); f2=cos(2*t2)+sin(100*t2);
  t3=t3(find(t3~=0));f3=exp(sin(100*t3))+exp(cos(t3))+sin(100*t3)+cos(2*t3);
  t4=t4(find(t4~=0)); f4=ones(size(t4));
elseif gt(nargin,1) && lt(nargin,4) % Two or Three Inputs
    warndlg('t series need to be in four separate ranges or one united
    !!!')
    warndlg('Your entries are ignored and example data taken instead!!!')
    t1=-20:.001:-13; t2=-13.001:.001:-3; t3=-3.001:.001:3; t4=3.001:1e-3:6;
    t = [t1,t2, t3, t4];
    f1=exp(sin(2*t1))+exp(cos(100*t1)); f2=cos(2*t2)+sin(100*t2);
    f3=exp(sin(100*t3))+exp(cos(2*t3))+sin(100*t3)+cos(2*t3);
    f4=ones(size(t4));
elseif eq(nargin, 4)               % Four Inputs
    t1 = varargin{1}; t2=varargin{2}; t3=varargin{3}; t4 = varargin{4};
    t = [t1,t2, t3, t4];
    f1=exp(sin(2*t1))+exp(cos(100*t1)); f2=cos(2*t2)+sin(100*t2);
    f3=exp(sin(100*t3))+exp(cos(2*t3))+sin(100*t3)+cos(2*t3);
    f4=ones(size(t4));
else                               % More than Four Inputs
warndlg('Check your entries: input arguments for [t] or [t1, t2, t3, t4]')
    warndlg('Your entries are ignored and default data taken instead!!!')
    t1=-20:.001:-13;t2=-12.999:.001:-3; t3=-2.999:.001:3; t4 = 3.001:1e-3:6;
    t = [t1,t2, t3, t4];
    f1=exp(sin(2*t1))+exp(cos(100*t1)); f2=cos(2*t2)+sin(100*t2);
    f3=exp(sin(100*t3))+exp(cos(2*t3))+sin(100*t3)+cos(2*t3);
    f4=ones(size(t4));
end
plot(t1, f1, 'r', t2, f2, 'b', t3, f3, 'm',t4, f4, 'go--'), grid on; shg,
legend('toggle')
```

```
% Set number of output variables is verified
MIN_outs = 0; MAX_outs=9;
if nargout>MAX_outs
fprintf('Asked %3g outputs that are more than set outputs!!! \n', nargout);
warndlg('This function file is assigned to have max. of 9 outputs!!!')
fprintf('Asked %3g outputs are beyond the set outputs !!! \n', nargout);

end

% NARGOUTCHK; Checks and prompts error if the number of outputs requested
% by the user is beyond 9!
% nargoutchk(MIN_outs, MAX_outs) % Can be also employed

end
```

It should be noted that in the `Ex7Var.m` function file, the **nargin** function will determine how many input arguments are specified while calling this function file for Task 1. The function's input arguments are taken for simulations depending on the number of the input arguments verified within the `if... elseif... elsefif ... elseif ... else ... end` conditional operators addressing all points in Task 1 for input arguments.

Moreover, the `for ... end` loop operator with another internal `if ...elseif ... elseif...else...end` conditional operator set after a first (`elseif ...`) splits up the given one input argument (t) value for four separate sets of values for t_1, t_2, t_3, t_4 according to the given exercise statements.

The logical indexing operations of **find( )** with `~= 0` define which elements (taken from `t` within the `for ... end` loop and `if ...elseif ... elseif...else...end` operations) of t_2, t_3, t_4 are to be taken for simulations in order to skip overlapping points. When the user enters two or three input arguments, **nargin** and `elseif ...` verify that entry. Two warning message dialogs will pop up informing the user that all entries are ignored and the default values are taken instead.

In case of four input arguments, all input arguments are considered in the order of t_1, t_2, t_3, t_4 and simulations are performed. If the number of input arguments is beyond four, then again two warning message dialogs will pop up informing the user that all entries are ignored and the default values are taken instead. The **plot()** command plots all of the simulation results. This completes all verifications.

The specified `MIN_outs = 0; MAX_outs=9;` along with `nargout` verify the number of requested outputs. At this area of the script, the **nargoutchk** function can also be employed to detect the wrong number of requested output variables. If the requested number of outputs is more than nine, the warning message is displayed in the Command Window, a plot figure is displayed, the warning dialog is displayed, and no output variables are obtained. The conditional statements (`if ... elseif... ... end`) with **nargout** determines which output corresponds to which simulation output.

Let's test the `Ex8Var.m` script with different numbers of input arguments and output variables.

1) No input and no output:

```
>> Ex7Var();
No outputs!
```

There will be no computation results except for the plot shown in Figure 2-7.

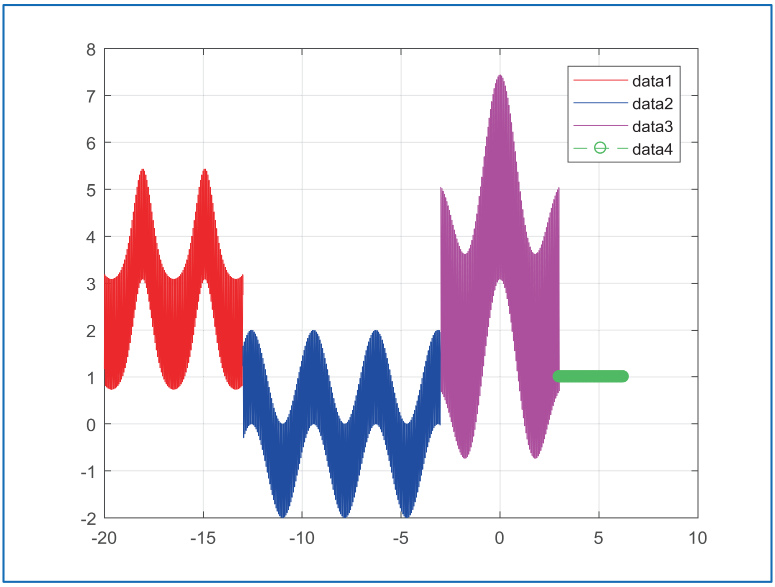


Figure 2-7. *Plot of the simulation results*

2) One input `t` is given and four outputs are requested:

```
>> t=linspace(-20, 6, 1000);
>> [f1, f2, f3, f4]=Ex7Var(t);
>> whos
Name      Size                Bytes  Class     Attributes

 f1        1x269               2152  double
 f2        1x385               3080  double
 f3        1x230               1840  double
 f4        1x116                928  double
 t         1x1000              8000  double
```

Four outputs (`f1`, `f2`, `f3`, and `f4`) are saved in the workspace and the plot figure (shown in Figure 2-7) is displayed.

3) No input and nine output variables are requested:

```
>> clearvars;
>> clearvars; [f1, f2, f3, f4, t_all, t1, t2, t3, t4]=Ex7Var();
>> whos
  Name          Size                Bytes  Class     Attributes

  f1            1x7001              56008  double
  f2            1x10000             80000  double
  f3            1x6000              48000  double
  f4            1x3000              24000  double
  t1            1x7001              56008  double
  t2            1x10000             80000  double
  t3            1x6000              48000  double
  t4            1x3000              24000  double
  t_all         1x26001            208008  double
```

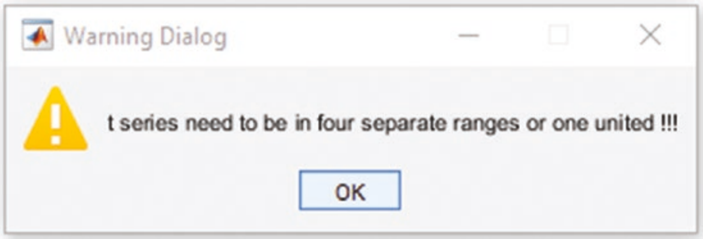
Nine outputs (`f1`, `f2`, `f3`, `f4`, `t1`, `t2`, `t3`, `t4`, and `t_all`) are obtained in the workspace.

4) Three input arguments and no output variables are requested:

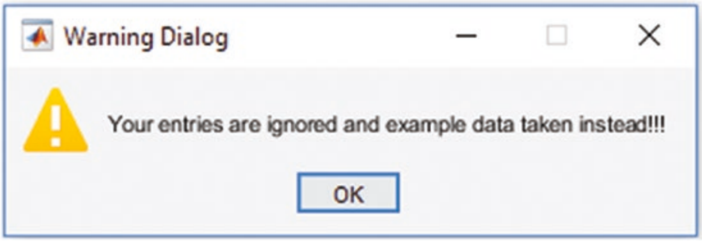
```
>> clearvars; t=-20:.005:6; t3=-2.999:.002:3; t4 = 3.001:1e-2:6;
>> Ex7Var(t, t3, t4);
No outputs!
```

```
>> whos
  Name        Size                 Bytes  Class     Attributes

  t           1x5201               41608  double
  t3          1x3000               24000  double
  t4          1x300                 2400  double
```

There is no output in the workspace from the simulations except for entries (input arguments t, t3, and t4) and there is a plot figure, as shown in Figure 2-7. In addition, the following two warning dialog boxes are displayed, stating that the mismatch of the entries with the necessary t series and as input entries are ignored:



t series need to be in four separate ranges or one united !!!



Your entries are ignored and example data taken instead!!!

5) Three input arguments and three output variables are requested:

```
>>clearvars; t_all=-20:.001:6; In2=-2.999:.001:3; In3 = 3.001:1e-3:6;
>> [Out1, Out2, Out3]=Ex7Var(t_all, In2, In3);
>> whos
  Name        Size                   Bytes  Class     Attributes

  Out1        1x7001                 56008  double
  Out2        1x10000                80000  double
  Out3        1x6000                 48000  double
  t_all       1x26001               208008  double
  In2         1x6000                 48000  double
  In3         1x3000                 24000  double
```

Three outputs (Out1, Out2, and Out3 representing f1, f2, and f3, respectively) from the simulations and entries (input arguments t_all, In2, and In3) are obtained in the workspace. There is a plot figure as well. In addition, there are two warning dialog boxes displayed, as shown in the previous case.

Note While calling/executing the function files, output variable names and input argument names can be altered. That would not cause any errors. This is one of the useful attributes of function files over M-files.

---

6) Four input arguments representing t1, t2, t3, and t4, and nine output variables representing f1, f2, f3, f4, t_all, t1, t2, t3, and tD4 are requested.

```
>> clearvars
t1 = -20:.0025:-13; t2=-12.999:.005:-3; t3=-2.999:.002:3; t4 =
3.001:5e-3:6;
>> [f1, f2, f3, f4, T_all, time1, time2, time3, time4]=Ex8Var(t1, t2, t3, t4);
>> whos
  Name       Size               Bytes  Class     Attributes

  T_all      1x8401             67208  double
  f1         1x2801             22408  double
  f2         1x2000             16000  double
  f3         1x3000             24000  double
  f4         1x600               4800  double
  t1         1x2801             22408  double
  t2         1x2000             16000  double
  t3         1x3000             24000  double
  t4         1x600               4800  double
  time1      1x2801             22408  double
  time2      1x2000             16000  double
  time3      1x3000             24000  double
  time4      1x600               4800  double
```

All requested output variables are obtained in the workspace along with the plot figure, as shown in the first case. Moreover, it should be noted that the input entries `t1`, `t2`, `t3`, and `t4` match the output variables `time1`, `time2`, `time3`, and `time4`, respectively.

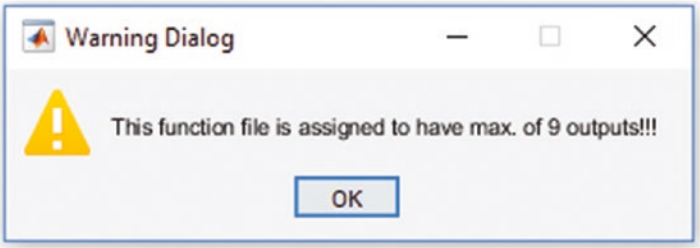
7) Nine output variables representing `f1`, `f2`, `f3`, `f4`, `t_all`, `t1`, `t2`, `t3`, and `t4` are requested without any input arguments.

```
>> clearvars
>> [f1, f2, f3, f4, T_all, time1, time2, time3, time4]=Ex7Var();
>> whos
  Name        Size                 Bytes  Class     Attributes

  T_all       1x26001             208008  double
  f1          1x7001               56008  double
  f2          1x10000              80000  double
  f3          1x6000               48000  double
  f4          1x3000               24000  double
  time1       1x7001               56008  double
  time2       1x10000              80000  double
  time3       1x6000               48000  double
  time4       1x3000               24000  double
```

With no input arguments and only nine output variables, the function file produces nine output variables with the assigned output variable names in the workspace. It does so by using the default set values for t_1, t_2, t_3, t_4 and the plot figure, as shown in the first case.

8) With one input for `t` and 10 outputs:

```
>> clearvars
>> tall=-20:.001:6;
>> [f1, f2, f3, f4, T_all, time1, time2, time3, time4, T_all]=Ex7Var(tall);
Asked  10 outputs are beyond the set outputs !!!
One or more output arguments not assigned during call to "varargout". >>
whos
  Name       Size                 Bytes  Class     Attributes

  tall       1x26001             208008  double
```



This function file is assigned to have max. of 9 outputs!!!

The tenth output variable is beyond the number of assigned output variables. Therefore, the function file does not produce an output variable in the workspace except for two warning messages, the warning dialog, and the plot figure.

There are many other possible scenarios to test this function file for robustness. The robustness of this function file is considerably high within the predefined conditions of the exercise for input arguments in particular. But there are a few other cases in which the output of this function file may not be accurate due to inaccurately chosen entries for `t` or t_1, t_2, t_3, t_4 . For example, one input for `t` might contain just two elements ([-20, 6]) of a row or column vector. Or similarly, the entries for t_1, t_2, t_3, t_4 might also be [-20, -13], [-12.999, -3], [-2.999, 3], [3.001, 6]. In order to improve the robustness of this function file, you can use MATLAB's built-in tool/operator called `nargchk` to verify the allowable number of input arguments along with `nargin`. Likewise, another built-in tool/operator called `nargoutchk` can be used to verify the number of defined output variables along with `nargout`.

Nested and Sub-Functions of Function Files

Function files can contain one or more nested functions (sub-functions) that perform specific computations and analyses. The general syntax of these types of function files is:

```
function [Output1, Output2, Output3, ...] = MAIN(In1, In2, In3, ...)
% MAIN.m contains several nested sub-functions
Output1 = Nest1;
Output2 = Nest2;
Output3 = Nest3;
...
function Out1Nest1()
Out1 = [do sth]
end
```

```
function Out2=Nest2(In2)
Out2 = [do sth]
end

function Out3=Nest3(In3)
Out3 = [do sth]
end
```

Let's look at several examples of employing nested functions in a function file.

Example 8

This example generates three different square matrices (of Pascal, Cauchy, and Krylov) within three nested functions with respect to the main function's input arguments (only integers). Note that input arguments define the size of the output matrices.

Here is the solution script; it's a function file called Ex8_MAIN:

```
function [Output1,Output2, Output3] = Ex8_MAIN(In1,In2, In3)
% Ex8_MAIN.m generates three square matrices within three nested functions
%   Input arguments: In1, In2, In3 are integers defining the size of the
%   output matrices.
%   Nest1 generates the square matrix of Pascal
%   Nest2 generates the square matrix of Cauchy
%   Nest3 generates the square matrix of Krylov
disp(['This is ' num2str(In1)  '-by-' num2str(In1) ' Pascal matrix'])
Output1 = Nest1; disp(Output1)
disp(['This is ' num2str(In2)  '-by-' num2str(In2) ' Cauchy matrix'])
Output2 = Nest2; disp(Output2)
disp(['This is ' num2str(In3)  '-by-' num2str(In3) ' Krylov matrix'])
Output3 = Nest3; disp(Output3)

    function Out1=Nest1()
%   Nest1 generates the square matrix of Pascal
        Out1 = pascal(In1);
    end
    function Out2=Nest2()
```

```
%   Nest2 generates the square matrix of Cauchy
        Out2 = gallery('cauchy',In2);
    end
    function Out3=Nest3()
%   Nest3 generates the square matrix of Krylov
        Out3 = gallery('krylov',In3);
    end
end
```

Now, let's test the function file with three nested functions.

```
>> Input1 = 2; Input2=3; Input3=4;
>> [Output1,Output2, Output3] = Ex8_MAIN(Input1,Input2, Input3);
This is 2-by-2 Pascal matrix
     1     1
     1     2
This is 3-by-3 Cauchy matrix
    0.5000    0.3333    0.2500
    0.3333    0.2500    0.2000
    0.2500    0.2000    0.1667
This is 4-by-4 Krylov matrix
    1.0000    0.6731    3.8494   -2.7613
    1.0000    1.7986    2.3591    4.8361
    1.0000   -0.8049    1.0833   -0.5926
    1.0000    1.7760   -1.1635    1.2842
>> whos
  Name         Size            Bytes  Class     Attributes

  Input1       1x1                 8  double
  Input2       1x1                 8  double
  Input3       1x1                 8  double
  Output1      2x2                32  double
  Output2      3x3                72  double
  Output3      4x4               128  double
```

Function Files Within M-Files

In recent versions of MATLAB, the function files (including nested and sub-functions) can also be employed within M-files. This makes the computation process more efficient and saves memory space in the workspace. Take a look at the following example.

Example 9

Given $y(x, t) = y_0 \sin(kx - \omega t)$, which is the solution of the wave equation, where $k = \frac{2\pi}{\lambda}$ is the wave number, $\omega = 2\pi f$ is the angular frequency, and λ is the wavelength.

Write an M-file with two nested functions to compute the values of $y(x, t)$ and four input arguments of k, ω, x, and t.

Given: $y_0 = \frac{1}{2}; \lambda = 495 * 10^{-9}; k = \frac{2\pi}{\lambda}; f = 5000; \omega = 2\pi f; x = [0,1], t = [0,5];$ $N = 10^6$ (the number of data points for x and t).

The solution script called Ex9_wFUN.m, with two embedded functions, follows:

```
y0=1/2;          % Magnitude of the wave
lambda=495e-9;   % Visible light wave length
k=2*pi/lambda;   % Wave number
f=5000;          % Frequency
omega=2*pi*f;    % Angular frequency
x=[0, 1e-5];     % Length
t=[0, 1e-3];     % Time length
N=1e5;           % Number of data points to be computed & simulated
xs=linspace(x(1), x(2), N);    % Wave Length series
time=linspace(t(1), t(2), N);  % Time series

y1 = F1(y0, k, xs, omega, time);
y2 = F2(y0, k, time, omega, xs); % NB: it is vital the order of the
variables: xs vs. time && V5 vs. V3
plot(xs, y1, 'bo', xs, y2, 'rx-'), grid on, hold on
title('Nested Function files within an M-file')
ORG0 = y0*sin(k*xs-omega*time);
plot(xs, ORG0, 'k', 'linewidth', 2)
legend('Fun1','Fun2','Original'), shg
```

```
function out1 = F1(var1, var2, var3, var4, var5)
% M-file nested function 1 called: F1
out1=var1*sin(var2*var3-var4*var5);
end

function out2 = F2(V1, V2, V3, V4, V5)
% M-file nested function 2 called: F2
% NB: it is vital the order of the variables: xs vs. time && V5 vs. V3
out2=V1*sin(V2*V5-V4*V3);
end
```

This M-file can be executed by clicking Run in the menu panel or can be called from another M-file or from the Command Window. If you executed the file, you would get the following output:

```
>> clearvars
>> Ex9_wFUN
>> whos
  Name         Size                Bytes  Class     Attributes

  N            1x1                     8  double
  ORG0         1x100000           800000  double
  f            1x1                     8  double
  k            1x1                     8  double
  lambda       1x1                     8  double
  omega        1x1                     8  double
  t            1x2                    16  double
  time         1x100000           800000  double
  x            1x2                    16  double
  xs           1x100000           800000  double
  y0           1x1                     8  double
  y1           1x100000           800000  double
  y2           1x100000           800000  double
```

In addition, the plot in Figure 2-8 is created.

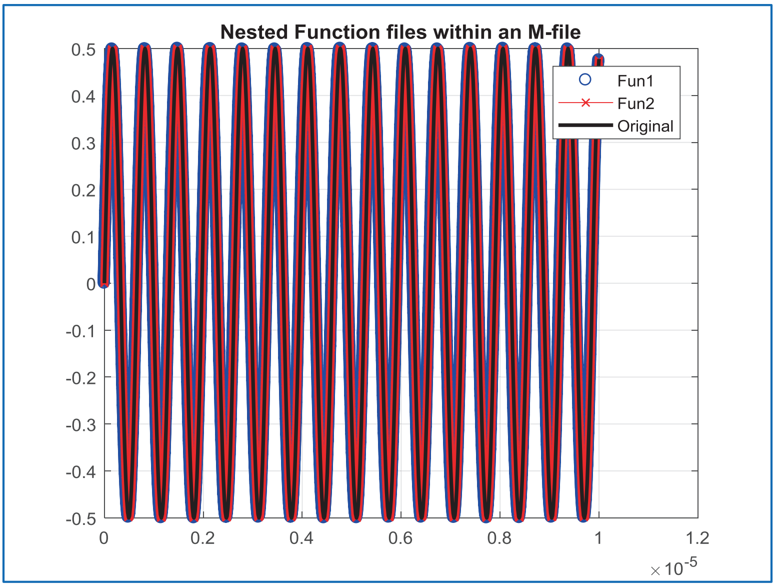


Figure 2-8. *Comparing the simulations obtained from the nested functions within the M-file*

The simulation results show that the M-file with the embedded functions work like any other M-file. Like function files, the order of the input arguments is vital for the embedded functions within M-files. If the order is mixed up, the outputs will be incorrect due to the fact that one variable's values will be used for another.

Summary of Scripts and Function Files

Warning and error messages included in M-files and function files are of great help not only to users but also to developers. Thus, while you're writing scripts (programs), it is important to add warning and error messages along with additional hints (as comments).

Consider these essential hints on how to write robust and efficient M-files and function files:

- Do as much checking (of input and output arguments, e.g., size and data type) as possible before executing a whole script to avoid the heavy burden of time-consuming calculations that may result in incorrect outputs.

- Always start with the most general checks (obvious ones) and then move to more complicated and complex ones. Add warning and error messages wherever necessary or appropriate. For example, add a unique example for display (e.g., `disp(‘This is ...'))` during the debugging process when you have varying numbers of input arguments and output variables. These messages not only help developers spot errors but also go through all anticipated scenarios in the code/script.
- Explain how to correct errors when they occur (e.g., `warning('Do something about this and that ...', A, B, C)`) and (e.g., `error('There is an error on this and that ...', Num(1), Den(3), u(1))`).
- Add warning message identifiers so users can turn warnings on and off.
- Add numerical examples to test your script with known correct solutions.
- Use the M-file profiler to check the efficiency of your M-files. To use the profiler, type `>> help profile` or `>> doc profile`.

Inline Functions

The MATLAB `inline` command lets users develop analytical expressions with input variables and assign the expressions to a variable. A general syntax of creating an `inline` function expression is as follows:

```
f=inline('[expression1; expression2, ...]','arg1','arg2','arg3', ...);
```

In this command syntax, `expression1`, `expression2`, and so on, are inline functions and `arg1`, `arg2`, `arg3`, and so on are input variables.

Example 1

$f(t)=\frac{sin(2t)}{e^{2t}}$ can be expressed via the `inline` function without any argument definition, since there is one variable that is identified automatically.

```
>> f = inline('sin(2*t)/exp(2*t)');
```

Example 2

$f(t, \theta) = [e^{-2t\theta}; sin(2t)]$ writes an `inline` function of $f(t, \theta)$. Arguments must be specified. In this case, there are two variables, $t \wedge \theta$. Thus, the given function expressions can be expressed via the following `inline` function:

```
>> F = inline('[exp(2*t.*theta); sin(2*t)]', 't', 'theta');
```

That can be tested with pre-defined arguments, such as `t` and `theta`.

```
>> t=0:pi/20:pi; theta=linspace(0,1,length(t));
```

You can run calculations with the `inline` function `F` simply by recalling the function with predefined input arguments.

```
>> fcalc=F(t, theta);
```

In addition, it is possible to perform computations and plot computed data from the `inline` function `F` simultaneously.

```
>> plot(t, log(F(t, theta)), 'linewidth', 1.5);
```

Example 3

Given a second order differential equation $\ddot{y}+\dot{y}+3y=t^2$, it can be expressed by two first order differential equations.

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = t^2 - (y_2 + 3y_1) \end{cases}$$

These two first order differential equations can be defined using the following `inline` function:

```
f =inline('[y(2); t^2-(y(2)+3*y(1))]', 't', 'y');
```

Based on these examples, you can see that it is very easy and straightforward to use the `inline` function tool not only in the Command Window but also in scripts that solve various computation problems.

Note The inline command will be removed in future releases of MATLAB so it is recommended that you use the anonymous functions instead.

Anonymous Functions with Handles (@)

An anonymous function is a function that is not stored in a program file. It is associated with a variable and is called with a function handle (@). In general, the function handles accept input variables and return output variables, similar to function files (as demonstrated previously). One major difference between a function file and an anonymous function expression is that an anonymous function with a handle (@) contains a single executable statement. The general syntax of anonymous functions with handles is:

```
Fun_handle=@(arg1, arg2, arg3, ...)([expression1; expression2;...])
```

Example 1

$$f(t)=\frac{sin(2t)}{e^{2t}}$$

The given function *f*(*t*) can be expressed via a function handle with the following command.

```
>> f_handle = @(t)(sin(2*t)./exp(2*t));
>> t=linspace(0,2*pi, 100);
>> f=f_handle(t); plot(t, f, 'o-')
```

It is also feasible to use function handles within loop iterations of scripts; for instance, Fun_handle.m.

```
% Fun_handle.m
f_handle = @(t)(sin(2*t)./exp(2*t));
t=linspace(0,2*pi, 100);
for ii=1:length(t)
    f(ii)=f_handle(t(ii));
end
plot(t, f, 'o-')
title('plot: f(t)=(sin(2*t)./exp(2*t)')
```

Example 2

Given $f(t, \theta) = [e^{-2t\theta}; \sin(2t)]$.

The given function $f(t, \theta)$ can be expressed via one function handle:

```
>> F_handle = @(t, theta)([exp(2*t.*theta); sin(2*t)]);
```

Let's specify data values for the input arguments `t` and `theta`.

```
>> t=0:pi/20:pi; theta=linspace(0,1,length(t));
```

In order to obtain computation results from the function handle `F_handle`, it needs to be recalled with input arguments `t` and `theta`.

```
>> Fvalues=F_handle(t, theta);
```

Example 3

Given $\ddot{y} + \dot{y} + 3\mathrm{y} = t^2$

It can be expressed using two first order differential equations in a state-space form.

$$\begin{cases} \dot{\mathrm{y}}_1 = y_2 \\ \dot{y}_2 = t^2 - (y_2 + 3y_1) \end{cases}$$

This derived system of two first order differential equations can be defined via the following function handle (`@`):

```
f =@(t, y)([y(2); t^2-(y(2)+3*y(1))]);
```

The function handle (`@`) tool of an anonymous function can also be implemented like an `inline` function to solve many kinds of problems, including differential equations.

Up to this point, I have demonstrated how to employ function files, `inline` functions, and anonymous function tools. There are some advantages and disadvantages of employing anonymous functions defined with handles (`@`) and `inline` functions in comparison to function files. Anonymous functions are easy to implement and design, and there is a very small chance of getting confused about function names and input arguments. On the other hand, function files are much more efficient and flexible in terms of computation time. This is very important when numerical simulation space is large. Moreover, function files can be employed within Simulink models.

How to Improve Script Performance

So, how do you speed up the simulation/computation processes? When you are dealing with large data sets, you may face memory problems. In order to prevent memory problems or errors in MATLAB, or just to improve computation efficiency, the following tips are recommended:

- Avoid large temporary variables, break variables into smaller components, and clear variables (using `clearvars`) when they are no longer needed.
- Use lower precision data types whenever there's no need for higher precision.
- Pre-allocate memory for arrays of fixed size to reduce defragmentation by using standard matrices such as `zeros()` and `ones()`.
- Use the `pack()` function to defragment memory.
- Consider writing data to a disk periodically.
- Increase the size of the swap file. That can be done via My Computer ➤ Properties ➤ Advanced ➤ Performance Settings. Note that it is recommended to set the swap file size to twice your computer's RAM.

Other techniques to speed up the simulation processes:

- Functions (function files) are faster than scripts
- Load and save are faster than file I/O functions when you need to import or export data sets
- Avoid large background processes
- Replace argument checking with `try-catch`
- Use `switch-case` over `if-elseif-else`
- Use sparse arrays for sparse data

That concludes the discussion of basic programming issues.

CHAPTER 3

Graphical User Interface Model Development

MATLAB has a number of graphical user interface (GUI) tools and functions that can be employed while building GUI models, either by using the GUI tools or by writing scripts. In this chapter, we cover how to build GUI models with the GUI development environment (GUIDE) and how to write scripts to generate popup and dialog boxes.

GUIDE

There are three principal groups of elements necessary to build a MATLAB GUI model. They are components, figures, and callbacks.

- MATLAB GUI components are graphical controls (push buttons, edit boxes, lists, sliders, and popup menus), static elements (frames, text strings), menus, and axes. Graphical control elements and static elements can be created by using the `uicontrol` function. Menus can be generated by using the `uimenu` and `uicontextmenu` functions. The `axes` function is used to display graphical data.
- The `figure` function is used to create figures that accommodate various combinations of components.
- Callbacks or callback functions are used to perform actions (simulations and displays) with respect to user entries, such as clicks with a cursor (mouse or touch screen) or entries from the keyboard.

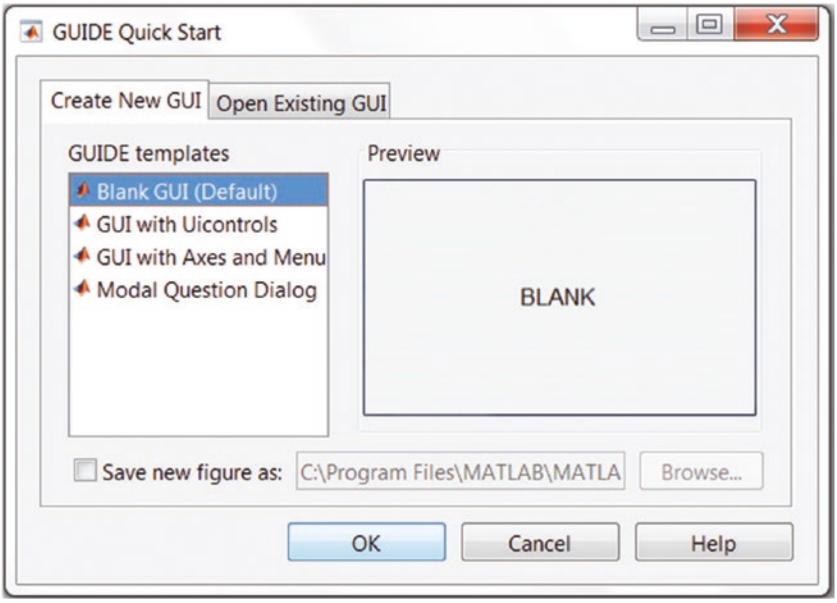
S. Eshkabilov, *Beginning MATLAB and Simulink*, https://doi.org/10.1007/978-1-4842-5061-7_3

All of these components and figures and their properties and behaviors can be created by writing scripts, functions, and callback functions using MATLAB's built-in functionality tools. However, this approach is rather cumbersome and long, and it requires a considerable amount of code and programming. An alternative and rather straightforward approach to designing a GUI model is MATLAB's user-friendly GUI development environment (GUIDE) tool.

GUIDE contains templates, including GUI with UI controls, GUI with axes and menus, and the modal question dialog. By using this tool, you can rather easily create a GUI model layout with all the necessary components, such as push buttons, menus, static elements, text, and so forth, and adjust their properties (size, color, type, etc.) and location with respect to the GUI model design. This can be done by using the mouse and keyboard, without any programming. Note that MATLAB (starting from MATLAB R2016a) introduced a user friendly drag-and-drop application development environment called App Designer. It's similar to GUIDE and in future releases, App Designer and GUIDE will be integrated. Here we demonstrate how to use GUIDE tools to create GUIs.

Many GUI controls in GUIDE are quite straightforward and building a GUI model by employing GUI control tools is not difficult. We first launch the GUIDE tool and become familiar with its components and tools. Then we move on to working with GUI's Property Inspector tools and modifying the operational behavior of the GUI blocks used in our model. Using this approach, after completing GUI modeling with GUI blocks, we save the model. MATLAB will automatically generate a script function file (M-file) that we modify according to our given GUI model requirements.

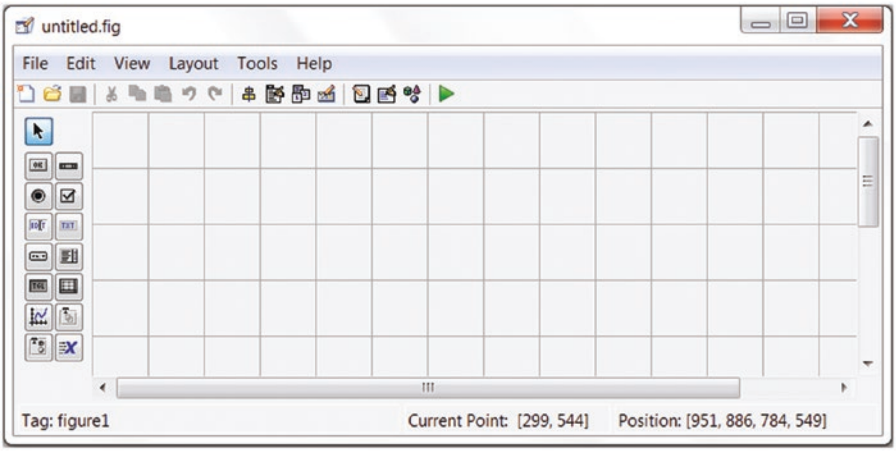
We can launch the GUIDE application by typing `guide` in the command window or choosing New ➤ App ➤ GUIDE from the menu bar. Subsequently, the GUIDE Quick Start window appears, as shown in Figure 3-1.



□ Save new figure as: C:\Program Files\MATLAB\MATLA

Figure 3-1. *GUIDE Quick Start window*

From these options, we can select a blank GUI (the default) or the ready-to-use example models of MATLAB, which are GUI with UIcontrols, GUI with Axes and Menu, or the Modal Question Dialog. When we choose Blank GUI (Default), the blank window shown in Figure 3-2 (named `untitled.fig` by default) will open.



Tag: figure1 Current Point: [299, 544] Position: [951, 886, 784, 549]

Figure 3-2. *GUI figure window components without names*

From this GUI layout window (Figure 3-2), we can start working. We can see the name tags of the GUI components and tools by selecting Show Names on Component Palette from the the MATLAB GUIDE Preferences. With this adjustment, the components will appear with their name tags, as shown in Figure 3-3.

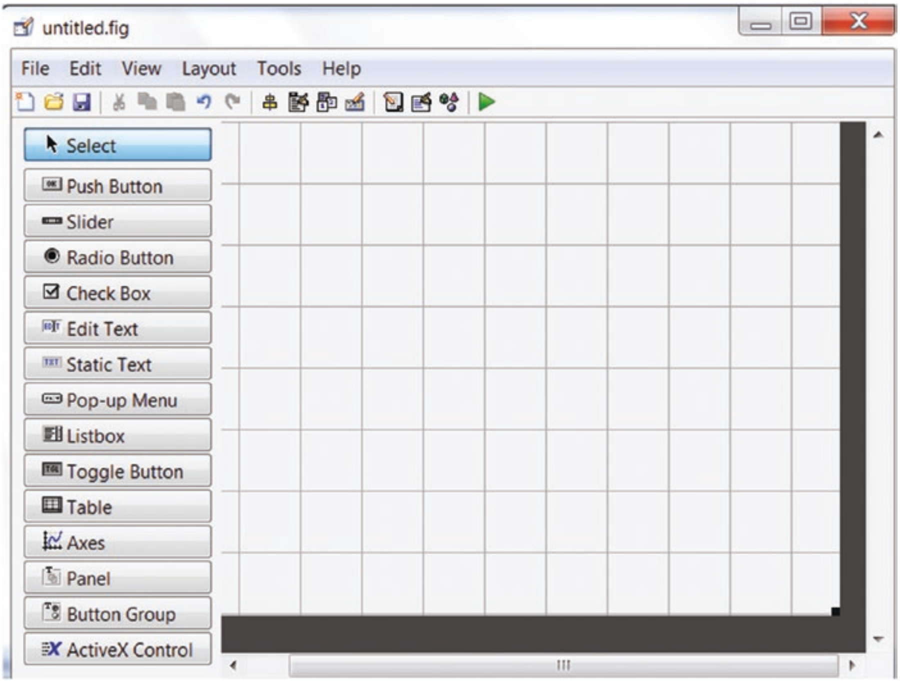


Figure 3-3. *GUI components with name tags displayed*

The GUI window shown in Figure 3-4 is composed of the GUI components, the design area, and the menu tools called Align Objects, Menu Editor, Tab Order Editor, etc. Once the GUI window is open, we can start building our GUI model by dragging the desired GUI components and dropping them into the design area (see Figure 3-4). We can resize and align the GUI components in our design area by clicking and dragging with a mouse or by using the right mouse button options or the Property Inspector, Align Objects, and other menu bar tools. Moreover, we can edit properties of the GUI components by clicking on the Editor and add menu tools to the GUI by clicking on the Toolbar Editor.

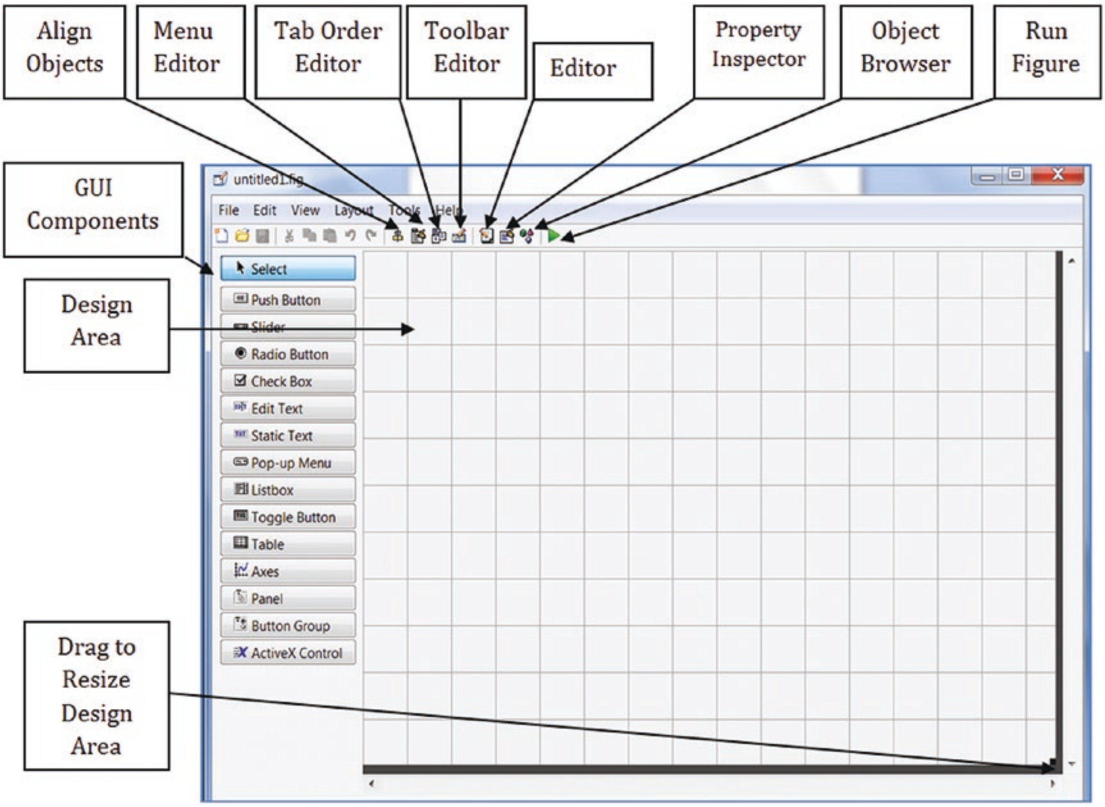


Figure 3-4. *The GUI GUIDE tool window and its menu bar*

There are several basic steps required to build a GUI model:

1. Clearly define the specific functions that the anticipated GUI model should have and which tools and components you need. As always, it is best to start with a draft sketch of the GUI model on a piece of paper.

2. Drag and drop all the necessary GUI components into the design area, and then modify and align them according to your needs and project requirements. Once every component is in place and adjusted with respect to each other, you can pull them into a Button Group (there is a component called Button Group), which gives you good flexibility for moving and aligning objects in the design area. Note that grouping components into a Button Group is optional.

3. Rename or provide recognizable name "tags" to all components used in the model so you can easily identify them in later stages while editing scripts and M-files. Moreover, you may need to adjust and modify the color, font, and size of the components. Note that changing the color, font size, text type, size, and position of components in the model will not affect the execution/simulation results of the GUI model.

4. Save the created work. It will be saved in two different file formats, one of which is `*.fig` (the Figure file) and the other is `.m` (the function file), containing a complete script of the model, including all nested callback functions for each GUI component.

5. Write/edit a script that implements the performance of each GUI component in association with the other components. This step requires some additional programming work, which will be discussed in the following examples.

Example 1: Building a 2D Plot

Let's take a very simple example of building a 2D plot of a cardinal sine function for a user's input value ranges: $sinc(\pi x) = \frac{sin(\pi x)}{\pi x}$.

By dragging and dropping the Push Button, Static Text, Edit Text, and Axes blocks, we start building the interface of the GUI model, as shown in Figure 3-5. After adjusting all the blocks in the GUI model with respect to each other, we can edit their properties by double-clicking on each block or using the Property Inspector option via the right mouse. Note that Static Text serves as information to show the input entry names: Xmin and Xmax. The values of Xmin and Xmax represent boundary values of the variable and can be defined in the Edit Text blocks.

Note There is one major difference between Static Text and Edit Text. The user can edit an entry in an Edit Text block, but cannot do so in Static Text block.

In addition, we add an Axes block that displays a plot of computed values of the function *sinc*(x) according to our entries for x_{min} and x_{max} in the Edit Text blocks. We add a Push Button block that makes the built model (see Figure 3-5) compute the values of the function and display the results in the plot area.

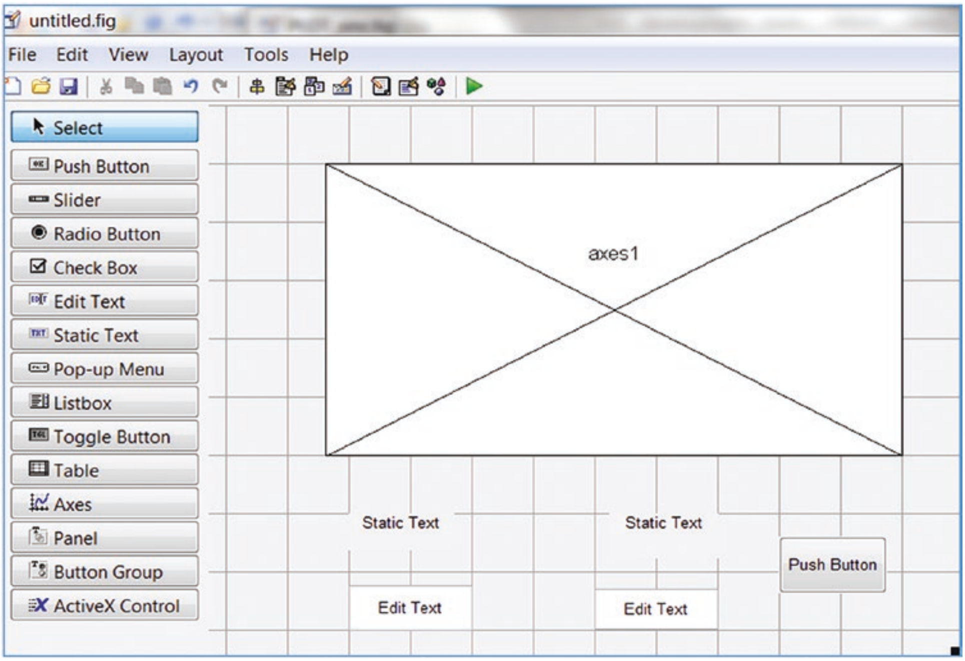
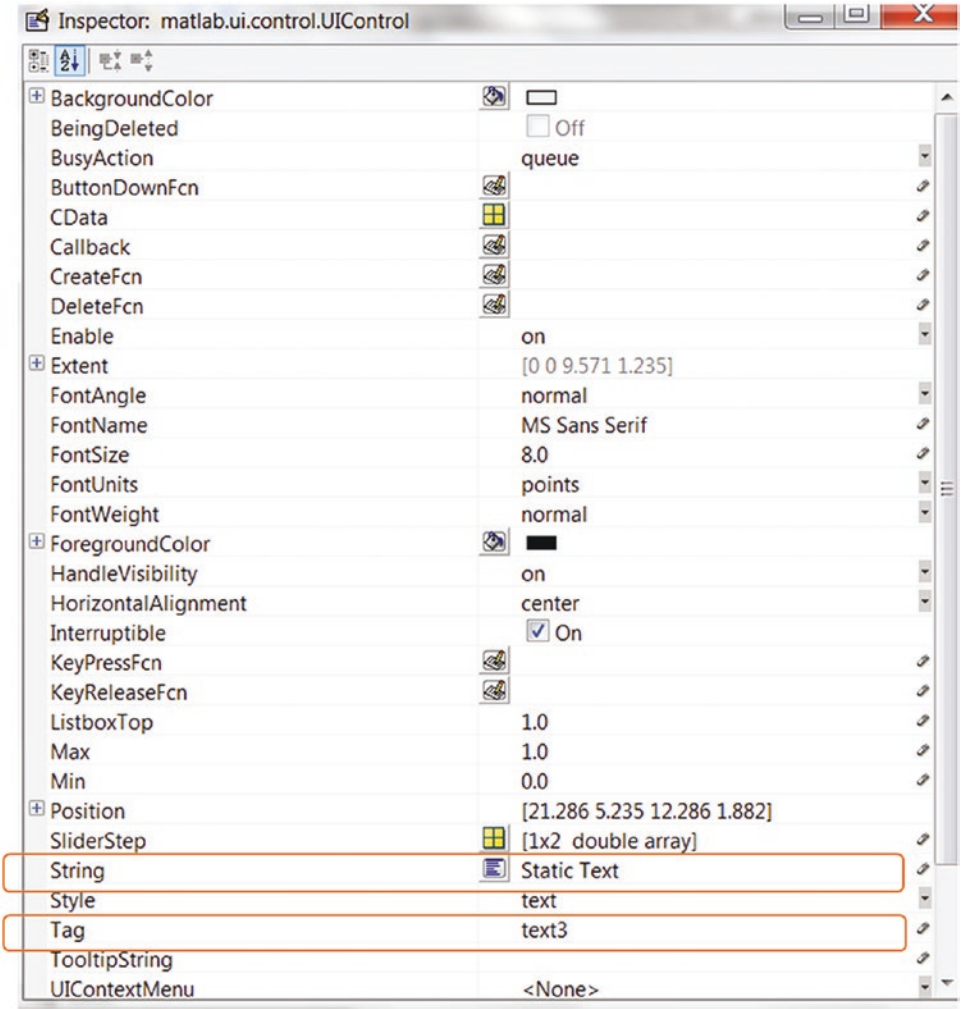


Figure 3-5. *Necessary blocks selected and placed onto the GUI model area*

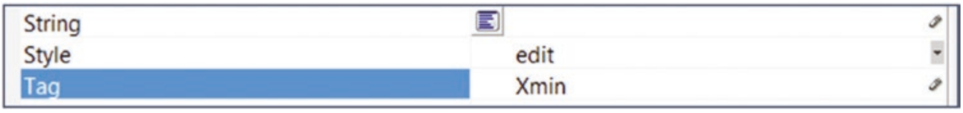
In order to make our GUI model more informative, legible, and user friendly, we can edit each block by altering the font size, color, background color, and so forth. There are many properties for each individual block that can be altered. For instance, we can change the front color, background color, font type, size and color, position, size, string, name, and a few other properties (see Figure 3-6). Among these properties, there are two main ones (see Figure 3-6) that we have to pay close attention to. The first one is `tag` and it's the name of a block and has to be a distinctive name. The second property is `string` and it's used to display fixed text or an empty space. The `string` is displayed in the GUI model.



BackgroundColor	□
BeingDeleted	□ Off
BusyAction	queue
ButtonDownFcn	
CData	
Callback	
CreateFcn	
DeleteFcn	
Enable	on
Extent	[0 0 9.571 1.235]
FontAngle	normal
FontName	MS Sans Serif
FontSize	8.0
FontUnits	points
FontWeight	normal
ForegroundColor	■
HandleVisibility	on
HorizontalAlignment	center
Interruptible	☑ On
KeyPressFcn	
KeyReleaseFcn	
ListboxTop	1.0
Max	1.0
Min	0.0
Position	[21.286 5.235 12.286 1.882]
SliderStep	[1x2 double array]
String	Static Text
Style	text
Tag	text3
TooltipString	
UIContextMenu	<None>

__Figure 3-6.__ Property Inspector window to edit properties of blocks

Let's make some changes to our GUI model blocks by altering their background color, size, font size, color, and type, and adjusting the positions of the blocks. Note that in the Static Text blocks, we edit the strings to be "Xmin" and "Xmax" in bold, 12.0 font size, and red and blue. In the Edit Text blocks, we remove their strings and make them empty. Moreover, we change their font size to 12.0, make them bold, and have a yellow background color.



String	
Style	edit
Tag	Xmin

Note For the Edit Text blocks to have pre-defined values (e.g., -6, 0, 3.14, and 100), we need to insert these values into the "String" value of each Edit Text block.

The Push Button's background color, font size, type, string, and tag names are modified. Its string is altered to `"PLOT"` and its tag is renamed to `PLOTsinc`. Figure 3-7 shows the completed GUI model.

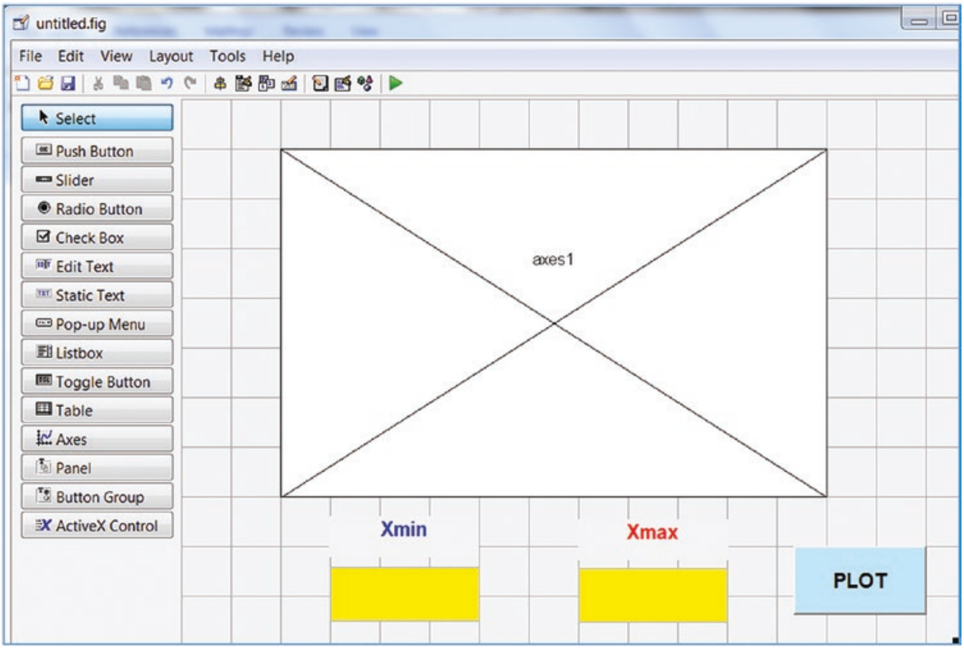


***Figure 3-7.** The completed GUI model with all modified buttons and components*

Save the model and click on the Run button or press Ctrl+T from the keyboard. The new model, as shown in Figure 3-8, does not reflect any results yet. There is one important step that is still missing in the model's coding part.

Note While building a GUI model and editing the properties of blocks, it is important to rename each block distinctively. The blocks' tags are required when modifying the callback functions of the GUI model.

In order to change the properties of several blocks simultaneously, select them all and then go to the Property Inspector to make changes.

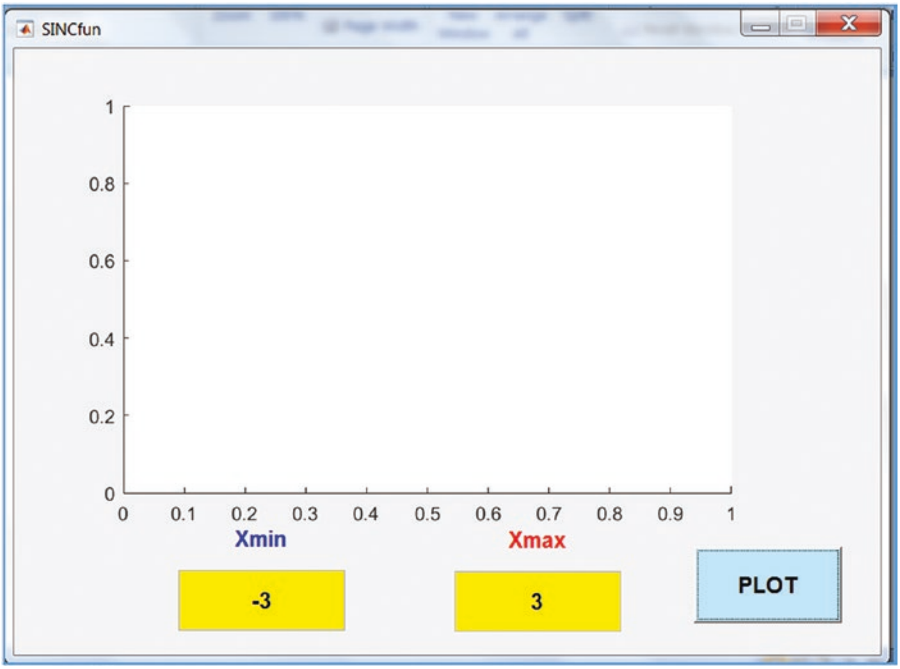


Figure 3-8. *The created SINC function plot GUI*

MATLAB saves the model in two file formats—`SINC_fun.m` (the function file) and `SINC_fun.fig` (the GUI figure). All the callback functions of the buttons used in `SINCfun.m` are shown here:

```
function varargout = SINCfun(varargin)
% SINCFUN MATLAB code for SINCfun.fig
% SINCFUN, by itself, creates a new SINCFUN or raises the existing
% singleton*.
...
% Begin initialization code - DO NOT EDIT
gui_Singleton = 1;
gui_State = struct('gui_Name', mfilename, ...
 'gui_Singleton', gui_Singleton, ...
 'gui_OpeningFcn', @SINCfun_OpeningFcn, ...
 'gui_OutputFcn', @SINCfun_OutputFcn, ...
 'gui_LayoutFcn', [] , ...
 'gui_Callback', []);
```

```
if nargin && ischar(varargin{1})
 gui_State.gui_Callback = str2func(varargin{1});
end

if nargout
 [varargout{1:nargout}] = gui_mainfcn(gui_State, varargin{:});
else
 gui_mainfcn(gui_State, varargin{:});
end
% End initialization code - DO NOT EDIT
% --- Executes just before SINCfun is made visible.
function SINCfun_OpeningFcn(hObject, eventdata, handles, varargin)
% This function has no output args, see OutputFcn.
...

function Xmax_Callback(hObject, eventdata, handles)
% hObject handle to Xmax (see GCBO)
...
% --- Executes on button press in PLOTsinc.
function PLOTsinc_Callback(hObject, eventdata, handles)
% hObject handle to PLOTsinc (see GCBO)
...
```

Note that most of this script (`SINC_fun.m` is automatically generated by the GUI model when it is saved) won't be changed and only one callback function will be edited. The callback functions have to be edited to make the GUI model perform the anticipated computations and display a plot figure. Note that every callback function has three parameter handles—`hObject`, `evendata`, and `handles`.

While editing/writing callback functions, we can look up the properties of handles using the `get()` function and assign or change any property of the handles by using the `set()` function. Note that in the `SINCfun.m` model, there is only one callback function that has to be edited—`PLOTsinc_Callback(hObject, eventdata, handles)`. The `PLOTsinc_Callback` callback function invokes the PLOT button, which is named with a tag name of `PLOTsinc`. The `PLOTsinc` tag name is given to the PLOT button via the Property Inspector while building the GUI model. This callback function computes $sinc(\pi x)$ by taking user entries for x_{min} and x_{max} and plotting the computed results.

To edit the `PLOTsinc_Callback(hObject, eventdata, handles)` callback function, we first convert the entries for Xmin and Xmax from the string format into the double format in the following way:

```
Xmin=str2double(get(handles.Xmin, 'string'));
Xmax=str2double(get(handles.Xmax, 'string'));
```

The `get()` command collects data from `handles.Xmin` and `handles.Xmax` as a string and then `str2double()` converts them into a numerical format. Double-formatted values of Xmin and Xmax are taken to generate equally spaced values of the variable x, which are then taken to compute f(x) = $sinc(\pi x)$ and plot the results using the following commands:

```
x=linspace(Xmin, Xmax, 200); f=sinc(x*pi); plot(x, f, 'b');
grid on
title('sinc(\pi*x) = sin(\pi*x)/(\pi*x) ')
xlabel('x'),ylabel('f(x)=sinc(\pi*x)'),
axis([Xmin, Xmax, -.25, 1.1])
```

Finally, the edited callback function `PLOTsinc_Callback(hObject, eventdata, handles)` contains the following:

```
function PLOTsinc_Callback(hObject, eventdata, handles)
Xmin=str2double(get(handles.Xmin, 'string'));
Xmax=str2double(get(handles.Xmax, 'string'));
x=linspace(Xmin, Xmax, 200);
f=sinc(x*pi);
plot(x, f, 'b'); grid on
title('sinc(\pi*x) = sin(\pi*x)/(\pi*x) ')
xlabel('x'),ylabel('f(x)=sinc(\pi*x)'),
axis([Xmin, Xmax, -.25, 1.1])
```

As noted, this is the only part of the M-file `SINCfun.m` that we have edited in order to make the model compute and plot the $sinc(\pi x)$ function with a single click in the GUI model. We save the `SINCfun.m` file and execute it by using ▷ Run in M-file editor or by pressing F5 from the keyboard. When we run `SINCfun.m`, the `SINCfun.fig` GUI model pops up. We enter two entries—$X_{min} = -5$ and $X_{max} = 6$—and click the PLOT button.

The results of the function are computed and plotted, as shown in Figure 3-9. Note that the value of Xmax must be larger than the value of Xmin; otherwise, there will be an error.

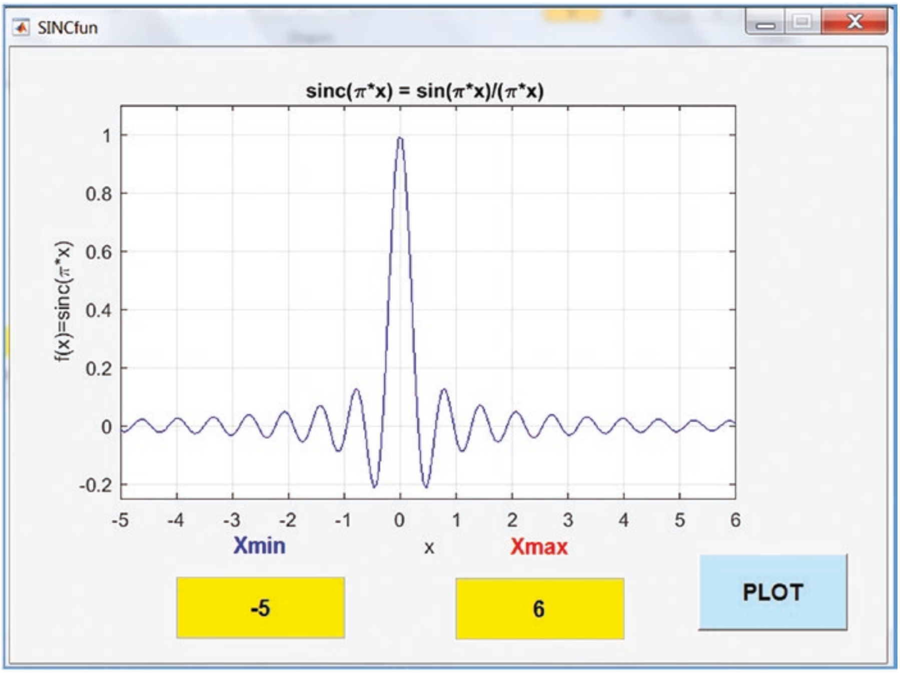


***Figure 3-9.** The GUI model to plot the sinc(πx) function*

This simple example shows how easy it is to use GUIDE tools to build GUI models. Once the GUI model interface layout is complete and the components' properties have been edited and saved, the M-file's (function file's) callback functions are generated. Subsequently, by editing only the required callback function or functions, we obtain the GUI model.

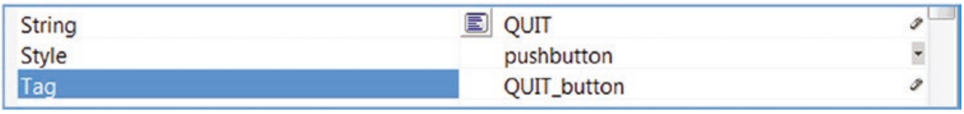
Example 2: Adding Functionality

Let's look at four more options for our GUI model (Figure 3-9):

- Create an Exit/Quit button to close a GUI model window
- Play a sound by recalling and executing another M-file (called `SOUND_hear.m`)

- Display an image (called GC2011.jpg) in *.jpg/*.jpeg format
- Display a message box with a message of "All Done Well Done!!!"

There are several ways to accomplish these tasks, one of which is to add a Push Button block and edit its callback function. We first go back to our GUI model editor and add one Push Button block. Change the Push Button's properties (background color, font size and type, and string and tag) as we did previously. Now we rename the Push Button block to QUIT by changing its string; its name tag is renamed QUIT_button. Before saving the updated GUI model SINCfun.fig, we need to alter the name of the window. To do that, we need to double-click on the design area (the area outside of any blocks that opens the Property Inspector window) and then change the name tag from figure1 (the default) to GUI_window.

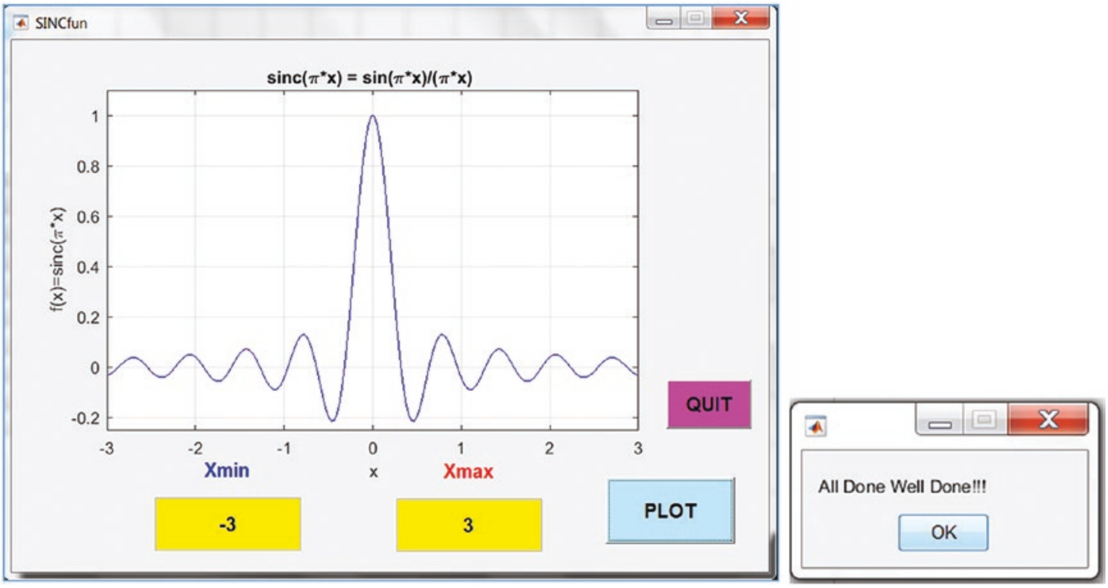


String	QUIT
Style	pushbutton
Tag	QUIT_button

We save the GUI model and execute it. After that, we edit the last automatically added callback function called QUIT_button_Callback(hObject, eventdata, handles) by adding the following to it. The built-in MATLAB function called delete() halts the whole process and shuts the window.

```
function QUIT_button_Callback(hObject, eventdata, handles)
delete(handles.GUI_window)
run('SOUND_hear.m'); A=imread('GC2011.jpg'); image(A);
msgbox('All Done Well Done!!!')
```

Figure 3-10 shows the updated GUI model. We can test it with two numerical entries for Xmin and Xmax and then click on the QUIT button. After pressing QUIT, the GUI window SINCfun.fig is closed.



All Done Well Done!!!

Figure 3-10. *The GUI model to display the sinc(πx) and quit options*

Subsequently, the built-in function `run()` executes the `SOUND_hear.m` file, which plays a sound wave. Another built-in function called `image()` displays the `GC2011.jpg` image, as shown in Figure 3-11. Finally, the `msgbox()` function displays the message: `All Done Well Done!!!`.

The separate M-file called `SOUND_hear.m` contains the following scripts:

```
% SOUND_hear.m
% Three CHIRP signals to make a sound
t=0:1/1e4:10;
D0=chirp(t, 0, 10, 3000, 'quadratic');
D1=chirp(t, 0, 10, 4000, 'q',[],'convex');
D2=chirp(t, .001, 10, 5000, 'logarithmic');
y=[D0, D1, D2]; sound(y);
```

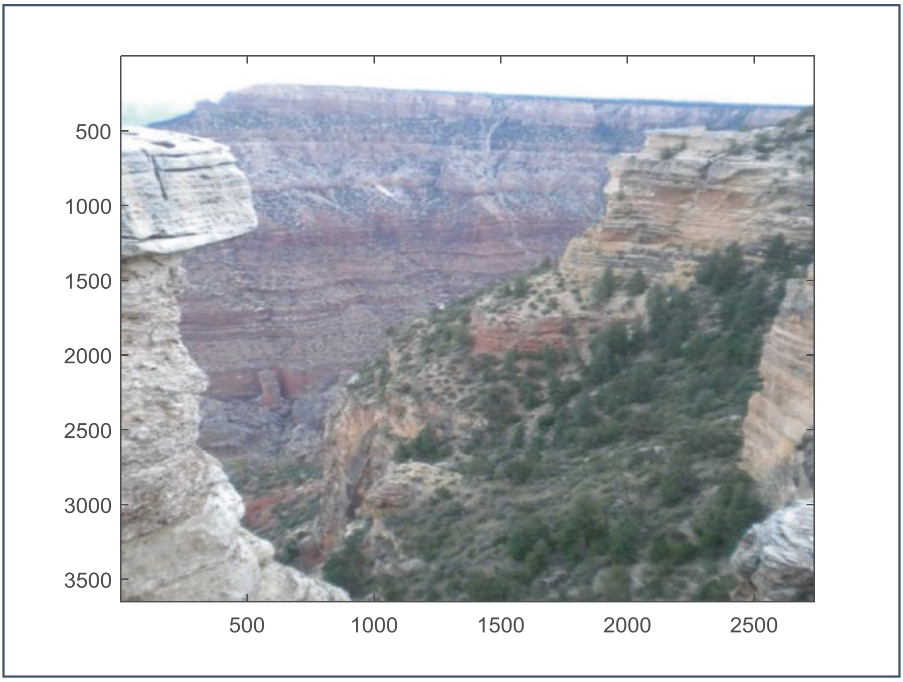


Figure 3-11. *Image file GC2011.jpg is displayed*

In this simple exercise, we have demonstrated how to associate GUI model buttons with other GUI blocks and M-files. Similarly, Simulink's files, as well as any compatible applications with a MATLAB package, can be linked to GUI models.

Example 3: Solving a Quadratic Equation

Let's take another example of creating a GUI model that solves a quadratic equation for any given coefficients for *a, b,* and *c* and displays a plot for the user's specified value ranges of the variable *x*. The quadratic equation is formulated by $ax^2 + bx + c = 0$ and if the numerical values of *a, b,* and *c* are given, we can compute the roots of the given equation numerically. This exercise is completed in two stages, building the GUI interface and editing the callback functions.

Building the GUI

We start again with a blank GUI window and choose a Panel block, onto which we drag and drop several blocks, such as Static Text, Edit Text, Push Button, and Axes. There is a good reason for employing the Panel block when building a GUI model. It helps you manipulate all the GUI blocks within it and around the Design Area with respect to each other.

We place the blocks in *a, b, c* order and then compute the results for *D* (discriminant), the root for x_1 and x_2, and the Push Button block to make the GUI model compute the discriminant, root x_1, and root x_2 values. After that, we edit the block properties by changing the font size, type, string, tag and background color of each block. Moreover, we edit the user's entry blocks for x_{min} and x_{max} and use a Push Button block to plot the computation results in 2D. In addition, we change the name of the panel window to "Quadratic Equation Solver" by double-clicking on the Design Area and changing the Title in the Property Inspector. Figure 3-12 shows our completed GUI model window.

Note In this GUI model (shown in Figure 3-12), the component names are not shown in order to save space in the palette window. You can turn them on/off by choosing File ➤ Preferences ➤ GUIDE ➤ Show Names in Component Palette.

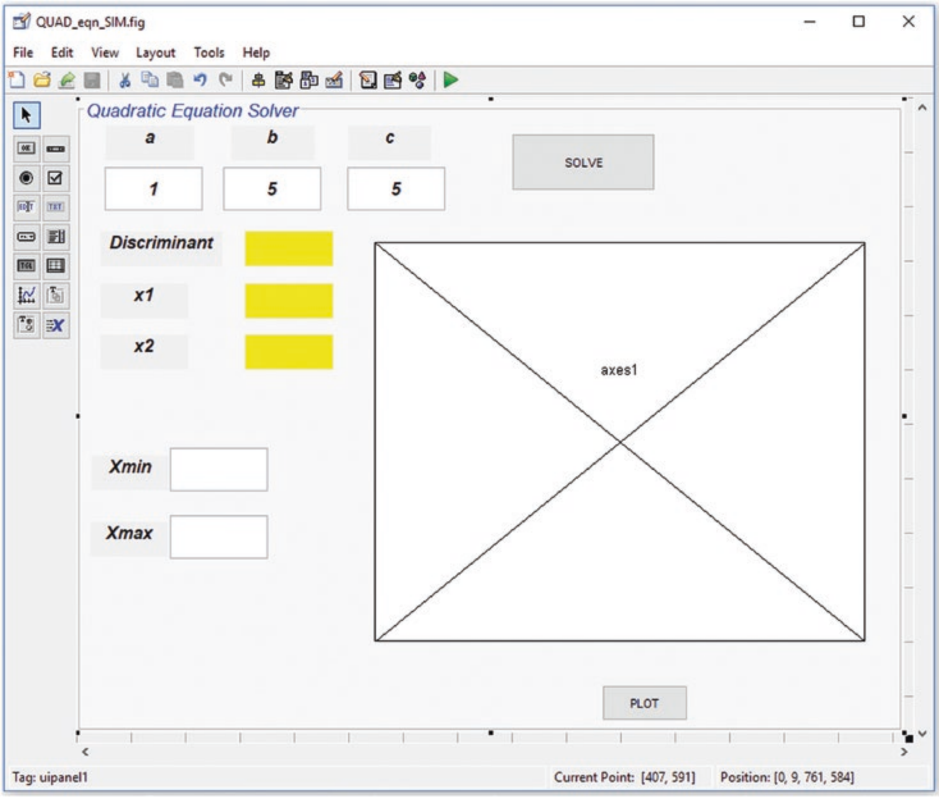


Figure 3-12. Complete design of the GUI model window

Note that in the GUI model shown in Figure 3-12, the following properties of the blocks are altered: in Static Text blocks, such as "a", "b", "c", "x1", "x2", "Xmin" and "Xmax", the string, font size and type, and background color are changed. The properties of the three Static Text blocks used to display the computation results for Discriminant, x1, and x2 are altered. For instance, for the Static Text block for the Discriminant value display, background color, font type, size, string and tag are also changed. Its string is changed to be an empty space and its tag is renamed to D.

The background color, font type, size, string, and tag of the Static Text blocks used to display the values of x1 and x2 are altered. The strings are set to be empty and the tags are renamed to x1 and x2, for x_1 and x_2, respectively. There are two Push Button blocks used to solve the given equation (based on a, b, c) and to display the plot of the given quadratic equation with respect to the user entries for Xmin and Xmax.

The first Push Button block's string is changed to `SOLVE` and its tag is `SOLVE_eqn`, and the second Push Button string is called `PLOT` and its tag is called `PLOT_eqn`. As stated above, tags are vital and require careful attention when editing or rewriting sub-functions in the M-file of the GUI model.

Up to this point, all we have worked with is within the GUI design window and the properties of our chosen blocks within the Property Inspector, which we did by double-clicking on each block individually. Another change that we made in this example was altering the panel window; we altered its background color, title, font type, and size. Finally, after completing all the changes in our GUI model, we saved it with the filename `QUAD_eqn_SIM.fig` (see Figure 3-13), which saved the `QUAD_eqn_SIM.m` automatically as well.

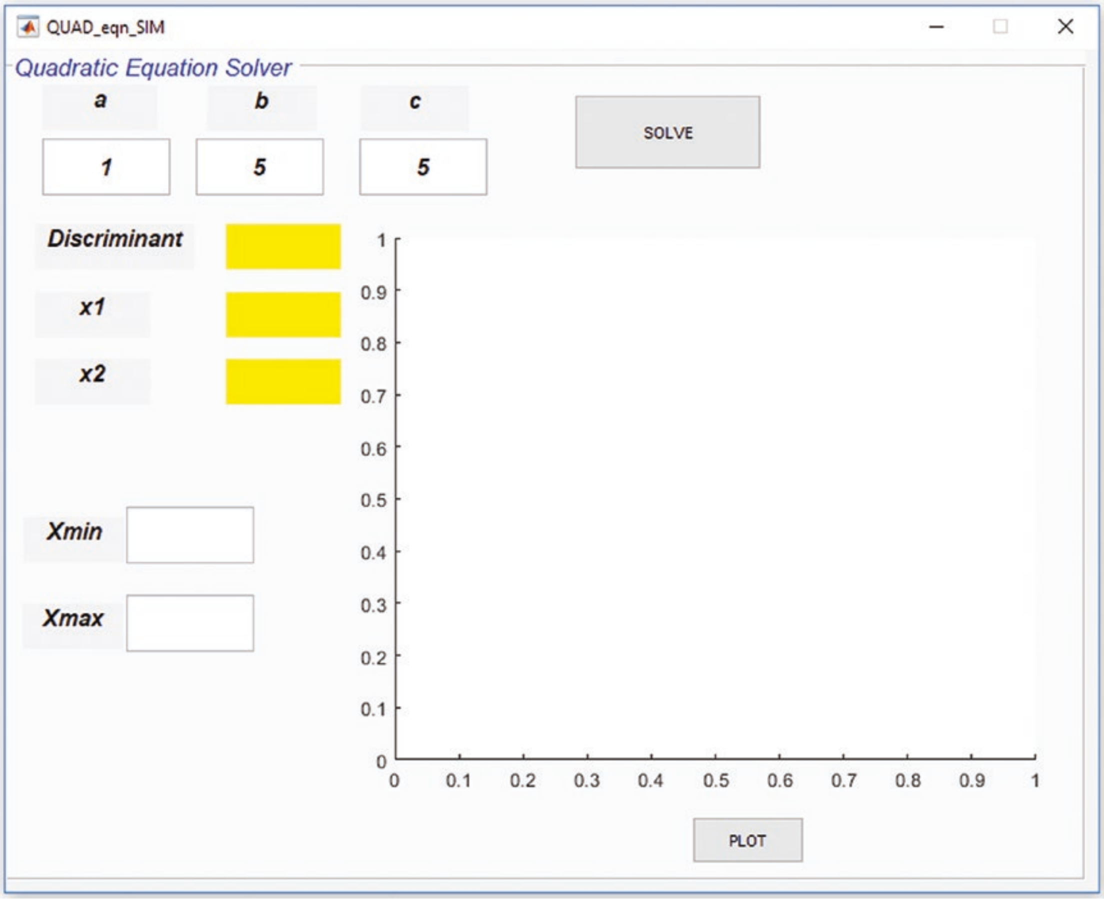


Figure 3-13. *Completed GUI model to compute roots and display a 2D plot of the quadratic equation*

Editing the Callback Functions

The model shown in Figure 3-13 does not reveal any results when we click on the SOLVE or PLOT buttons. We edit and rewrite two callback functions of the M-file—`QUAD_eqn_SIM.m`. Note that all of the automatically generated comments are removed from the script.

```
function varargout = QUAD_eqn_SIM(varargin)
% QUAD_EQN_SIM MATLAB code for QUAD_eqn_SIM.fig
...
gui_Singleton = 1;
```

```
gui_State = struct('gui_Name', mfilename, ...
 'gui_Singleton', gui_Singleton, ...
 'gui_OpeningFcn', @QUAD_eqn_SIM_OpeningFcn, ...
 'gui_OutputFcn', @QUAD_eqn_SIM_OutputFcn, ...
 'gui_LayoutFcn', [] , ...
 'gui_Callback', []);
if nargin && ischar(varargin{1})
 gui_State.gui_Callback = str2func(varargin{1});
end
...
handles.output = hObject;
guidata(hObject, handles);

function varargout = QUAD_eqn_SIM_OutputFcn(hObject, eventdata, handles)
...
varargout{1} = handles.output;

function a_Callback(hObject, eventdata, handles)
...
function a_CreateFcn(hObject, eventdata, handles)
...
function b_Callback(hObject, eventdata, handles)
...
function b_CreateFcn(hObject, eventdata, handles)
...
function edit3_Callback(hObject, eventdata, handles)
...
function edit3_CreateFcn(hObject, eventdata, handles)
...
if ispc && isequal(get(hObject,'BackgroundColor'), get(0,'defaultUicontrol
BackgroundColor'))
 set(hObject,'BackgroundColor','white');
end

function Xmin_Callback(hObject, eventdata, handles)
...
```

```
function Xmin_CreateFcn(hObject, eventdata, handles)
...
function Xmax_Callback(hObject, eventdata, handles)
...
function Xmax_CreateFcn(hObject, eventdata, handles)
...
if ispc && isequal(get(hObject,'BackgroundColor'), get(0,'defaultUicontrol
BackgroundColor'))
 set(hObject,'BackgroundColor','white');
end

function SOLVE_eqn_Callback(hObject, eventdata, handles)
...
function PLOT_eqn_Callback(hObject, eventdata, handles)
...
```

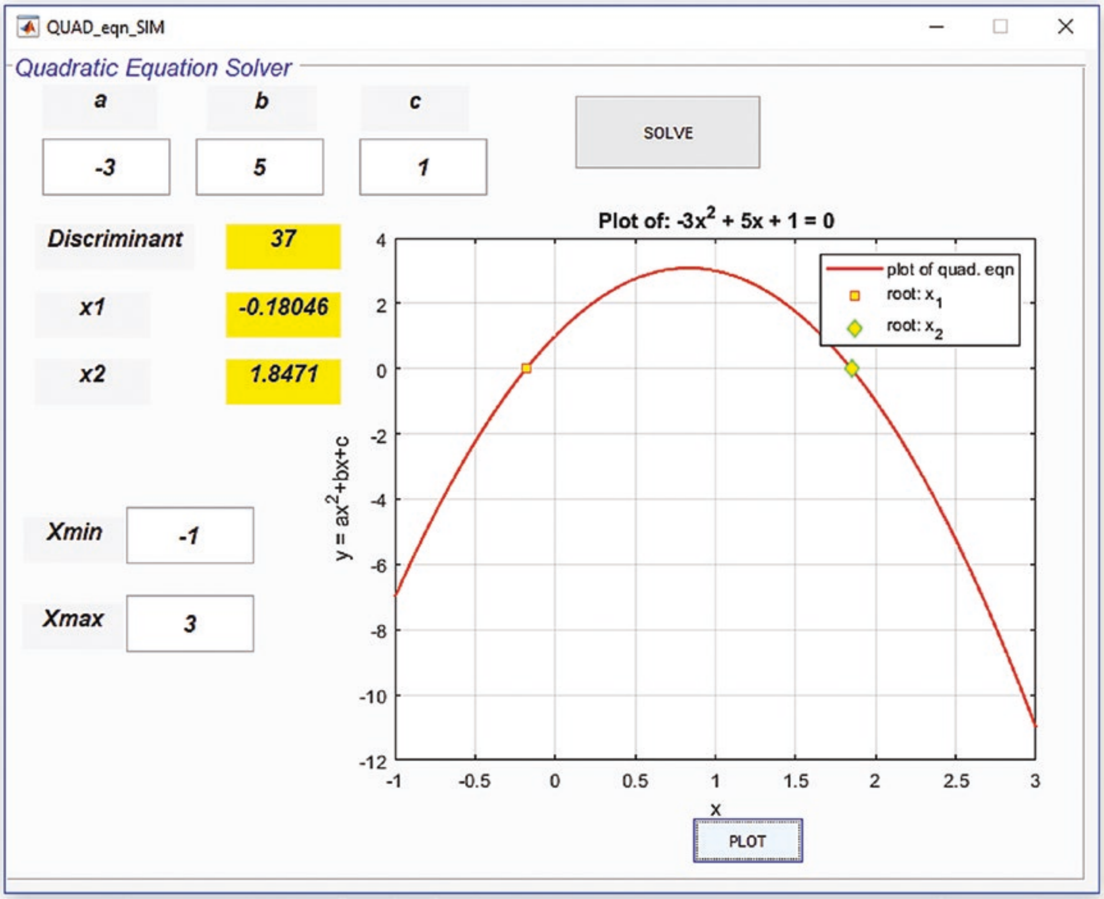
The callback functions that we will edit are `PLOT_eqn_Callback(hObject, eventdata, handles)` and `SOLVE_eqn_Callback(hObject, eventdata, handles)`. These callback functions make the GUI model compute and display discriminant, two real-valued roots in pre-defined blocks and display a 2D plot according to our entries for Xmin and Xmax. Here are the edited callback functions for the two push buttons called `SOLVE` and `PLOT`:

```
...
function SOLVE_eqn_Callback(hObject, eventdata, handles)
a=str2double(get(handles.a, 'string'));
b=str2double(get(handles.b, 'string'));
c=str2double(get(handles.c, 'string'));
D=b^2-4*a*c;
if D>0 % There are two real valued roots;
x1=(-b+sqrt(D))/(2*a);
x2=(-b-sqrt(D))/(2*a);
elseif D==0 % There is a unique root;
 x1=-b/(2*a);
 x2=x1;
else % No real valued roots exist;
 x1='No Root';
 x2='No Root';
end
D=num2str(D); set(handles.D, 'string', D);
x1=num2str(x1); set(handles.x1, 'string', x1);
x2=num2str(x2); set(handles.x2, 'string', x2);
function PLOT_eqn_Callback(hObject, eventdata, handles)
a=str2double(get(handles.a, 'string'));
b=str2double(get(handles.b, 'string'));
c=str2double(get(handles.c, 'string'));
D=str2double(get(handles.D, 'string'));
x1=str2double(get(handles.x1, 'string'));
x2=str2double(get(handles.x2, 'string'));Xmin=str2double(get(handles.Xmin,
'string'));
Xmax=str2double(get(handles.Xmax, 'string'));
x=linspace(Xmin, Xmax, 200); y=a*x.^2+b*x+c;
plot(x, y, 'r-', 'linewidth', 1.5); xlabel('x'), ylabel('y = ax^2+bx+c')
if D>=0   % Roots x1 and x2 will be plotted if the equation has real roots
hold on  % Plot is held if D>=0
plot(x1, 0, 'rs', x2, 0, 'gd', 'markersize', 7, 'markerfacecolor', 'y')
legend('plot of quad. eqn', 'root: x_1', 'root: x_2')
end
title(['Plot of:  ' num2str(a)  'x^2 + '  num2str(b)  'x + '  num2str(c)  ' = 0']);
grid on
```

Note that in the sub-function SOLVE_eqn_Callback, we use the get() command to obtain string values of *a*, *b*, *c* and convert them into numerical data with str2double(). Using the set() command, the computed values of D, x1, and x2 are assigned to D, x1, and x2 in order to display them in their respective Static Text blocks called "D", "x1", and "x2". In the latter sub-function, we obtain the numerical values of *a*, *b*, and *c* with the get() command. We run the whole M-file called QUAD_eqn_SIM.m by clicking the Run button or calling the file from the command window (for example $-3x^2 + 5x + 1 = 0$) and then plot the given equation for $x = -1\ldots3$.

Note that in the SOLVE_eqn_Callback callback function, if the discriminant is $D \geq 0$, then the computed roots will be also plotted. Moreover, there are several plot tools used here (hold on, legend, markersize, and markerfacecolor) that are explained in detail in Chapter 6.

Figure 3-14 shows the final GUI model with its results for the given example $-3x^2 + 5x + 1 = 0$ and for $x = -1\ldots3$.



a: -3

b: 5

c: 1

Discriminant: 37

x1: -0.18046

x2: 1.8471

Xmin: -1

Xmax: 3

***Figure 3-14.** The GUI model to compute roots of a quadratic equation and plot its values with user entries for Xmin and Xmax*

Let's review some of the most common errors made while editing and rewriting sub-functions. They are:

- Misspelled tag names of blocks that are recalled within sub-functions or set values to display in a GUI model window
- Necessary conversion operators (such the `get()` and `set()` operators) are not employed appropriately within sub-functions for numerical calculations

- Editing the wrong sub-functions (callback functions) of buttons to make a GUI model perform its aimed operations

In order to avoid these common mistakes, it is recommended to choose tag names carefully and double-check their names while writing and editing nested callback functions. Another recommendation is to write down all the tag names and type them in with care. When converting variable values from one type to another with the `get()` and `set()` operators, look at the task specifics. For instance, what type of data is needed for processing, and by which handle names (such as `handles.WHAT`) are values of variables or data obtained. There are several ways to avoid editing the wrong callback functions. One is to look up an assigned name tag for each button that makes the GUI model perform its anticipated operations. MATLAB automatically assigns a sub-function name to every operational button and entry block with its given name tag (a given name tag by a user) with a underscore sign and `callback()`. For instance, if an operational button's name tag is `PLOT_all`, it will have an assigned sub-function called `PLOT_all_Callback(hObject, eventdata, handles)`. Another approach, after saving the GUI model and the automatically generated M-file, is to click on an operational button (e.g., a push button) in a GUI model design window. Then right-click and choose View Callbacks ➤ Callback to directly reach the right sub-function subject to edit.

GUI Dialogs and Message Boxes

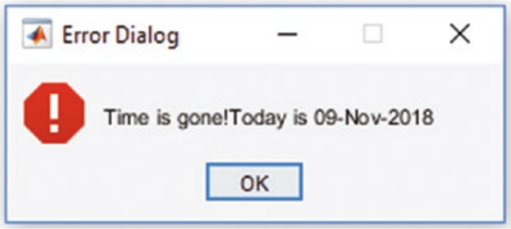
MATLAB has a few of commands that create/call ready-to-use popup dialog boxes, including Error, Warning, Help, Information, User Entry Input dialog, and so forth. They can be created without writing scripts and callback functions or building GUIs. One of the most common purposes of these message boxes is to deliver messages to the users. In general, creating these popup message boxes is straightforward and requires only one command. The dialog and message boxes (except for the user input box) can all be called and generated with one general command: `errordlg()`, `warndlg()`, `helpdlg()`, and `msgbox()`. On the other hand, a user input dialog box requires several commands, such as the name of the input dialog, the input dialog window title/message, and the input dialog window. Let's look at a few examples to see how to create these message boxes, a user entry dialog window, and an entry status box.

Error Dialog

General command syntax: `errordlg('Add Notes')`

Example:

```
errordlg(['Time is gone!' 'Today is ' date])
```



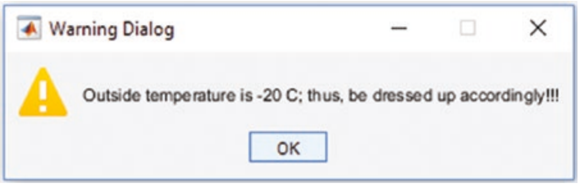
Time is gone!Today is 09-Nov-2018

Warning Message

General command syntax: `warndlg('Add Notes')`

Example:

```
warndlg(['Outside temperature is ' num2str(-20) ' C; thus, be dressed up
accordingly!!!'])
```



Outside temperature is -20 C; thus, be dressed up accordingly!!!

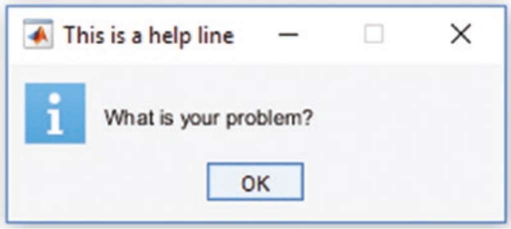
F1 Help/Message Box

General command syntax:

```
helpdlg('Add Notes')
```

Example:

```
helpdlg('What is your problem?', 'This is a help line');
```

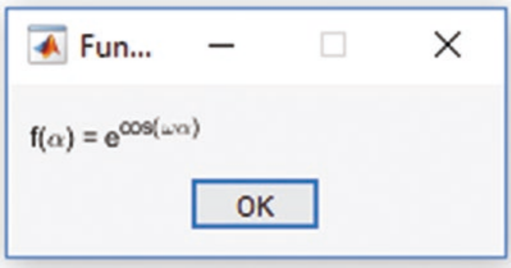

This is a help line

What is your problem?

A second example:

```
CreateStruct.WindowStyle='replace';
CreateStruct.Interpreter='tex';
msgbox('f(\alpha) = e^{cos(\omega*\alpha)}','Function: ',CreateStruct);
```

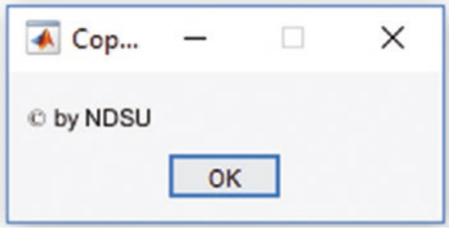
Note that `CreateStruct`, used in this example, is a variable type of `struct` that creates a structure type of variable with two fields of a character type —called `WindowStyle` with the value of `replace` and `Interpreter` with the value of `tex`. They are recognized automatically by MATLAB. Without creating the `CreateStruct` variable with its two fields, `WindowStyle` and `Interpreter`, the equation would be displayed as shown in the message box: $f(\alpha) = e^{\cos(\omega\alpha)}$


Fun...

$f(\alpha) = e^{\cos(\omega\alpha)}$

Finally:

```
msgbox('\copyright by NDSU', 'Copyright',CreateStruct)
```


Cop...

© by NDSU

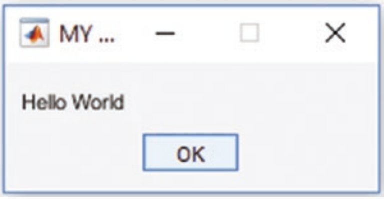
General Syntax

There are alternative ways to create help/message/error/warning messages. The general command syntax is:

```
HM = msgbox('Message', 'Box Title', 'Icon')
```

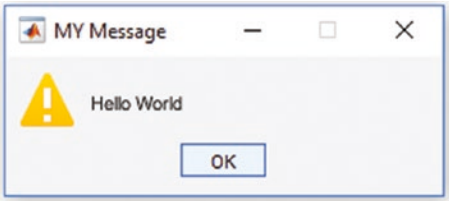
Here is just a message box:

```
msgbox('Hello World', 'My Message')
```



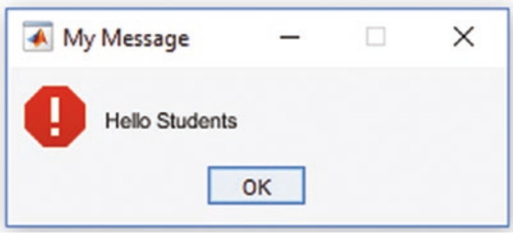
Here's a warning message:

```
msgbox('Hello World', 'My Message', 'warn')
```



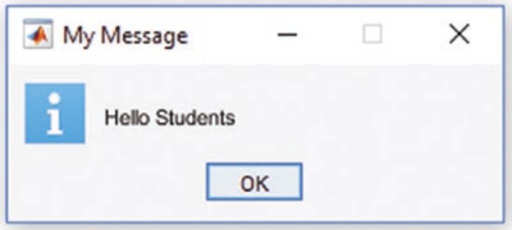
Here's a message box with an error sign:

```
msgbox('Hello Students', 'My Message', 'error')
```



Here's a message box with an information sign:

```
msgbox('Hello Students', 'My Message', 'help')
```



My Message

Hello Students

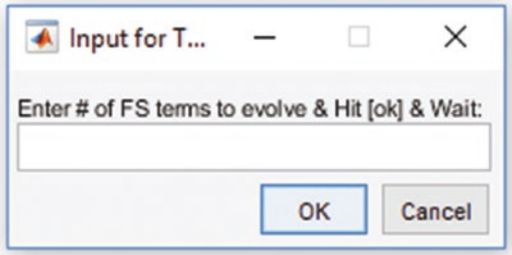
Input Dialog

General command syntax:

```
ANSWER = inputdlg(PROMPT,NAME)
```

Here is a short script:

```
Enter1={'Enter # of FS terms to evolve & Hit [ok] & Wait: '}; % Message
Name1='Input for TERMS of Fourier Series'; % Dialog box name
Numlines = 1; % Number of lines for Input dialog
ANSWER=inputdlg(Enter1, Name1, Numlines); % Generate and Read Input Dialog
[n status] = str2num(ANSWER{1}); % Picks up the user entry under variable
name of n
```



Input for T...

Enter # of FS terms to evolve & Hit [ok] & Wait:

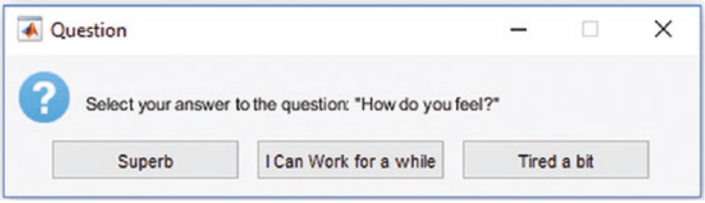
Question Dialog

General command syntax:

```
|ButtonName = questdlg('Your Question ...', 'Question?', 'Option1',
'Option2', ....)
```

An example:

```
YA= questdlg('Select your answer to the question: "How do you feel?"
','Question', 'Superb', 'I Can Work for a while', 'Tired a bit', ' ')
```



Let's look at more involved example that has several choices.

Making a Choice

You are in a café and would like to make a purchase. You would like to choose from the menu: coffee, tea, or some sweets. The café has some shortages that you are not aware of. The message and dialog boxes such as warning, help, input, or error messages need to be displayed according to your order and the café shortages. Here are the steps to be implemented in this script.

Step 0. You are in a café and a waiter comes up to take your order.

Step 1. You decide between coffee, tea, and chocolate cake.

Step 2. The waiter responds that they don't actually carry tea and are out of chocolate cake.

Step 3. You then choose your coffee type: Normal, Strong, or Special (your taste).

Step 4. You taste and grade the coffee.

Step 5. You pay the bill and leave a tip.

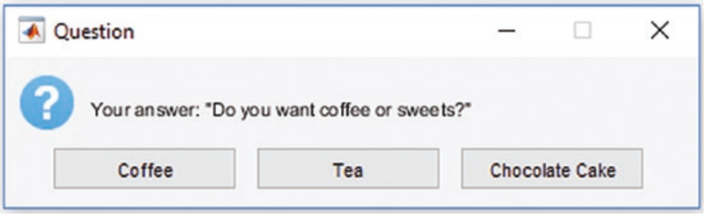
Here is the answer script:

```
clearvars
% It is time for coffee, let's go to a cafe ...
% Step 0
H1 = msgbox('You are in a cafe and ordering coffee of your taste', 'Cafe');
```

```
pause(2), delete(H1)
% Step 1. Ordering your coffee
H2 = msgbox('What can I do for you today? Do you want coffee or sweets',...
 'Waiter has arrived');
pause(3), delete(H2)
% Answer what you like to have
YA= questdlg('Your answer: "Do you want coffee or sweets?" ','Question', ...
 'Coffee', 'Tea', 'Chocolate Cake', ' ');
switch YA
 case 'Coffee'
 CA = questdlg('Your answer: "OK. What kind of coffee?" ','Question', ...
 'Normal', 'Strong', 'Special', ' ');
 case 'Tea'
 H3=warndlg(['Sorry! We do not serve' ' Tea. Maybe something else?']); pause(2)
 delete(H3)
 case 'Chocolate Cake'
 H4=errordlg(['Very Sorry!' 'No cakes Left ']); pause(2)
 delete(H4)
end
Response=exist('CA', 'var');
if Response ==1
 switch CA
 case 'Normal'
 HN=warndlg(['Normal coffee is', ...
 ' 3 tea-spoons of coffee+3 Table-spoons of Milk+1 tea-spoon of sugar']);
 pause(3), delete(HN)
 case 'Strong'
 HS=warndlg(['Strong coffee is', ...
 ' 4 tea-spoons of coffee+2 Table-spoons of Milk+2 tea-spoon of sugar']);
 pause(3), delete(HS)
 case 'Special'
HS = msgbox('Select: how much coffee, milk and sugar to add?', 'Selection');
 pause(3), delete(HS)
 Call = 'Order your coffee';
 Selection = {'how many tea-spoons of coffee: ',...
 'how many tablespoons of milk: ', 'how many teaspoons of sugar: '};
 n_lines=[1, 13];
 ANS=inputdlg(Selection, Call, 1, {' ', ' ', ' '});
 end
 OK = msgbox('You have had your coffee','You are about to leave ');
 pause(5), delete(OK)
 PB= questdlg('Waiter asks: "How was the coffee?" ','Question', ...
 'Just Superb', 'Nice', 'Umm...but OK', ' ');
 switch PB
 case 'Just Superb'
 helpdlg('Thank you so much! 4$+1$ (tips)', 'Here is the payment');
 case 'Nice'
 helpdlg('Thanks! 4$+0.50$ (tips)', 'Here is the payment');
 case 'Umm...but OK'
 helpdlg('Thank you! 4$ (No-tips)', 'Here is the payment');
 end
else
 H5 = msgbox('Bye - Bye! See you next time!', 'Waiter leaves');
 pause(4), delete(H5)
end
```

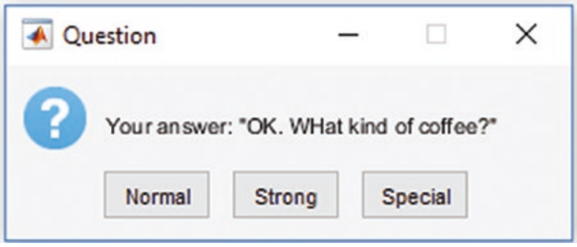
Note that in this script, we used MATLAB's built-in function `exist`, which verifies whether a specific variable exists in the workspace. It uses the command syntax of `exist('N', 'var')` to determine whether the variable `N` is available in the workspace. It can also be used to verify whether any specific file exists in the current directory. For example, `exist('MY_fun.c', 'file')` checks whether the file called `MY_fun.c` is present in the current directory of MATLAB. The output of the built-in function `exist()` can be 0, 1, or 2. The 0 means the variable or file does not exist, 1 means the variable exists, and 2 means the file exists in the current directory.

When the script is executed, two message boxes (`H1` and `H2`) are displayed for two to three seconds, and then closed automatically. Then the next question dialog box pops up and you need to choose Coffee, Tea, or Chocolate Cake.



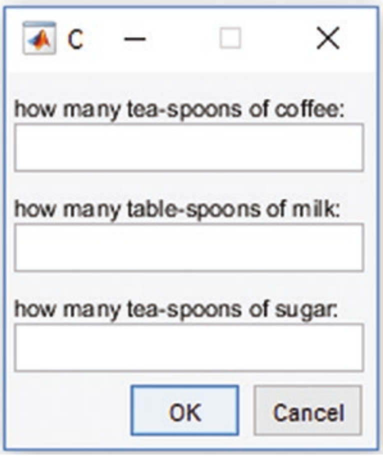
Your answer: "Do you want coffee or sweets?"

Depending on your selection, other options and responses will appear. If you select Coffee, the question dialog (CA) will pop up and ask you to select your coffee type: Normal, Strong, or Special.



Your answer: "OK. WHat kind of coffee?"

If you select Normal or Strong in the dialog box (CA), there will be popup box displaying the contents of your selected coffee in terms of coffee, milk, and sugar. If you select the Special type of coffee, you need to input (input dialog ANS) the amount of coffee, milk, and sugar you want in your coffee.

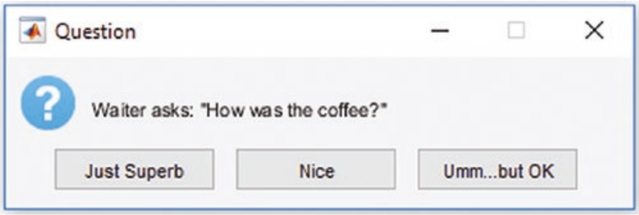


how many tea-spoons of coffee:

how many table-spoons of milk:

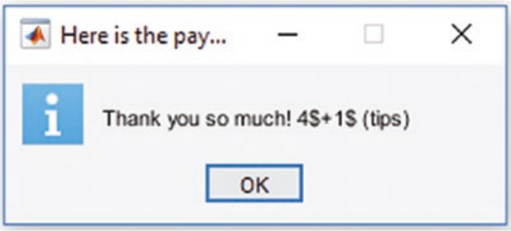
how many tea-spoons of sugar:

The message box (OK) pops up and closes down automatically after five seconds. Finally, the waiter asks you how your coffee tasted.



Waiter asks: "How was the coffee?"

Based on your answer, you get the help dialog box indicating how much to pay, including the tip.



Thank you so much! 4$+1$ (tips)

If you select Tea, the warning dialog box (H3) will pop up and close down automatically after two seconds,. If you choose Chocolate Cake, the error dialog (H4) will pop up and close down automatically in two seconds.

The last message box (H5) is displayed if you select Tea or Chocolate Cake in the first step.

These message and dialog boxes can also be employed within function files in a very similar manner.

Providing Input to an Equation

This simple numerical simulation exercise computes the values of a mathematical equation by writing a number to the function file: $F = e^{\cos(\omega t)}$.

The varying inputs are as follows:

- No input, whereby the default values $\omega_n = 13; t = 0 : \frac{\pi}{100} : \pi;$ are taken
- One input, ω_n or t, whereby the missing argument takes the default values
- Two inputs, ω_n and t, whereby an error dialog pops up.

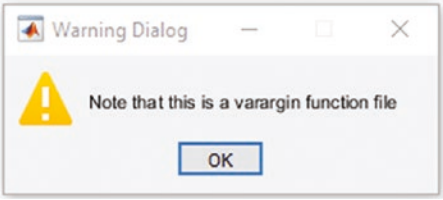
Here is the complete answer script of this exercise:

```
function F = ERR(varargin)
% HELP. ERR.m simulates how to employ ERROR dialog box
warndlg('Note that this is a varargin function file ');
if nargin == 0
 omega = 13; t = 0:pi/100:pi;F = exp(cos(omega*t)); plot(t, F, 'bo--'), shg
elseif nargin == 1 && numel(varargin{1})==1
omega=varargin{1}; t = 0:pi/100:pi;F = exp(cos(omega*t)); plot(t, F, 'bo--')
elseif nargin == 1 && numel(varargin{1})>1
 t = varargin{1}; omega=13;F = exp(cos(omega*t)); plot(t, F, 'bo--')
elseif nargin ==2 && numel(varargin{1})==1
omega=varargin{1}; t =varargin{2};F = exp(cos(omega*t)); plot(t, F, 'bo--')
elseif nargin == 2 && numel(varargin{1})>1
omega=varargin{2}; t = varargin{1};F = exp(cos(omega*t)); plot(t, F, 'bo--')
else
errordlg(['Wrong Number of Inputs!' num2str(nargin) ' are too many INPUTs'])
 disp('No OUPUTS')
F=warndlg(['No OUTPUTS because ' num2str(nargin) ' are too many INPUTS']);
end
end
```

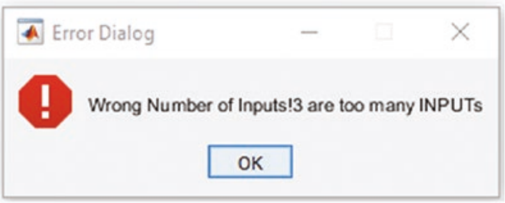
Let's simulate the above function file (ERR.m) from the command window:

```
>> omega = 13; t = 0:pi/100:pi; OmegaN=13;
>> F = ERR(omega, OmegaN, t);
No OUPUTS
```

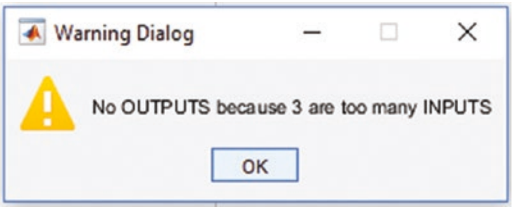
These Warning and Error dialog boxes are displayed.



Note that this is a varargin function file



Wrong Number of Inputs!3 are too many INPUTs



No OUTPUTS because 3 are too many INPUTS

This concludes our brief discussion of GUI tools and functions. These tools are very handy and can be associated with M/MLX-files and function files to make scripts more user friendly with GUI tools.

CHAPTER 4

MEX Files, C/C++, and Standalone Applications

MEX stands for *MATLAB executable* and it's a way to call custom C, C++, or FORTRAN routines/code directly from MATLAB, thereby treating them as if they were MATLAB built-in functions. Since MATLAB is compatible with several programming languages, including C/C++, FORTRAN, C#, and Java, the MEX files can be called exactly like M-files or M-functions within the MATLAB environment. It is also possible to create standalone applications in MATLAB as self-executable files.

MATLAB's Coder toolbox is very powerful and is capable of creating standalone and self-executable C code. It is very beneficial to know how to create C code from MATLAB's M-files since the C programming language is free and does not have any licensing requirements. It can be employed to create executable files, dynamic link libraries, and libraries of large applications. In fact, C can be used to solve a wide range of tasks with myriad types of applications.

There are several reasons that we would create or employ MEX files. One of them is that they are capable of calling large existing C, C++, or FORTRAN programs directly from MATLAB without rewriting them as M-files. The other reason is to speed up computation processes where M-files have bottlenecks. For instance, MEX files can be employed on large loop-based computations in order to avoid bottlenecks with time-consuming computation processes. The MEX files are better than M-files for several reasons. They are called and executed as executable files without having to be compiled, unlike M-files. They can call large existing C or FORTRAN routines directly from MATLAB without having to be rewritten as M-files. Moreover, the MEX files allow parallelism, so we can write multi-threaded C codes for various costly (in terms of machine time) computations and simulations that cannot be vectorized.

S. Eshkabilov, *Beginning MATLAB and Simulink*, https://doi.org/10.1007/978-1-4842-5061-7_4

Verifying Compilers

Before we start generating MEX files, it's a good idea to verify which compilers are installed in the current MATLAB package. This can be done by typing the following command in the command window prompt:

```
>> mex -setup
```

Then we follow the instructions to select the appropriate compiler from the list of installed compilers. If there is no compiler installed, we need to install the one that is compatible with MATLAB in order to generate MEX files (a list of compatible compilers can be found on the MathWorks website: https://www.mathworks.com/support/requirements/previous-releases.html). A general flowchart of C/C++ code generation in MATLAB is shown in Figure 4-1.

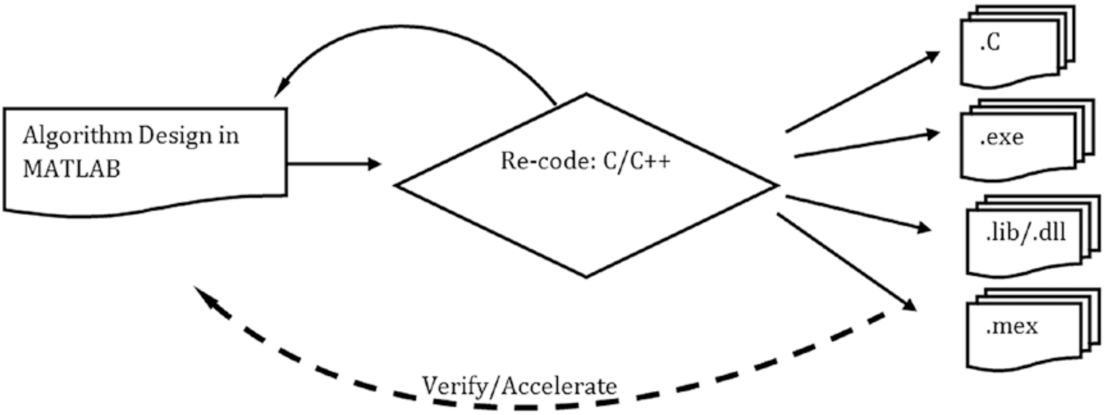


Figure 4-1. *Flowchart of code generation in C/C++*

Hereafter, we demonstrate several examples of converting M-files into C/C++, MEX, and executable standalone applications, compiling C code, and testing it in the MATLAB environment.

Generating C Code

This simple example will generate C code in MATLAB to compute the sum of odd numbers based on the user's specified integer number. We first write an M-file and then convert it into a MEX file in MATLAB using a code generator application tool called

MATLAB Coder[1]. It is used to generate C and C++ code from MATLAB code for a variety of hardware platforms, embedded systems, and desktop platforms. The generated code can be integrated in existing programs and projects as source code, libraries, and/or dynamic libraries.

Note MATLAB Coder is a separate application that does not come with the base MATLAB package and has to be installed separately.

Let's take this `SUModd.m` M-file function, which computes the sum of odd integers up to the user-specified integer, `N`.

```
function SUModd(N)
%#codegen
Sum=0; ii=1;
while ne(ii,N)
    if ne(mod(ii,2),0)
        Sum=Sum+ii;
    else
        Sum=Sum;
    end
    ii=ii+1;
end
Sum
```

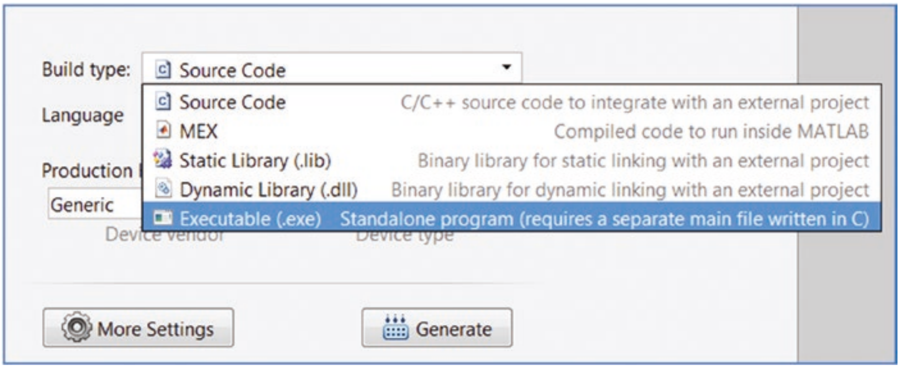
Note that the second line of the `SUModd.m` function is a command (comment), `%#codegen`. It's recommended that we include this comment with M-files that are subject to conversion into MEX files. The conversion of the M-file can be achieved in three ways. With this command:

```
>> mex SUModd.m
```

[1] MATLAB Coder is the registered trademark of The MathWorks Inc.

Or by typing >> `coder` in the command window, or by using the MATLAB Coder[2] option, MATLAB Coder, which is under the APPS tab in the main menu. When we click on the MATLAB Coder icon, the GUI window of the MATLAB Coder pops up. We then specify the name of the M-file to be converted into C/C++ and MEX in the Generate Code for Function box and input the variable type N (single, double, int8, uint8, int16, uint16, … structure, complex number, etc.) and size (1x1, … mxn).

After we press Enter, the MATLAB Coder will automatically generate the MEX file with all source code in `*.c` and `*.h` if there are no issues converting the MATLAB functions. We start the code-generation process by selecting from the drop-down options which source code type we want to generate. In this case, we can choose from Source Code in C/C++, MEX, Static Library (`.lib`), Dynamic Library (`.dll`), and Executable (`.exe`), as shown in Figure 4-2. Note that these options are operating system and platform dependent. In our case, we are working in the Windows OS environment, so these libraries are applicable to Windows OS.



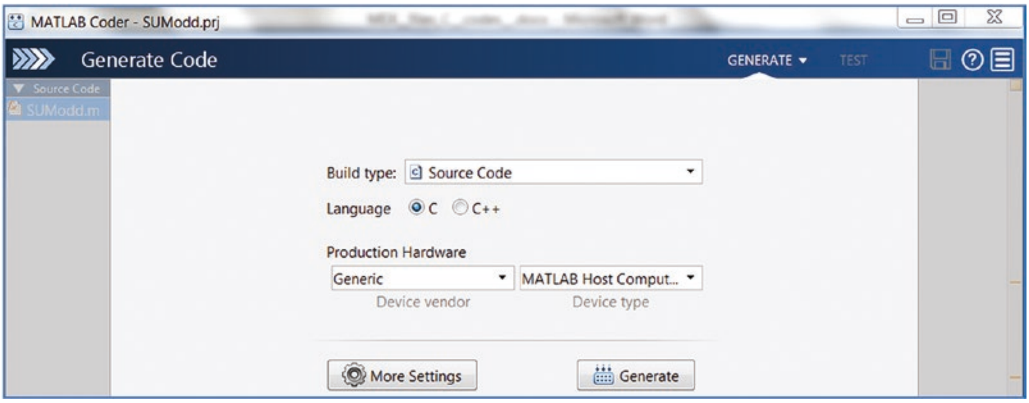
Build type: Source Code

Source Code C/C++ source code to integrate with an external project
MEX Compiled code to run inside MATLAB
Static Library (.lib) Binary library for static linking with an external project
Dynamic Library (.dll) Binary library for dynamic linking with an external project
Executable (.exe) Standalone program (requires a separate main file written in C)
Language
Production
Generic
Device vendor
Device type
More Settings
Generate

Figure 4-2. *Source code type selection generated in C/C++*

Next, we need to choose in which language—C or C++—we want to create the MEX file (see Figure 4-3).

[2] MATLAB Coder is the registered trademark of The MathWorks Inc.



Build type: Source Code

Language ● C ○ C++

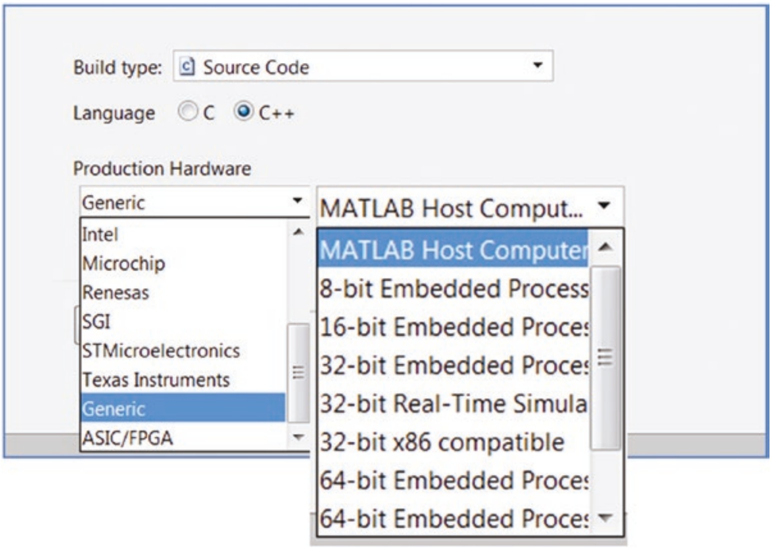
Production Hardware

Generic (Device vendor)

MATLAB Host Comput... (Device type)

***Figure 4-3.** GUI window to generate C/C++ from MATLAB scripts via the MATLAB Coder*

Then we select the device vendor and type—Production Hardware, Host Computer Type (see Figure 4-4).



Build type: Source Code

Language ○ C ● C++

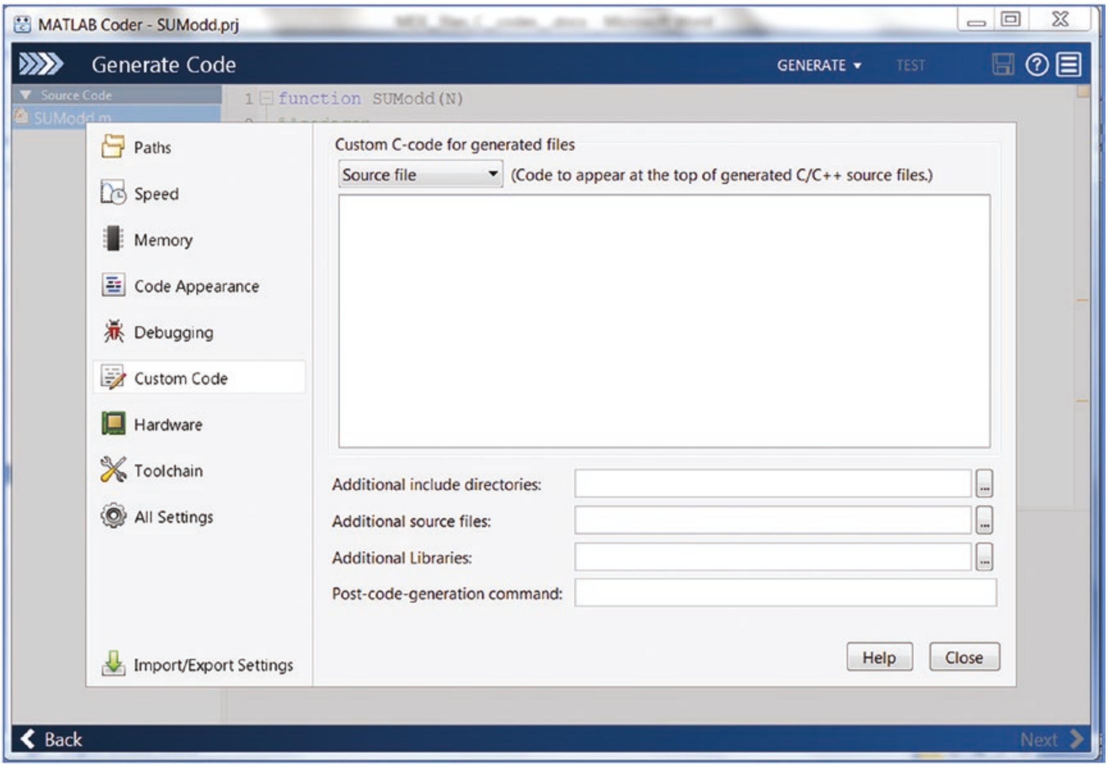
Production Hardware

Generic

MATLAB Host Comput...

***Figure 4-4.** Production hardware and processor type selection options*

We can make a few more adjustments from the More Settings area, as shown in Figure 4-5.



Custom C-code for generated files

Source file (Code to appear at the top of generated C/C++ source files.)

Additional include directories:

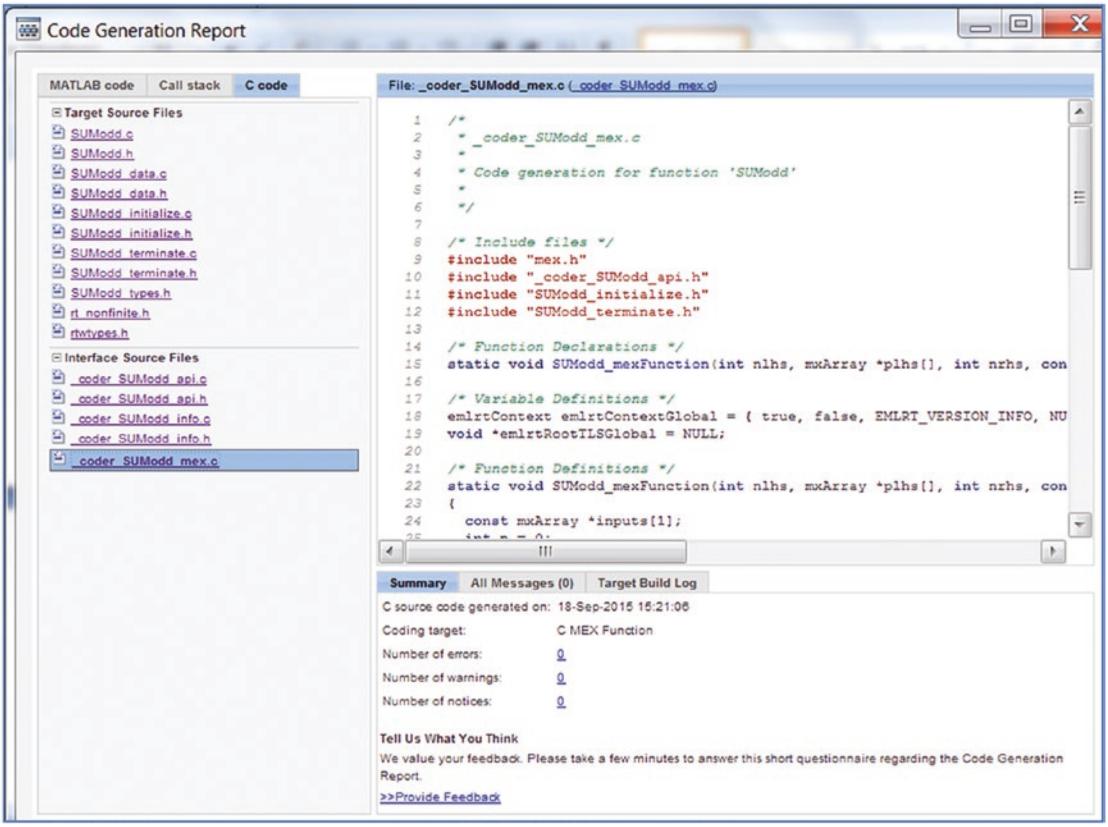
Additional source files:

Additional Libraries:

Post-code-generation command:

***Figure 4-5.** More Settings option of the Coder's GUI window tools*

If there are no errors or warnings, the C-generated code with the target source code will appear (see Figure 4-6). The MEX file can be opened in the MATLAB workspace or within any M-file, like any MATLAB function file or M-file would be.



Target Source Files
- SUModd.c
- SUModd.h
- SUModd_data.c
- SUModd_data.h
- SUModd_initialize.c
- SUModd_initialize.h
- SUModd_terminate.c
- SUModd_terminate.h
- SUModd_types.h
- rt_nonfinite.h
- rtwtypes.h

Interface Source Files
- _coder_SUModd_api.c
- _coder_SUModd_api.h
- _coder_SUModd_info.c
- _coder_SUModd_info.h
- _coder_SUModd_mex.c

File: _coder_SUModd_mex.c (_coder_SUModd_mex.c)

```
/*
 * _coder_SUModd_mex.c
 *
 * Code generation for function 'SUModd'
 *
 */

/* Include files */
#include "mex.h"
#include "_coder_SUModd_api.h"
#include "SUModd_initialize.h"
#include "SUModd_terminate.h"

/* Function Declarations */
static void SUModd_mexFunction(int nlhs, mxArray *plhs[], int nrhs, con

/* Variable Definitions */
emlrtContext emlrtContextGlobal = { true, false, EMLRT_VERSION_INFO, NU
void *emlrtRootTLSGlobal = NULL;

/* Function Definitions */
static void SUModd_mexFunction(int nlhs, mxArray *plhs[], int nrhs, con
{
  const mxArray *inputs[1];
  int n = 0;
```

C source code generated on: 18-Sep-2015 15:21:06
Coding target: C MEX Function
Number of errors: 0
Number of warnings: 0
Number of notices: 0

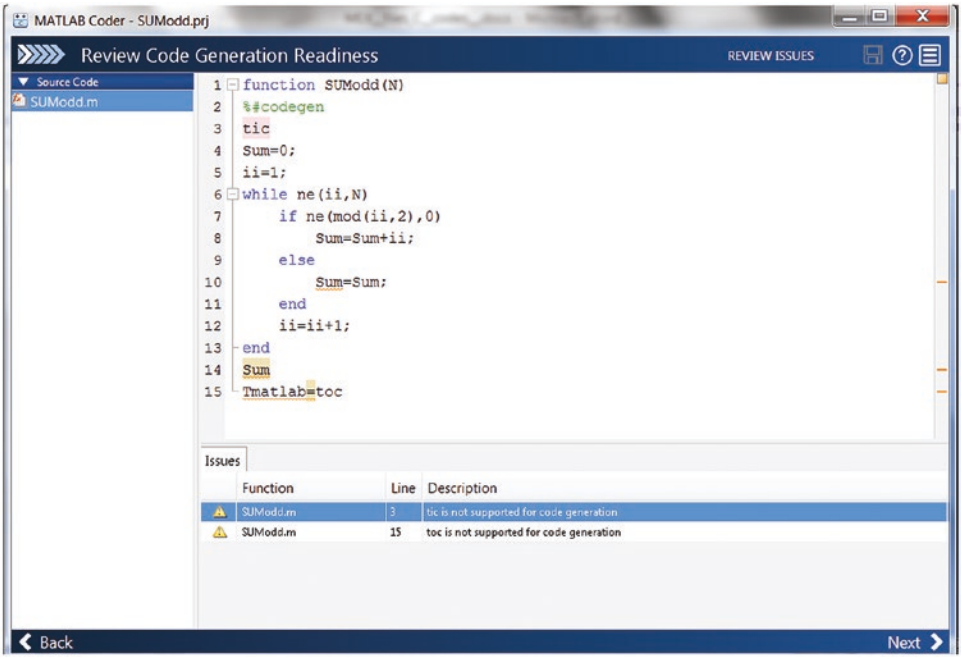
Tell Us What You Think
We value your feedback. Please take a few minutes to answer this short questionnaire regarding the Code Generation Report.
>>Provide Feedback

Figure 4-6. *Generated MEX file contents and libraries from the source code called SUModd.m*

Figure 4-6 shows the successfully generated MEX file called `SUModd_mex.c`. It is saved in the current (working) directory of MATLAB.

There might be issues linked to the conversion and encoding processes of M-files to MEX files, as a number of MATLAB's functions are incompatible with C. As an example, let's modify the M-file (in our initial M-file called `SUModd.m`) by including the `tic` and `toc` MATLAB commands, which compute elapsed time, and then try to recreate the MEX file.

With these modifications in the M-file, the Coder prompts the two compatibility issues illustrated in Figure 4-7. After removing the two commands—`tic` and `toc`—from our M-file and saving the updated version, we re-encode and then obtain no warnings. Subsequently, the converted code works flawlessly.



```
function SUMOdd(N)
%#codegen
tic
Sum=0;
ii=1;
while ne(ii,N)
    if ne(mod(ii,2),0)
        Sum=Sum+ii;
    else
        Sum=Sum;
    end
    ii=ii+1;
end
Sum
Tmatlab=toc
```

Function	Line	Description
SUMOdd.m	3	tic is not supported for code generation
SUMOdd.m	15	toc is not supported for code generation

Figure 4-7. *Two errors/issues occurred with the tic and toc commands while converting the M-file to the MEX file*

The process of recalling and using MEX files is similar to any M-file. For this example, the created MEX file can be recalled as follows:

```
>> SUMOdd_mex(N)
```

It also can be recalled within any M-file in a similar way. Note that `N` is pre-defined in the MATLAB workspace according to the variable type and size used in the MEX file during the initial step. One of the most important advantages of MEX files over M-files when dealing with `for` and `while` loops is their speed, in other words, computation efficiency. For instance, from the previous example, we can demonstrate the efficiency of the MEX file over its mother M-file.

```
% Compare computation efficiency of MATLAB and MEX files
clearvars; N=1e+8;
Tmatlab=cputime; SUMOdd(N), TM=cputime-Tmatlab;
fprintf('M-file Comp. Time is %5.5f sec \n', TM);
clearvars; N=1e+8;
Tmex=cputime; SUMOdd_mex(N), TMEX=cputime-Tmex;
fprintf('MEX-file Comp. Time is %5.5f sec \n', TMEX);
```

The computation (elapsed) time for each file is:

M-file comp. time is 9.26115 sec

MEX file comp. time is 1.06168 sec

So the computation time of the MEX file is about nine times shorter than the original M-file. Also, it must be noted that MEX files take precedence over M-files when the same filenames exist in both M-files and MEX functions.

Creating MEX Files from Other Languages

MEX files can be created from C/C++ or FORTRAN source code as well. That can be done with a single command:

```
>> mex HELLO.c
```

An alternative command is:

```
>> mex -v HELLO.c
```

In the latter case, the `-v` flag displays the compiling and linking process. Note that the source code `HELLO.c` in C (given below) is already created and put in the working directory of MATLAB.

```
#include "mex.h" /* Always include this */
void mexFunction(int nlhs, mxArray *plhs[], /* Output variables */
int nrhs, const mxArray *prhs[]) /* Input variables */
{
    /* Do something interesting or fun */
mexPrintf("Hello WORLD:)...:), This is SALOMON ... world!\n");
return;
}
```

The created MEX file, called `HELLO_mex.mexw64` from the source code `HELLO.c`, can be executed by recalling it in the command window:

```
>> HELLO_mex
```

That command outputs the following:

```
>> Hello WORLD:)...:), This is SALOMON ... world!
```

Note that the file extension of the MEX files, either `mexw32` or `mexw64`, indicates which processor type (OS system) is running on the user computer.

Here, we have covered the generic procedures for creating MEX files; however, the exact procedures are system dependent. In other words, they depend on which operating system is installed, which MATLAB version and compilers are installed and used, and so forth. Thus, it is necessary to recompile MEX files for every platform. One of the most essential challenges in developing MEX files is that the problem statement that should be in vector form. If it's not, then the created MEX application will be rather slow.

Building Standalone Applications

Standalone applications can be installed and run on machines/computers that don't have the MATLAB package. To compile/create a standalone application from MATLAB code/script, the MATLAB Application Compiler[3] is required along with the MATLAB package. Generating standalone applications from existing MATLAB scripts is a straightforward process. It can be accomplished in several ways. We can run this command:

```
>> mcc
```

Or we can use the MATLAB Application Compiler with GUI tools, which can be accessed by choosing the APPS tab ➤ APPLICATION DEPLOYMENT ➤ MATLAB Application Compiler Application Compiler.

When we click on the Application Compiler icon, the MATLAB Application Compiler's GUI window will open, as shown in Figure 4-8. We can perform the following actions, as shown in the number callouts on Figure 4-8. (1) We can add a main filename (M-file) by clicking on the add + button. (2) We may also need to add information, such as the application's name, author's name, email, company, summary, description, and so forth, to the application. (3) We can add/change the custom splash screen by clicking on the placeholder and adding a photo/screenshot.

[3] MATLAB Compiler is the registered trademark of the MathWorks Inc.

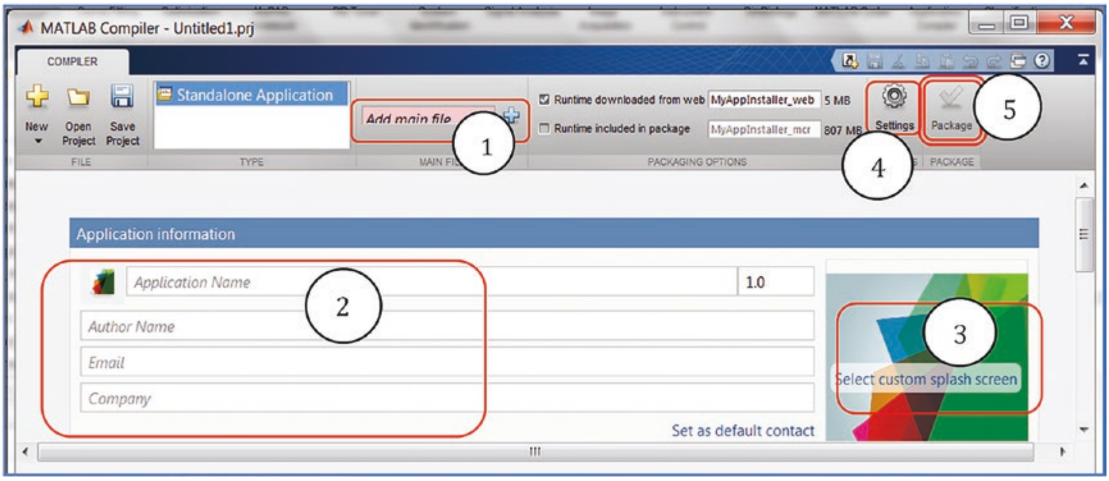
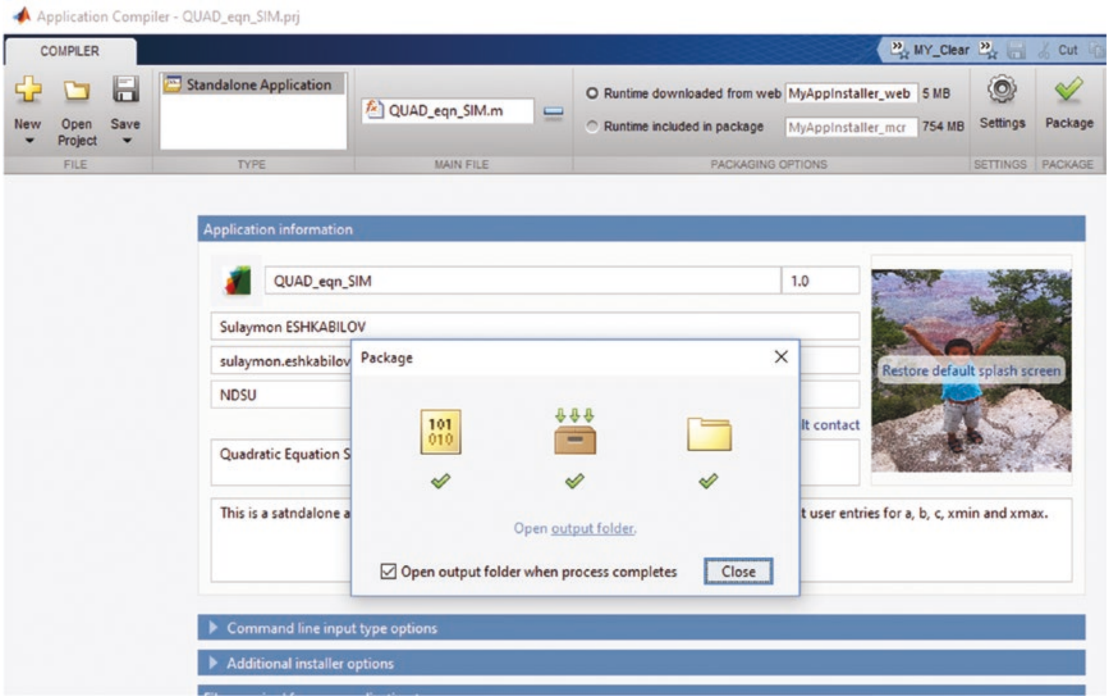


Figure 4-8. *MATLAB Compiler's standalone application GUI developer*

(4) We can also adjust settings, such as any additional parameters passed to the MATLAB Compiler, the location of the output folders for test and end user files, and the package installers via the Settings icon. (5) After we add a filename in step 1 to create the executable standalone application, the Package icon will become active (see Figure 4-9).

Note that in this example, we are using the function file called `QUAD_eqn_SIM.m` (the GUI model created in Chapter 3) to generate a standalone application. Moreover, we have added some information about the project and added a photo for the custom splash screen. After completing all of these steps, we click on the ✓ Package icon and indicate where to save the standalone application project.

The process of creating the standalone application then starts, as shown in Figure 4-9. This may take a few minutes depending on the complexity of the original M-file and whether it also calls/uses other M-files and data (`.mat` files), uses GUI tools, and so forth. If no problematic issues or errors are encountered during the packaging process of a standalone application, the new application will be created and packaged in three folders—`for_redistribution`, `for_redistribution_files_only`, and `for_testing`—and in one log file called `PackagingLog.txt`. These are generated automatically by the compiler. In order to verify the generated standalone application, we open the folder called `for_testing` (or click on the hyperlinked Open Output folder) and within this folder, we run `QUAD_eqn_SIM.prj` QUAD_eqn_SIM by clicking on it.



Application Compiler - QUAD_eqn_SIM.prj

Standalone Application

QUAD_eqn_SIM.m

○ Runtime downloaded from web MyAppInstaller_web 5 MB

○ Runtime included in package MyAppInstaller_mcr 754 MB

Application information

QUAD_eqn_SIM 1.0

Sulaymon ESHKABILOV

sulaymon.eshkabilov

NDSU

Quadratic Equation S

This is a satndalone a … t user entries for a, b, c, xmin and xmax.

Package

☑ Open output folder when process completes

Figure 4-9. Note indicating successful completion of the compiled standalone application project QUAD_eqn_SIM.prj

Note When the packaging/compiling process of the standalone application is completed, there will be three folders—`for_redistribution`, `for_redistribution_files_only`, and `for_testing`—and one log file called `PackagingLog.txt`. These are generated automatically by the MATLAB Application Compiler.

The created standalone application opens, as shown in Figure 4-10. Now it can be tested with some entries.

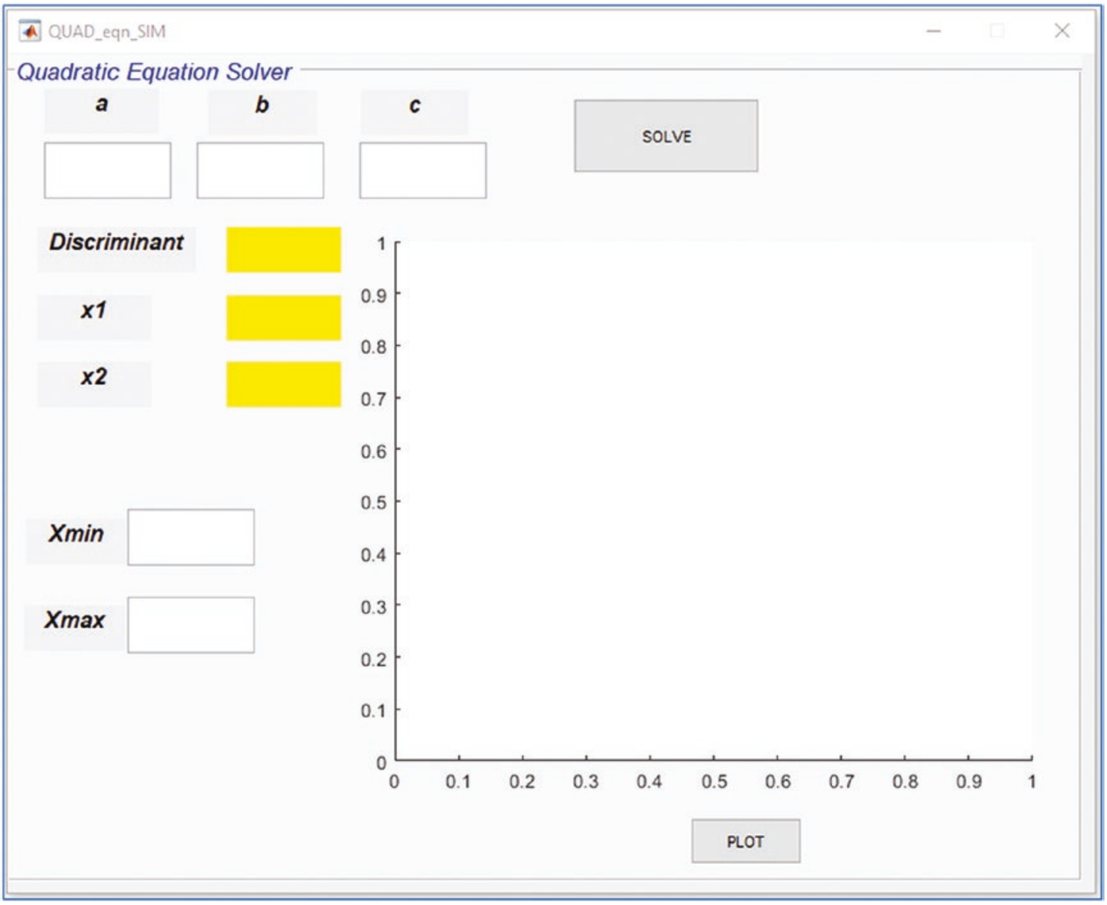
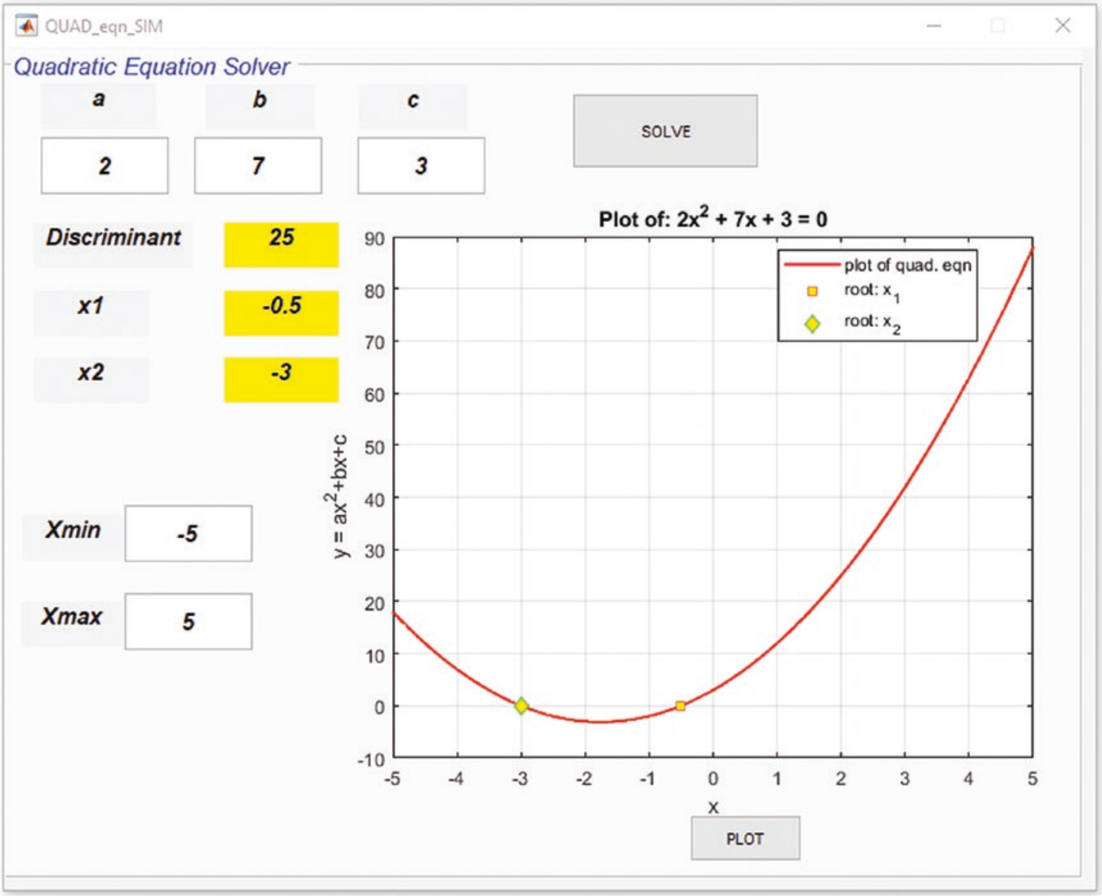


Figure 4-10. *Complete standalone application*

We can test our newly created application with the entries:—a = 2, b = 7, and c = 3—and plot for xmin = -5 and xmax = 5. The results are shown in Figure 4-11. So now this standalone application can be installed on any computer and it works without the MATLAB package. The installation package of the standalone application is one executable file, called `MyAppinstaller_web.exe`. This `*.exe` file resides inside the folder called `for_redistribution`. This application can be installed by double-clicking this `*.exe` file on any computer. It runs without the MATLAB packages seamlessly, as shown in Figure 4-12. We simply select the folder where we want to install the application and follow several other standard steps for installation of software packages. Note that this installation process may take a few minutes.



QUAD_eqn_SIM

Quadratic Equation Solver

a: 2

b: 7

c: 3

Discriminant: 25

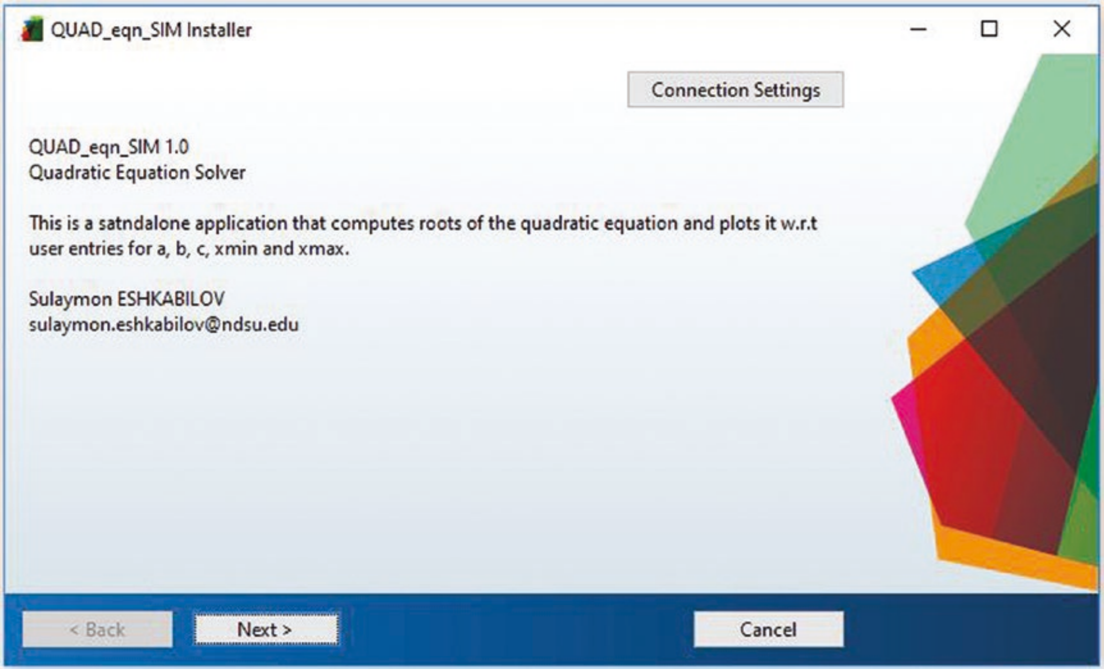
x1: -0.5

x2: -3

Xmin: -5

Xmax: 5

Figure 4-11. *Complete standalone application with simulation results*



QUAD_eqn_SIM 1.0
Quadratic Equation Solver

This is a satndalone application that computes roots of the quadratic equation and plots it w.r.t user entries for a, b, c, xmin and xmax.

Sulaymon ESHKABILOV
sulaymon.eshkabilov@ndsu.edu

Figure 4-12. *QUAD_eqn_SIM standalone application installation process*

Once the standalone application is installed, it appears in the program's list of computers and can be launched like any other program, by clicking on it or using its shortcut icon.

Note The only installation `*.exe` (called `MyAppinstaller_web.exe`) resides inside the folder called `for_redistribution`. It's the only file required to install the developed application on any machine.

Note that, like MEX files, standalone applications are also system dependent. In other words, if we create a standalone application on a 64-bit processor OS computer in MATLAB, it can be run only on 64-bit processor OS system computers. Thus, for each system type, we have to recompile the standalone application. In addition, it must be noted that in MATLAB, there are many functions and tools within various toolboxes and if we employ specific functions or tools from various toolboxes of MATLAB and/or packages and intend to create a standalone application, the MATLAB Compiler may fail to package these applications for obvious reasons. On the other hand, it is viable to create standalone applications from GUI models with some limitations based on MATLAB's incompatibility with those standalone applications.

CHAPTER 5

Simulink Modeling Essentials

Simulink[1] is the graphical programming package that works in association with MATLAB and interacts as one combined package. It is employed for modeling, simulating, and analyzing dynamic systems, control algorithm development, and so forth. It supports linear and non-linear systems, continuous and discrete systems, and multi-rate systems. With Simulink, you can model myriad types of systems, processes, and problems, and you can use top-down and bottom-up approaches.

The Simulink package, like MATLAB, is expandable. By using its standard blocks, you can develop your own library of blocks and subsystems and add to existing Simulink Libraries and expand them. In our Simulink models, we can combine continuous systems with discrete ones. One of the main advantages of using the package from a user's perspective is that it is much easier to model systems via block diagrams that doesn't require any preliminary programming skills or experience from the users. In Simulink, you simply drag, drop, and connect blocks, and of course, adjust parameters, solvers, and others in the model. Another sound advantage of the Simulink package is that its models can work interactively with MATLAB and can be manipulated/executed from the MATLAB Command Window and the M/MLX-files and function files.

Simulink Modeling

To launch the package from MATLAB, you just type `>> simulink` in the Command Window, click on the Simulink icon from the menu panel, or click New and Simulink Model. The window in Figure 5-1 will pop up. From the Simulink startup window, you can open existing models (recently worked ones) from the right-side pane (Open) or

[1] Simulink is a registered trademark of the MathWorks Inc.

S. Eshkabilov, *Beginning MATLAB and Simulink*, https://doi.org/10.1007/978-1-4842-5061-7_5

create a new model by clicking on Blank Model or the other options there. Also, from the Examples tab, users can open, study, and change existing examples. There are dozens of examples from different areas of engineering, physics, computing, image processing, code generation, and so forth.

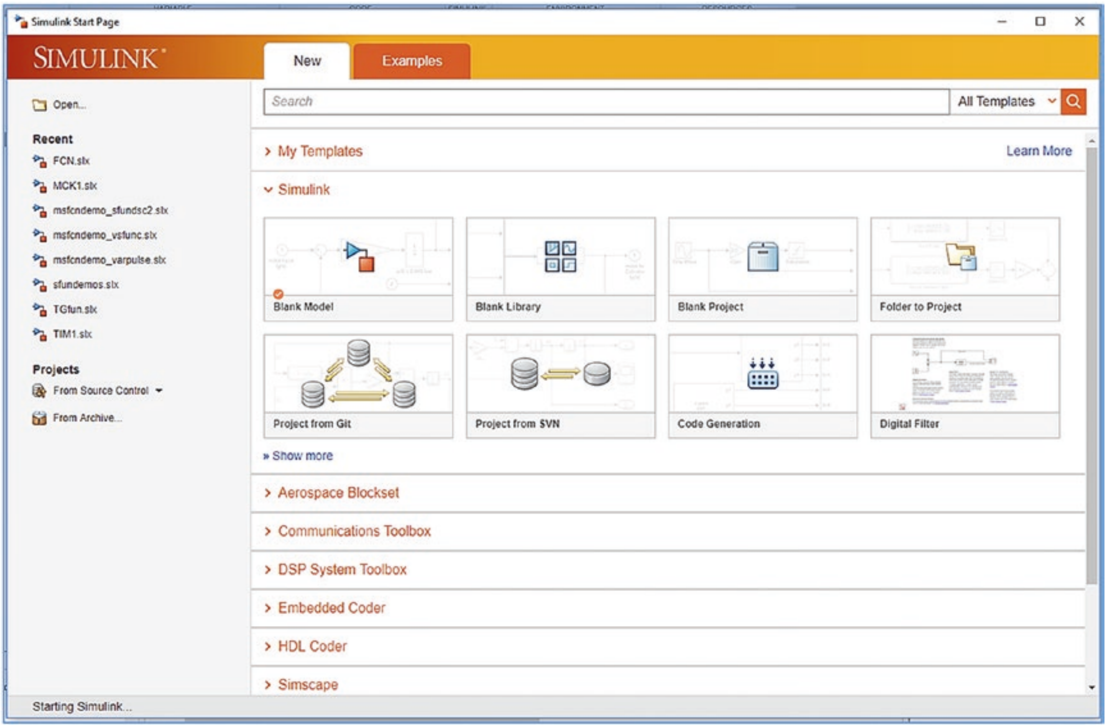


Figure 5-1. *Simulink's startup window*

Figure 5-1 shows the default startup window of the Simulink package. Note that Simulink is a standalone package and there are a few toolboxes and add-ons (see Figure 5-2) that can be installed. All of the blocks of the additional libraries and add-ons will be accessible once they're installed from the Simulink Library browser.

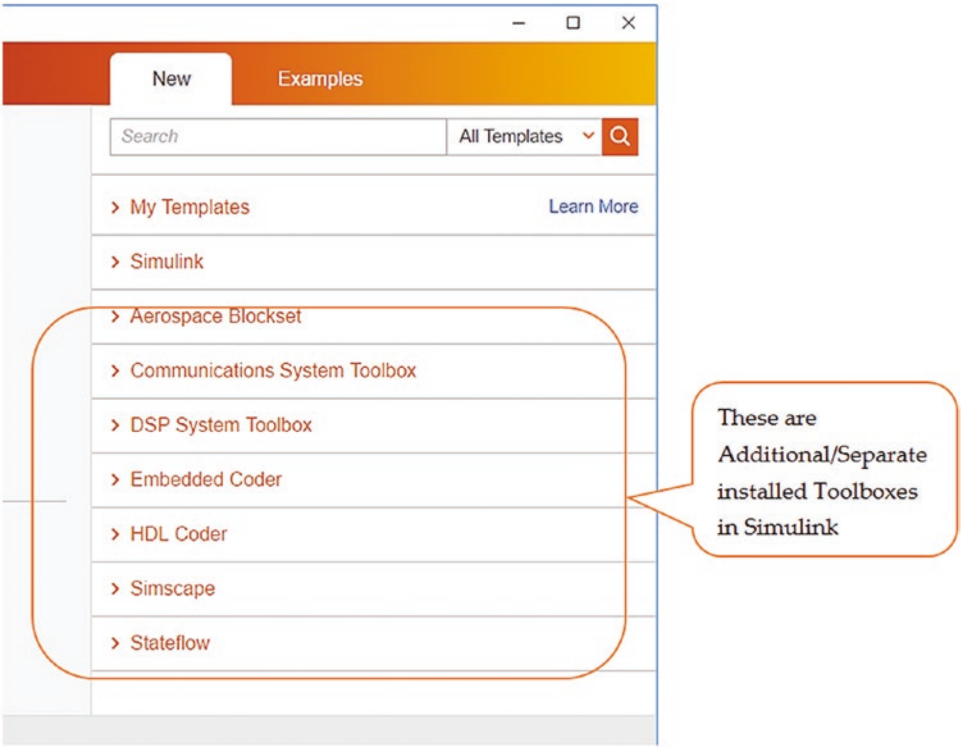


Figure 5-2. *Simulink's startup window and additional toolboxes installed in it*

Example: Arithmetic Calculations

Let's take the following simple to demonstrate how to build a Simulink model that computes addition, multiplication, and subtraction of four different scalar numbers. The complete model is shown in Figure 5-3. It's composed of Constant, Gain, Sum, and Display blocks. The scalars entered in constant blocks are 13, 22, 3i/2, and $\sqrt{5}$. When this model is executed, it displays its computed results in the Display block. Note that all of the blocks employed in this model are available in the Simulink Library.

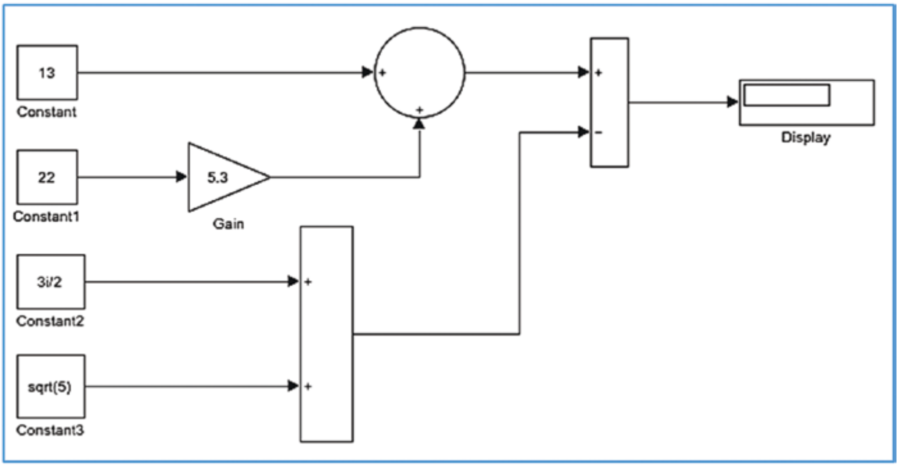


Figure 5-3. *Complete Simulink model example with arithmetic calculations*

To open a blank model, we click Blank Model, as shown in Figure 5-1. Note that a Blank Model can be opened from the already opened Simulink window by clicking or pressing Ctrl+N from the keyboard. After clicking the blank model icon (see Figure 5-1), the model shown in Figure 5-4 opens. This is called `untitled.slx` by default, just like MATLAB's M/MLX-files.

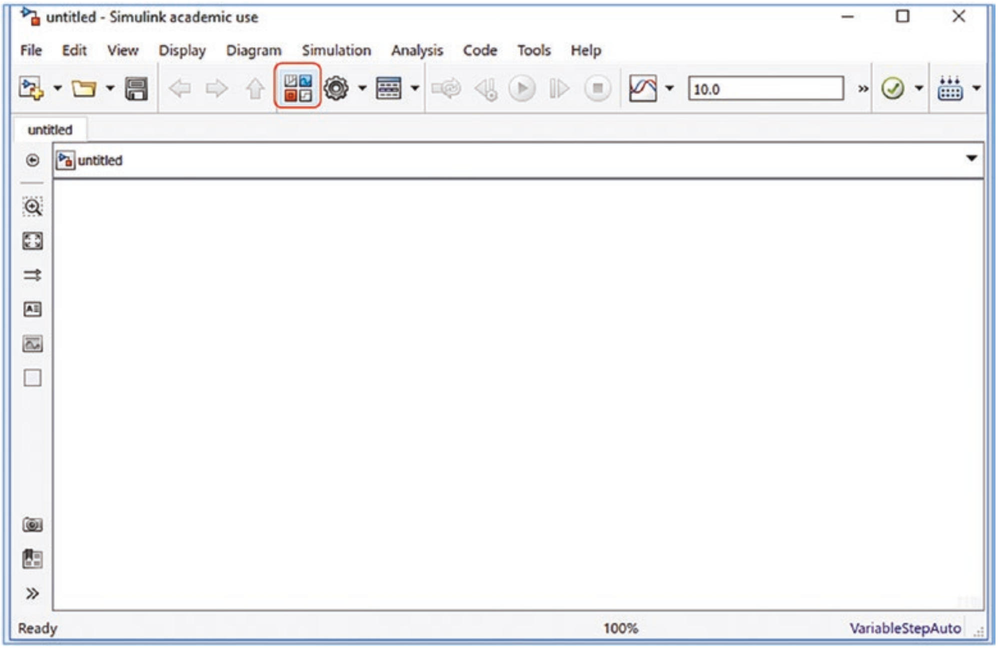


Figure 5-4. *New Simulink model window (untitled by default)*

From the opened blank model window, click on (the Simulink Library icon). That will open a Simulink Library window, as shown in Figure 5-5. It must be noted that the toolboxes that are available in the library are defined by which toolboxes are installed and which user developed/created custom libraries are installed. If necessary, you can also create a new blank model by clicking on the New Model icon in the Library Browser (see Figure 5-5). This also creates a blank model window.

The Simulink Library (see Figure 5-5) looks different in different versions but most of its general blocks function using the same principle. In this regard, it is worth pointing out that the package has been developed and subject to constant improvement with novel add-ons/blocks, and therefore, the models created in recent versions of Simulink are not fully compatible with its older versions. All of models are forward compatible; models created in older versions of Simulink work in later versions of the package.

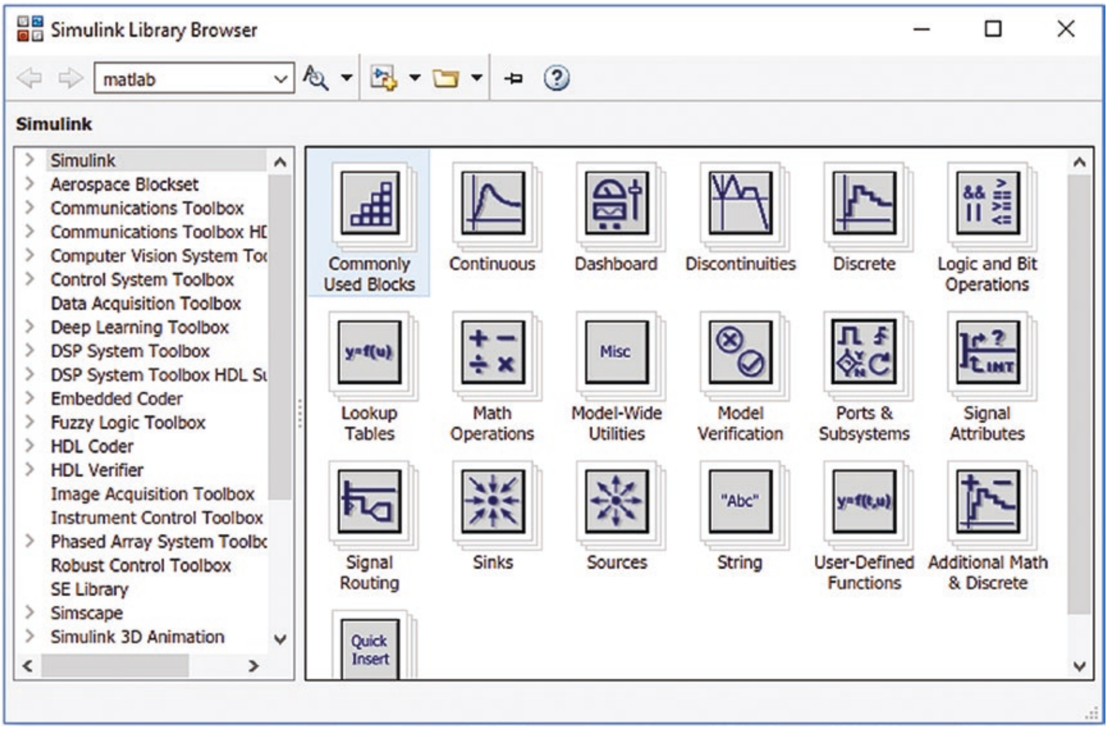


Figure 5-5. *Simulink Library browser*

Let's build a new model where we drag and drop all the necessary blocks from the library. Dragging and dropping a block into a new model area is the most common practice in Simulink based modeling or programming; however, in recent versions of Simulink (starting from MATLAB/Simulink 2018a), there is an alternative way of obtaining blocks within a model area. It is also possible to obtain any block by double-clicking on a desired spot of the model area. For example, if you want to use an Input block, you can double-click and type in the Create Annotation search box. The prompt drop-down options will appear, as shown in Figure 5-6. From the drop-down option, you can select the desired block name. There is another optional step to specify the port number in this case. For other blocks, this option differs.

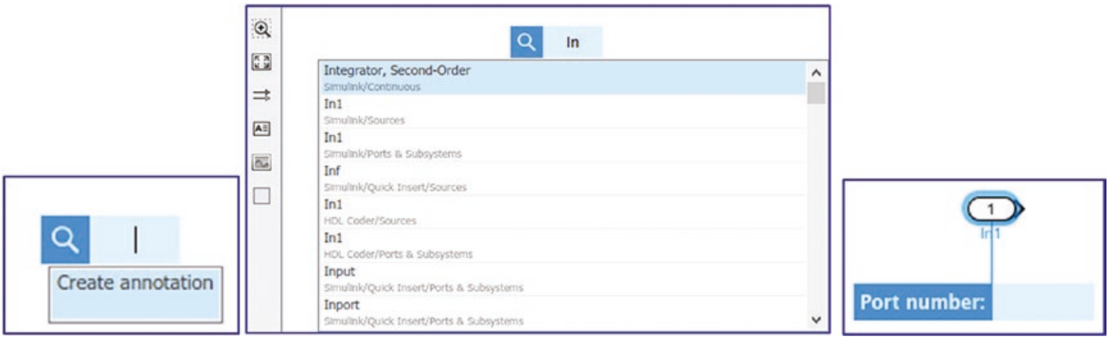

Create annotation
In
Integrator, Second-Order
Simulink/Continuous
In1
Simulink/Sources
In1
Simulink/Ports & Subsystems
Inf
Simulink/Quick Insert/Sources
In1
HDL Coder/Sources
In1
HDL Coder/Ports & Subsystems
Input
Simulink/Quick Insert/Ports & Subsystems
Inport
Simulink/Quick Insert/Ports & Subsystems
1
In1
Port number:

Figure 5-6. *Creating/obtaining blocks within the model area by using the annotation option*

Once all of the necessary blocks are placed in the model area, they need to be connected. There are two ways to connect blocks in the Simulink model window. You can connect a block in the blank model window by clicking the left mouse button and holding the Ctrl key from the keyboard. You then click the left mouse button on another block to connect the two. An alternative way of linking blocks or connecting signals in between blocks is to drag a signal arrow (see Figure 5-7) to the block you want to connect.

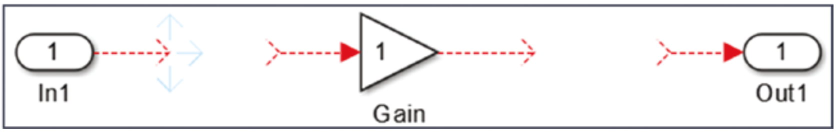


Figure 5-7. *Connecting blocks*

Some blocks have input and output ports, some only have input ports, and others only have output ports. You can only connect signals from their output port to another block's input port. In other words, it is not possible to connect signals from output port to output port or input to input.

Note Blocks can be resized easily. You click on the block to be resized and drag it from one of the four corners of the rectangular boundary around the block.

Before you start working with Simulink modeling, you must adjust one tool to ease the process of working with the library of blocks. To keep the Simulink Library on top of all the model windows, the 📌 icon must be clicked to a position of on top 📌. This is done with a single click.

Note To keep the Simulink Library on top of all the windows, including the model window, click the stay on top icon 📌.

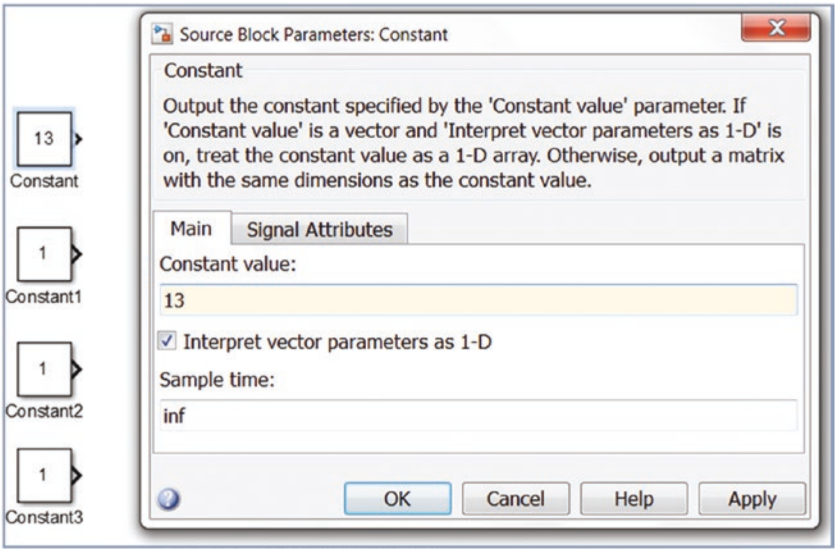
Moreover, at the beginning you need to work within Simulink Library. That can be accessed by clicking on the + before Simulink. The Simulink Library contains a number of block sets grouped in Commonly Used Blocks, Continuous, Dashboard, Discontinuities, Discrete, Logic, and Bit Operations. Let's look at several examples to explore the modeling tools and aspects in the Simulink environment.

Example: Modeling Simple Arithmetic Operations

Let's compute simple arithmetic operations such as $+, -, /, *, \sqrt{\ }$, etc. $13+22*5.3-\left(3i/2+\sqrt{5}\right)$ and display the results.

To model these arithmetic computations of the exercise, you need the following blocks from the library: Simulink/Commonly Used Blocks: four Constant blocks, three Sum blocks, one Gain block and one Display block from Simulink/Sinks to output the computation results.

1. Drag and drop all of these blocks in the model area.
2. All of the Constant blocks' constant values must be changed according to the given task (13, 22, 3i/2, $\sqrt{5}$)—see Figure 5-8. Similarly, add one Gain block (for 5.3) by double-clicking on each block, one after another (see Figure 5-9).
3. Link a Constant block called Constant (13) to a Sum block with the + sign. You do this by left-clicking the block and holding the Ctrl key from the keyboard. Then you left-click a Sum block.
4. Link a Constant block called Constant1 (22) to a Gain block, which is linked to the Sum block with + sign, as shown in Figure 5-6.
5. Link a Constant block called Constant2 (3i/2) to a second Sum block. This rectangular shape can be changed by double-clicking on a Sum block with + sign. Similarly, to this Sum block, the Constant block called Constant3 ($\sqrt{5}$) is linked using a + sign.
6. Connect two Sum blocks to a third Sum block with the + and – signs. Subsequently, link the third Sum block to the Display block.



Constant

Output the constant specified by the 'Constant value' parameter. If 'Constant value' is a vector and 'Interpret vector parameters as 1-D' is on, treat the constant value as a 1-D array. Otherwise, output a matrix with the same dimensions as the constant value.

Constant value:

13

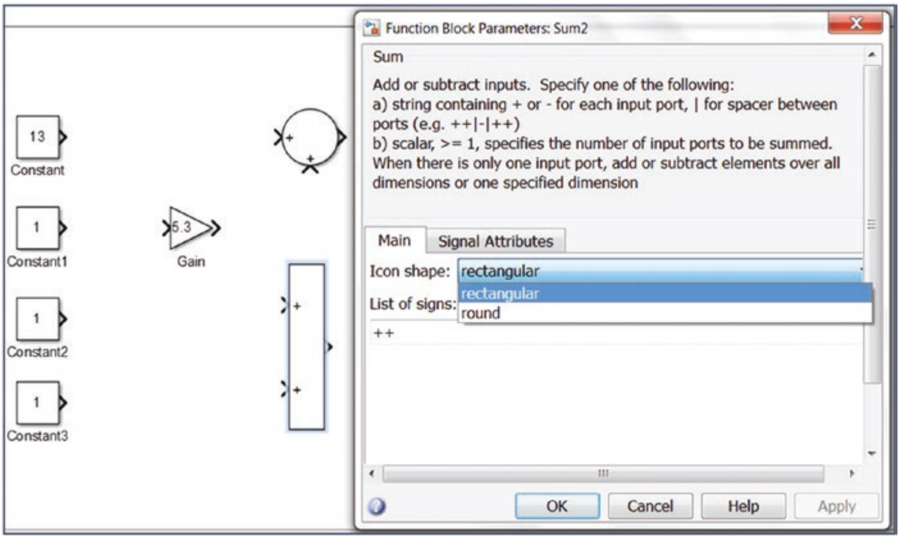
☑ Interpret vector parameters as 1-D

Sample time:

inf

Figure 5-8. *Blocks*

Note Any block can be copied numerous times by holding Ctrl and left-clicking the block, then dragging the block over any spot in a model space. This method is faster and more efficient than dragging the same block from the library.



Function Block Parameters: Sum2

Sum

Add or subtract inputs. Specify one of the following:
a) string containing + or - for each input port, | for spacer between ports (e.g. ++|-|++)
b) scalar, >= 1, specifies the number of input ports to be summed.
When there is only one input port, add or subtract elements over all dimensions or one specified dimension

Icon shape: rectangular

List of signs: ++

Figure 5-9. *Altering the Sum block's signs and icon shape*

That completes our simple computation Simulink model.

This model is saved with the filename of `Ex1_Arithmetic_operations.mdl`.

Note In the latest versions of Simulink, models can be saved either in `*.mdl` (backward compatible file format) or `*.slx` (supported in later versions). Any Simulink model can be exported to previous versions via File ➤ Export Model to ➤ Previous Version…. In the Save As Type option, you select the appropriate version from the drop-down.

To see computation results, click the Run button or press the Ctrl+T keys from the keyboard. The results are displayed in the display block. Figure 5-10 shows the complete model.

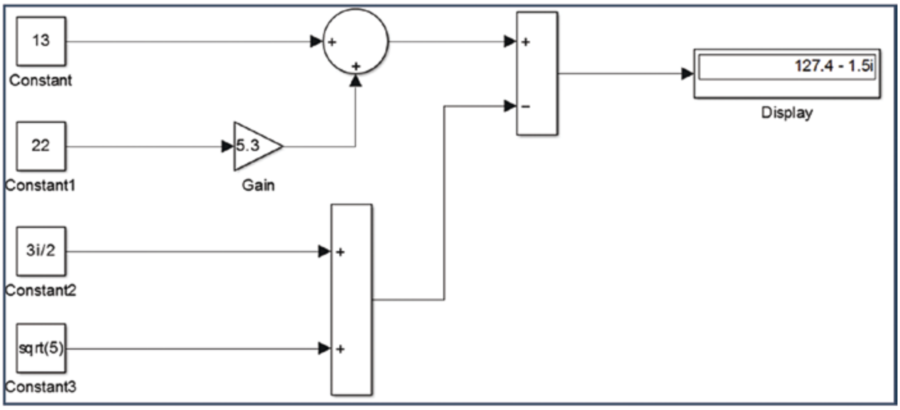


Figure 5-10. *Completed Simulink model called Ex1_Arithmetic_operations.mdl*

Note To view a model in full screen, just press the spacebar from the keyboard. To zoom in and out, press the Ctrl++ and Ctrl+- keys (in later versions) from the keyboard, or click on ⛶ to get a Fit View, or use 🔍 to zoom in.

Note that $\sqrt{5}$ is typed in as a value of the Constant block called Constant3 by `sqrt(5)` and 3i/2 is typed as 3i/2 since Simulink (like MATLAB) recognizes imaginary numbers via letters of `i` and `j` automatically. Any Simulink model can be simulated/executed from MATLAB via the workspace or within any M-file using the `sim()` command. The simulated model must reside in the current working directory. For instance, the previous model can be run with this command:

```
>> sim('Ex1_Arithmetic_operations.mdl')
```

Note A name of any block can be altered by clicking on its name tag.

Properties and parameters of any block can be altered by opening the property window (by double-clicking on it) and inserting necessary changes or selecting necessary options. For instance, in a Display block, you can format the data as short, long, short_e, long_e, hex, etc.

Performing Matrix Operations

We'll build a Simulink model that performs the following matrix operations:

$$\begin{bmatrix} 3 & 2.5i \\ -2i & 3.2 \end{bmatrix}^{T} * \frac{1}{5} + \begin{bmatrix} \sqrt{-3i} & \sqrt{2.2} \\ \sqrt{2.5} & \sqrt{5.4} \end{bmatrix}^{-1} * \sqrt{3.3i} \text{ and displays the results.}$$

The complete model is shown in Figure 5-11. The model performs matrix operations, such as, inverse, square root, transpose, sum, and division by a scalar (real and imaginary numbers). The matrix operations (transpose and inverse): $\begin{bmatrix} 3 & 2.5i \\ -2i & 3.2 \end{bmatrix}^{T}, \begin{bmatrix} \sqrt{-3i} & \sqrt{2.2} \\ \sqrt{2.5} & \sqrt{5.4} \end{bmatrix}^{-1}$ are performed by employing the Interpreted MATLAB Function block. Math Function and Product of Elements are compared with the computed results and shown via Display blocks.

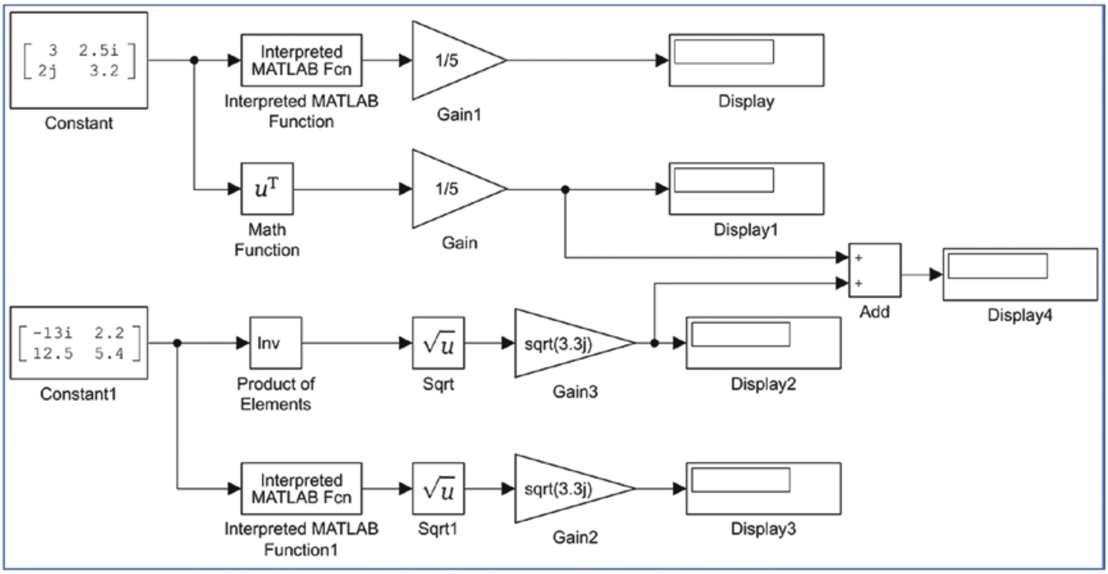


***Figure 5-11.** The complete model: example 2, matrix operations*

To model this exercise, you use the following steps.

1. For this model, you need two Constant blocks, four Gain blocks, five Display blocks, an Add block from Simulink/Commonly Used Blocks library, two Sqrt blocks, one Math Function block,

a Product of Elements block from the Simulink/Math Operations library, and two MATLAB Interpreted Function from the Simulink/User-Defined Functions (see Figure 5-12).

2. Enter elements of two matrices into two Constant blocks as: `[3 2.5j; 2j 3.2]` (see Figure 5-13) and `[-3i 2.2; 2.5 5.4]`. These will be connected with two separate Math Function blocks, one of which is Math Function (transpose of a matrix) and the other is Product of Elements (inverse of a matrix). These two blocks are edited accordingly; for instance, to obtain a transpose operator of the Math Function block, a Function type is chosen to be a transpose from drop-down options. In the Product of Elements block, the number of inputs option is changed to be a division (/) and multiplication option is selected to be a `matrix(*)` multiplication. .

3. Link the Math Function (transpose) block to the Gain block (1/5), which is subsequently linked to the Sum block. The Product of Elements block is connected to the Sqrt block, which is linked to the Gain block ($\sqrt{3.3i}$). And at last, signals from the Gain block (1/5) and the Gain block ($\sqrt{3.3i}$) are connected with the Sum block (see Figure 5-13), which is linked to the Display block.

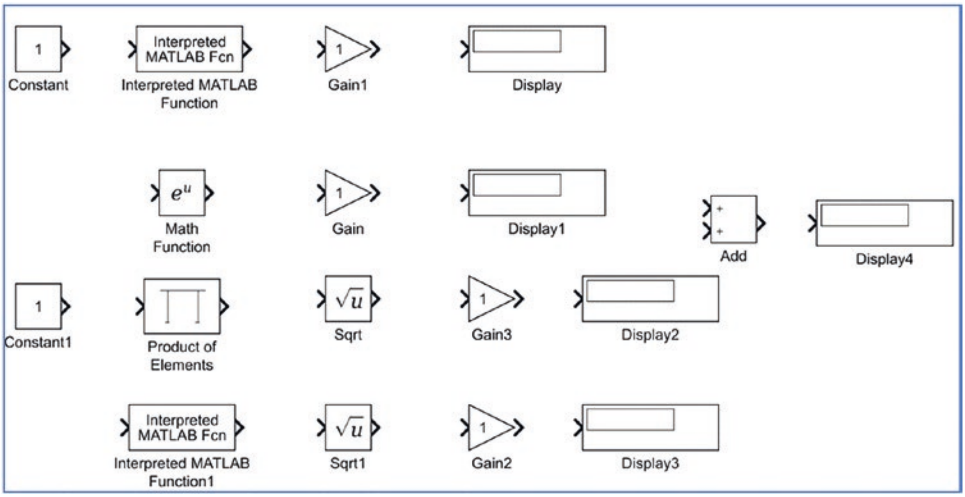
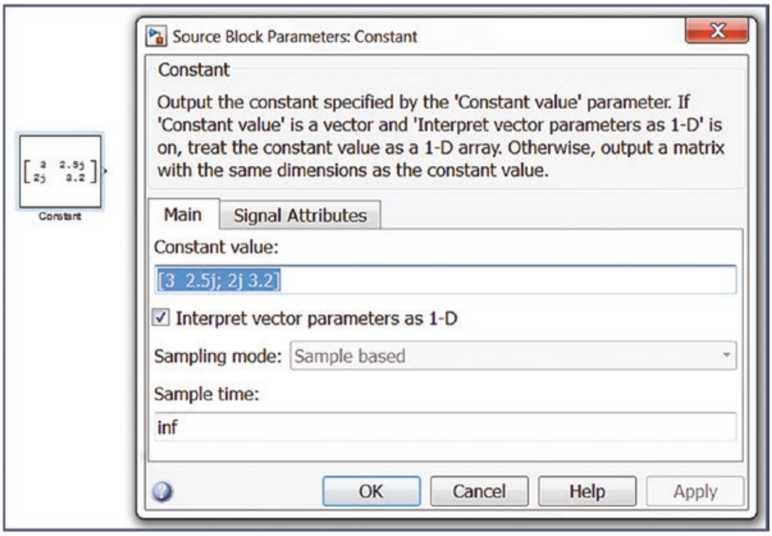


Figure 5-12. *All blocks necessary for this model*



Source Block Parameters: Constant

Constant

Output the constant specified by the 'Constant value' parameter. If 'Constant value' is a vector and 'Interpret vector parameters as 1-D' is on, treat the constant value as a 1-D array. Otherwise, output a matrix with the same dimensions as the constant value.

Main | Signal Attributes

Constant value: [3 2.5j; 2j 3.2]

☑ Interpret vector parameters as 1-D

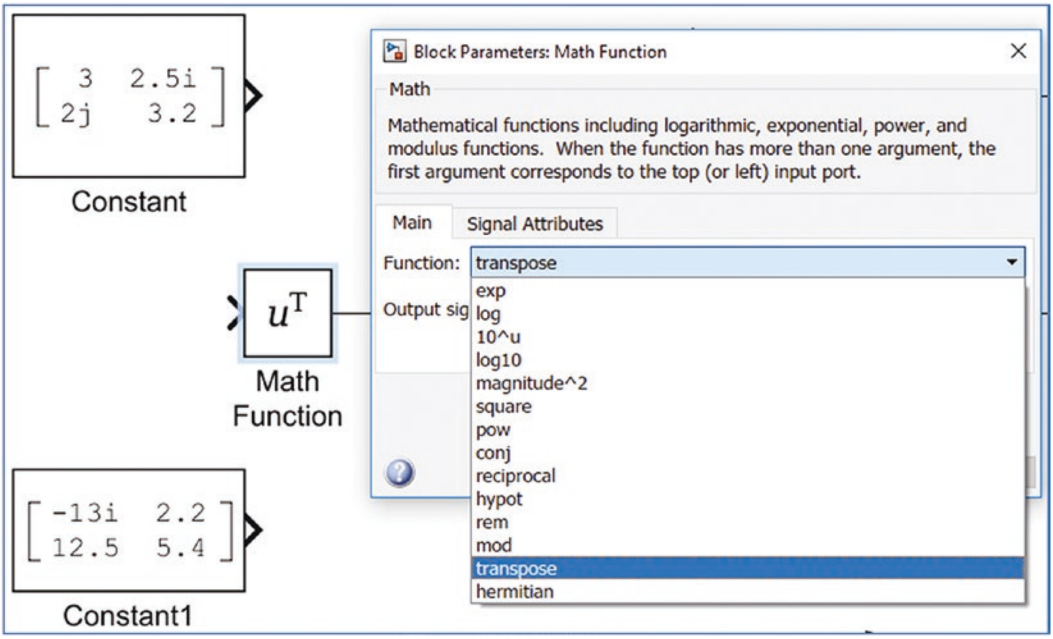
Sampling mode: Sample based

Sample time: inf

***Figure 5-13.** Inputting matrix elements into the Constant block's constant value*

Note In Simulink modeling, we can attain the same results by altering properties and parameters of blocks. For instance, matrix inverse can be obtained by inserting the MATLAB function called `inv(u)` in the Interpreted MATLAB Function block.

Matrix transpose is obtained from the Math Function block's options, as shown in Figure 5-14.



Block Parameters: Math Function

Math

Mathematical functions including logarithmic, exponential, power, and modulus functions. When the function has more than one argument, the first argument corresponds to the top (or left) input port.

Main | Signal Attributes

Function: transpose

Output sig

Figure 5-14. *Selecting Function type in the Math Function block*

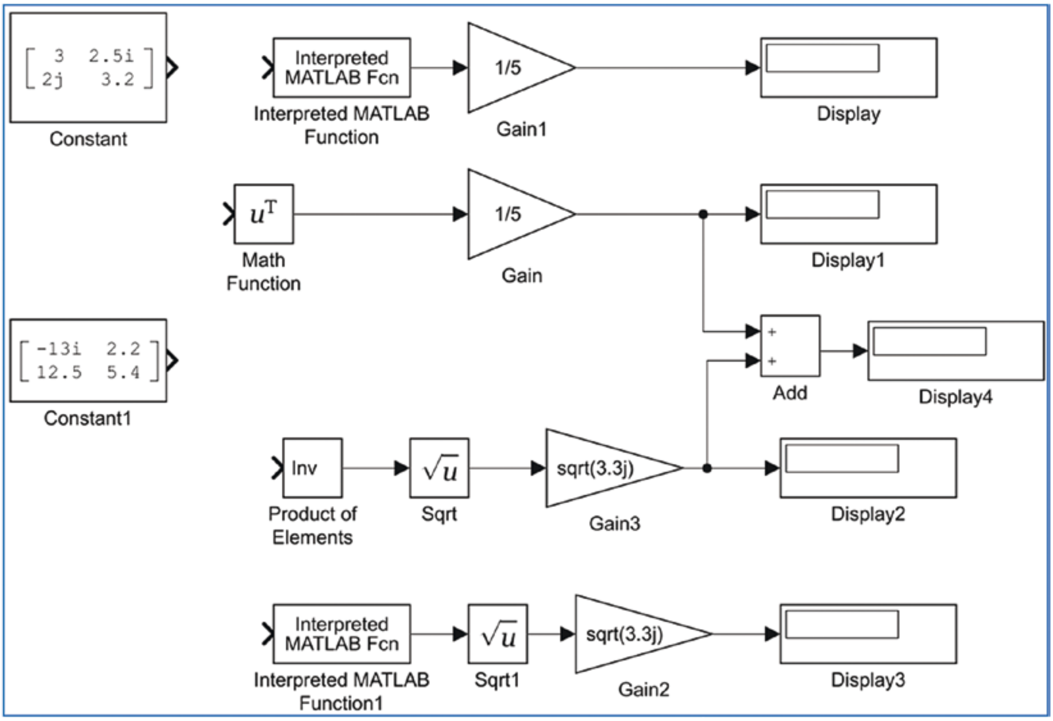


Figure 5-15. *All blocks with adjusted options and entered values/elements*

Note that there are two ways to obtain the inverse of matrices demonstrated in the completed model, as shown in Figure 5-16. By running the completed model, you can see that the results in both matrix inverse operations are the same.

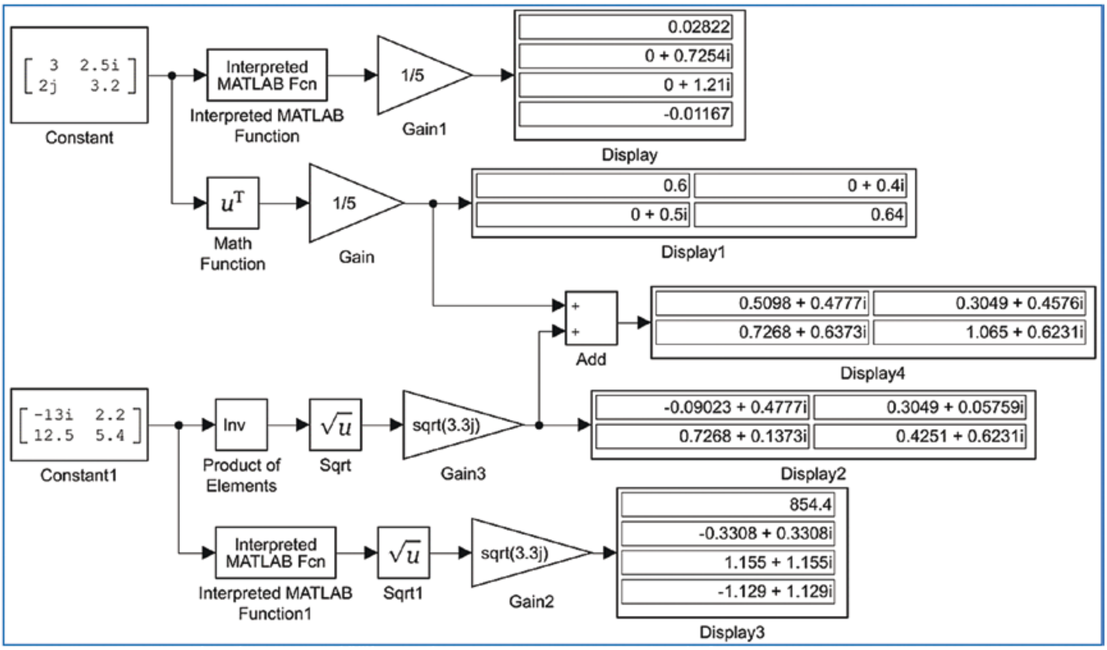


Figure 5-16. *Complete matrix operations and Simulink model called Ex2__MATRIX_operations.slx*

Note There are two file formats used to save Simulink models—`*.mdl` and `*.slx`. The latter model type is supported in later versions (starting with MATLAB 2010) of the Simulink package and the former format can be opened and simulated by most versions, depending on the blocks used. Via a Model Export option, we can save models in the previous versions of Simulink.

There are a few new blocks with different properties included in later versions of the Simulink package. Therefore, the models developed by employing such new blocks cannot be simulated by earlier versions of the package.

Computing Values of Functions

In this section, you learn how to compute values of the following math functions, save the computation results in a separate *.mat file and MATLAB workspace simultaneously, and display them in a plot figure. Given: $H(t) = e^{sinc(t)} + e^{\sin(250t)}$, $t = -3\pi \ldots 3\pi$, $\Delta t = \pi/3000$.

There are several ways to build a computation model of the given example. Let's start with a simple and straightforward way. You first take the necessary blocks from the Simulink Library, like the previous two examples. For that, you need the following blocks: Clock, Scope, Math Function, Gain, Trigonometric Function, To File, and Add To Workspace, which are from the Simulink/Sources, Simulink/Math Operations, Simulink/Sinks, and Simulink/User-Defined Functions. The modeling process starts with dragging all of the blocks from the Simulink Library and connecting them in the order of Clock+Interpreted MATLAB Function+Math Function1+Add+Scope2+To File and Clock+Trigonometric Function+Math Function+Scope1+Add+To Workspace. Figure 5-17 shows the completed model that is saved under a filename Ex3_Function_Compute.slx of the following math functions, save the.

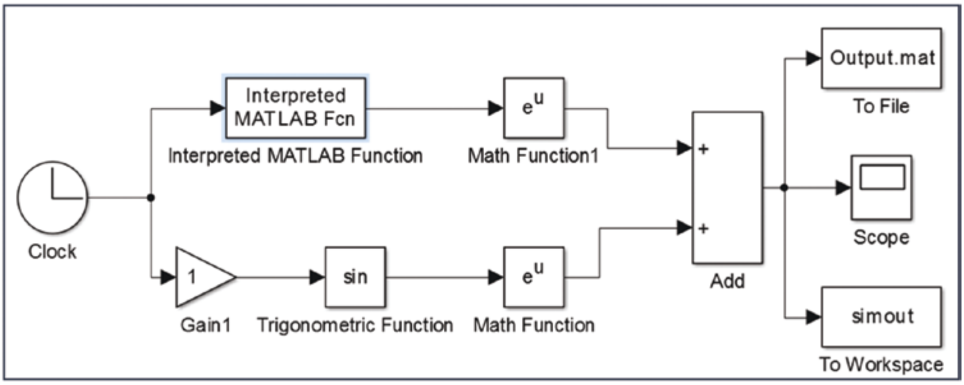
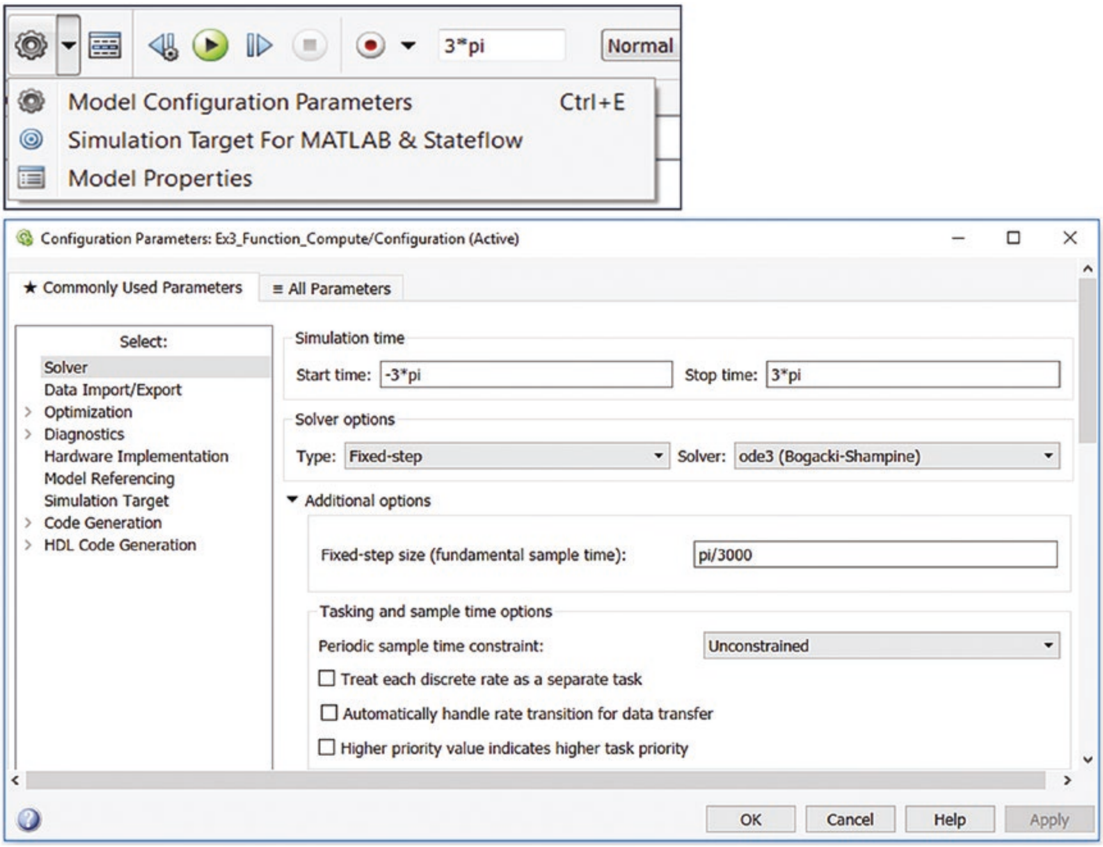


Figure 5-17. *The complete simulation model called Ex3_Function_Compute.slx*

This completed model, as it is, cannot be executed because the properties and parameters of several blocks used in the model need to be fixed according to the given tasks of this exercise.

For instance, the simulation period should be in the range of $t = -3\pi\ldots3\pi$ with the time interval of $\Delta t = \pi/300$. The MATLAB function of the block Interpreted MATLAB Function must be altered to *sinc*() and two optional editing points—the To File and To Workspace blocks—that should be renamed as external *.mat files and a variable saved in the workspace in structure format.

You can start adjusting the simulation time interval and the time step via the Model Configuration Parameters (see Figure 5-18). They can be accessed by clicking on the ⚙▾ icon or pressing the Ctrl+E keys from the keyboard. You insert the start time: `-3*pi` and the stop time: `3*pi`, and change the solver type to a fixed-step from the variable step that is chosen by default. In addition, you should specify from Solver options, the Type: Fixed-step and from the Additional options, the Fixed-step size (fundamental sample time): pi/3000. See Chapter 8 for a more detailed explanation on solvers and how to choose their types and parameters, including solver algorithm, step size, relative error, and absolute error tolerances.



Configuration Parameters: Ex3_Function_Compute/Configuration (Active)

Simulation time

Start time: -3*pi Stop time: 3*pi

Solver options

Type: Fixed-step Solver: ode3 (Bogacki-Shampine)

Additional options

Fixed-step size (fundamental sample time): pi/3000

Tasking and sample time options

Periodic sample time constraint: Unconstrained

☐ Treat each discrete rate as a separate task

☐ Automatically handle rate transition for data transfer

☐ Higher priority value indicates higher task priority

***Figure 5-18.** Adjusting configuration parameters*

Note For many models, the accuracy of the simulation results depends on the chosen solver type and step size.

Also, before proceeding with the simulations, you need to make the following adjustments to the model:

1. The MATLAB function of the Interpreted MATLAB Function block should be the `sinc(u)`. Note that the input variable name `u` is defined by default.

2. In both MATH Function blocks, the Function type (from the drop-down options) `exp` function should be used (it's the default).

3. In the Gain block, the Gain value should be 250 to obtain `250*t`.

4. In the Trigonometric function, make it the `sin` function by default.

5. Even though this step is optional, rename the output file `Output_Data.mat` and the output variable by output saved in the MATLAB workspace.

Note When you're saving the computation results via an output file in `*.mat` format and an output variable, there are several options (formats) to save data—Time-Series, Array, Structure, and Structure with Time Series.

After making all these adjustments, the model should be in the form shown in Figure 5-19.

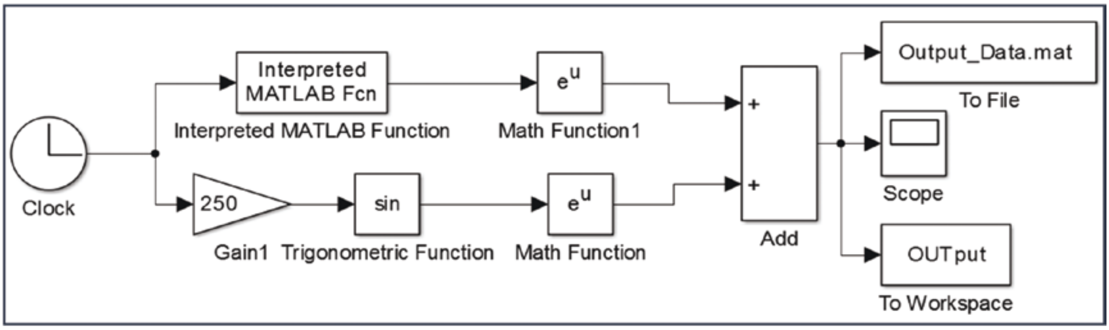


***Figure 5-19.** Finalized model with all necessary adjustments and tunings*

Finally, you can simulate the `Ex3_Function_Compute.slx` model by pressing Ctrl+T from the keyboard or clicking the Run button. You can view the simulation results by double-clicking on the Scope block. Figure 5-20 shows the simulation results, plotted in the Scope block.

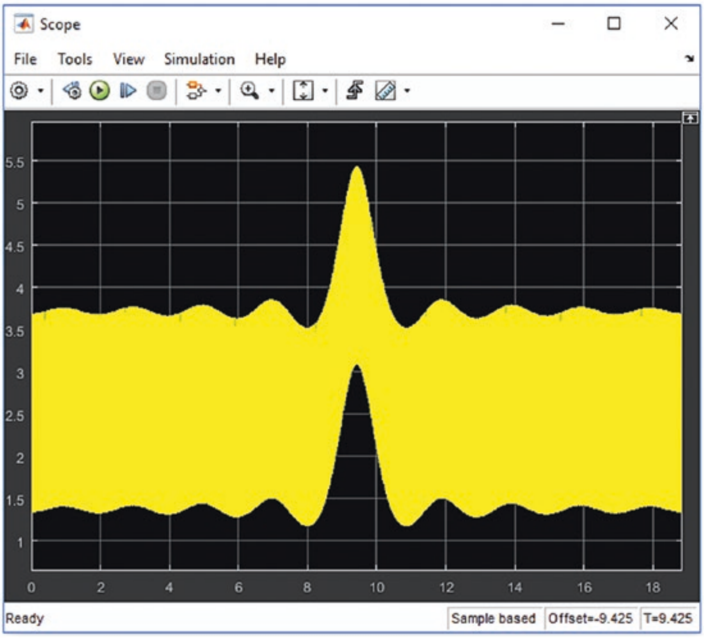
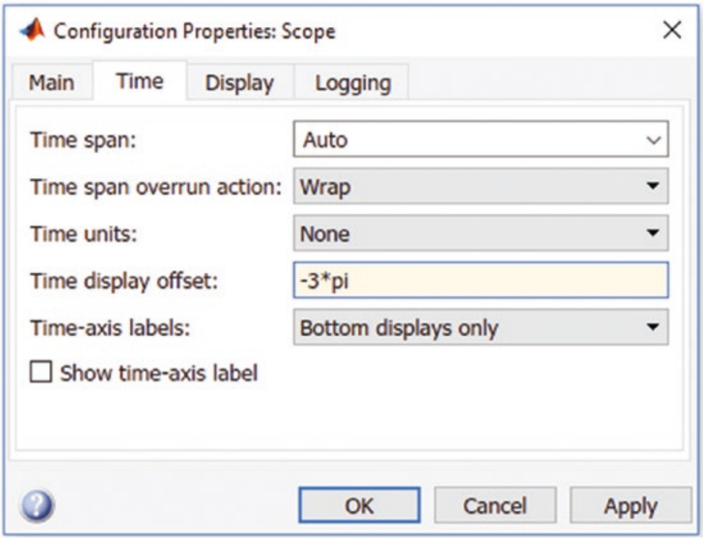


Figure 5-20. *Simulation results*

Note In the Scope display, the simulation results can be zoomed in or out proportionally, horizontally, and vertically. They can also be auto-zoomed in or out, all by using the 🔍, ↔, ↕, ⛶ tools.

The simulation results, shown in Figure 5-20, are not coherent with the set simulation period that is set to be within `-3*pi ... 3*pi`. The problem is in the Scope block's Time display offset, which is set to 0 by default. The time offset needs to be set to `-3*pi` and that can be fixed via the Configuration properties of Scope. The Configuration properties of Scope can be accessed by clicking on the ⚙ icon from its drop-down options of ⚙ Configuration Properties. After opening the Configuration Properties, click the Time tab and enter `-3*pi` in Time display offset. Subsequently, press Apply and OK (see Figure 5-21) and you'll obtain a correct display of the results in Scope, as shown in Figure 5-22. Besides, the simulation results displayed in the Scope can be saved as a variable in the format of structure with time series, array, or structure. You do this by ticking on the Log Data to Workspace option under the Logging tab (see Figure 5-21).

Moreover, the Scope block has many graphical display options that can be accessed via the ⚙ drop-down options, the choosing Style 🖌. Via the Style 🖌 options, you can adjust the figure color, axes colors, lines, and markers. See Figure 5-22.



Configuration Properties: Scope

Time span: Auto

Time span overrun action: Wrap

Time units: None

Time display offset: -3*pi

Time-axis labels: Bottom displays only

□ Show time-axis label

Figure 5-21. *Setting up the Time display offset at -3*pi in Configuration Properties: Scope*

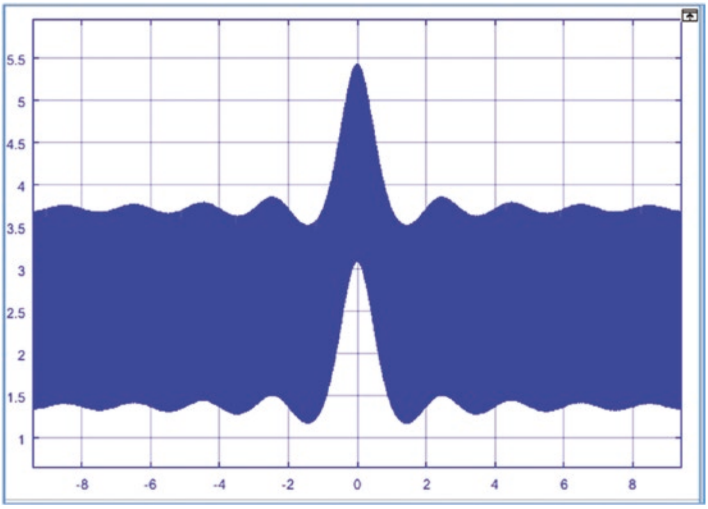


Figure 5-22. *A complete view of the simulation results*

In order to improve readability and manageability of large and complex Simulink models, a whole model or part of a model can be associated in one Subsystem block. It is easy and straightforward to create subsystems from existing models. The easiest way to create subsystems from existing models is to select the blocks (interlinked ones) by holding the left mouse button and dragging the cursor over the desired blocks, and then pressing Ctrl+G from the keyboard. In order to create one subsystem out of a whole model, press Ctrl+A first and then Ctrl+G from the keyboard.

Let's look at the complete model (see Figure 5-19) and create a subsystem out of the whole model, excluding the Input signal to Clock and the Output signal to blocks—To File, Scope and To Workspace. You first select the model blocks except for the Input block and all the Output blocks, by using the left mouse button, and then press Ctrl+G from the keyboard. The subsystem is created from the selected blocks of the model, as shown in Figure 5-23. To have access what is under the created subsystem, you need to double-click on it. See Figure 5-24.

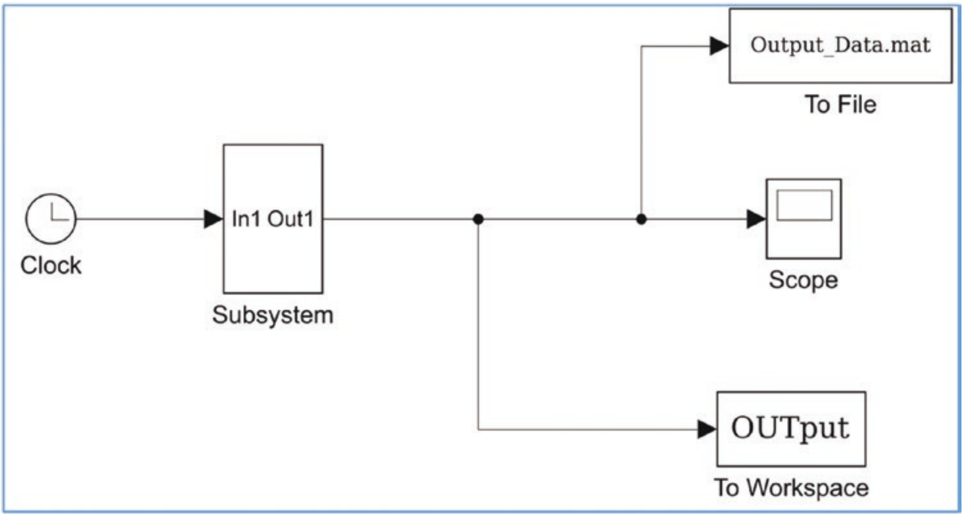


Figure 5-23. *Subsystem created out of the completed model (Figure 5-19), excluding Input and Output blocks*

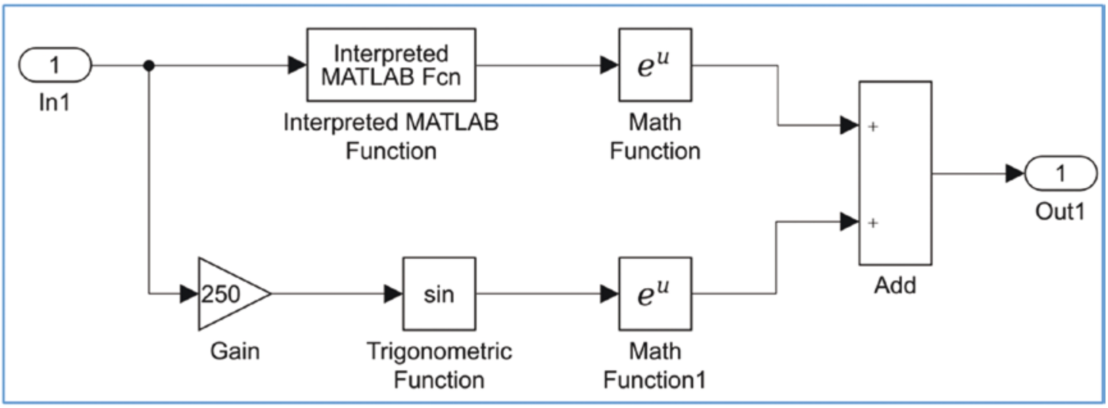


***Figure 5-24.** The created subsystem components*

Note that this subsystem in Figure 5-24 has one input block called `In1` and one output block called `Out1`. They are linked with the Input block, Clock, and Output blocks—To File, Scope, To Workspace—to receive and send signals, respectively. Note that if you create a subsystem from the whole model (see Figure 5-19) only excluding the Input block, then the subsystem will contain only one input block, called `In1`. If you create a subsystem from the whole model (see Figure 5-19) excluding the output blocks—To File, Scope, To Workspace—then it will contain one output block `Out1`. If you create a subsystem from the whole model including all blocks including Input and Output blocks, the created subsystem does not contain any input and output blocks.

The completed model can be simplified. In other words, the number of blocks used in this model can be reduced by employing the Interpreted MATLAB Function block with appropriately edited function formulation expressed by `exp(sinc(u))+exp(sin(250*u))` and adjusting Scope parameters that save simulation results in the MATLAB workspace as a structure with time series. Figure 5-25, together with the subsystem created from the model in Figure 5-23, produces the same results as the subsystem in Figure 5-19.

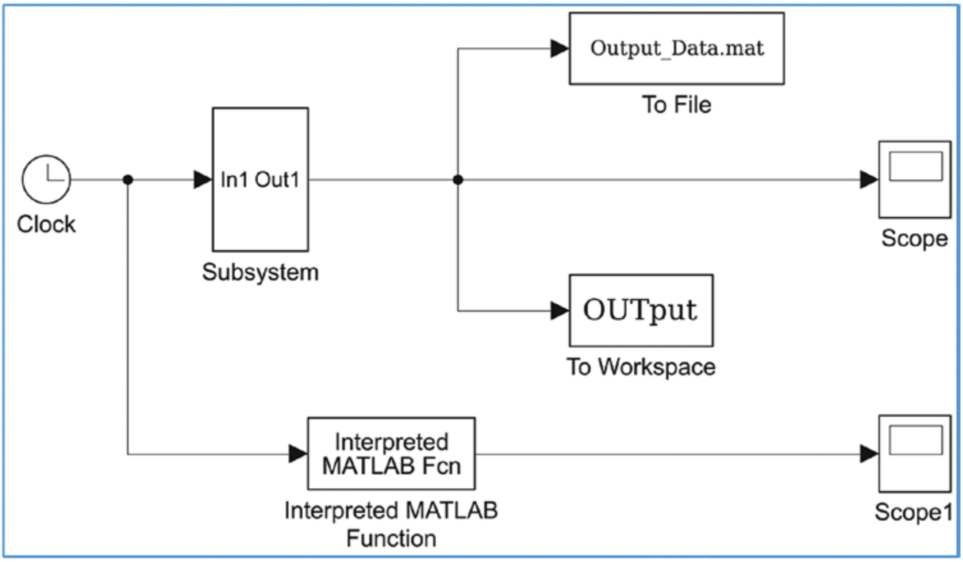


Figure 5-25. *Simplified model Ex3_Function_Compute_Simple.slx*

Input/Output Signals from/to the MATLAB Workspace

As stated, the Simulink model works interactively and flawlessly with the MATLAB workspace. For model development and simulation purposes, input signals can be generated in the MATLAB workspace or within an M-file and then transferred to the Simulink model environment. Similarly, all final simulation results of Simulink models can be sent to the MATLAB workspace. In order to have an input signal loaded from the MATLAB workspace and an output signal (simulation results) sent back to the MATLAB workspace at the same time, you use input and output blocks and adjust the model configuration parameters ⚙▾, which can be accessed by clicking on the ⚙▾ icon from the menu panel or pressing Ctrl+E from the keyboard. In the previous example shown in Figure 5-25 (simplified part is considered), we substitute the input signal Clock block with `In1` (the Input block) and the Scope block with `Out1` (the Output block); see Figure 5-26.

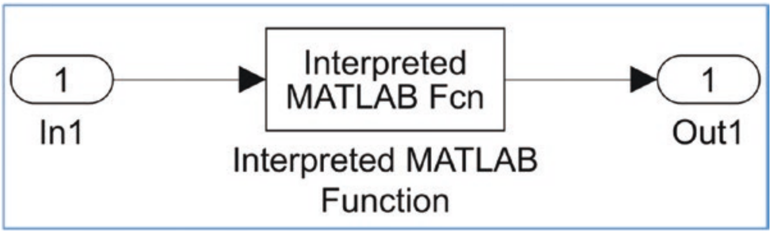
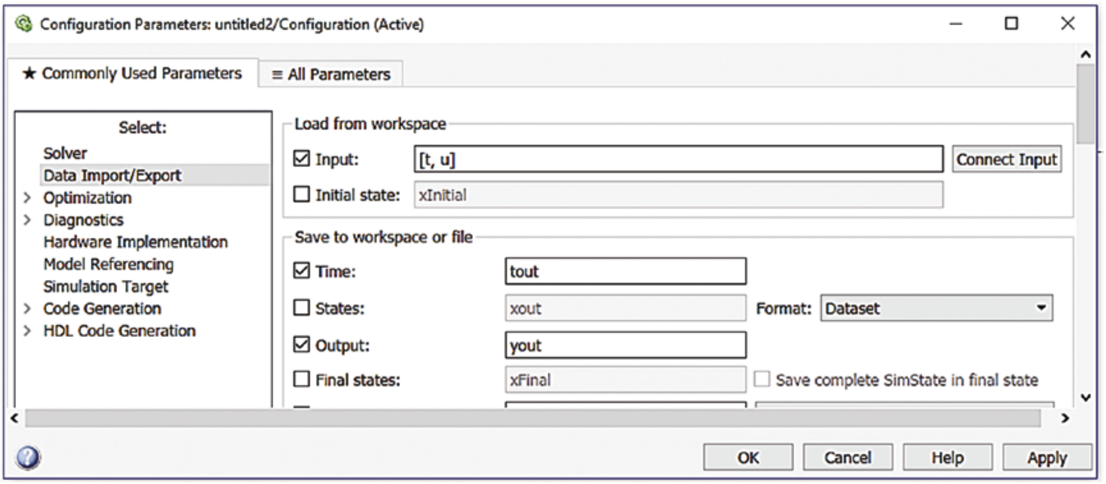


Figure 5-26. *Ex3_Function_Compute_In_Out.slx model with Input/Output from/to MATLAB workspace*

In addition, the Input and Output signal options in the model configuration parameters ⚙▾ are checked (see Figure 5-27) for input as `t`, `u` and for output as `tout` and `yout` respectively. Note that before starting the simulation, you must define the input signal in the form of t, u in the MATLAB workspace as two column vectors. That can be done for this example as follows:

```
>> t=[-3*pi:pi/3000:3*pi]'; u=t;
```

In this example, the input signal is defined by time only and thus, t = u. Moreover, we make several adjustments (Start time: -3*pi; End time: 3*pi; Type: Fixed-step; Fixed-step size (fundamental sample time): pi/3000) in the Solver options of this model via the Configuration Parameters, as shown in the example in Figure 5-18.


Configuration Parameters: untitled2/Configuration (Active)

★ Commonly Used Parameters | ≡ All Parameters

Select:
- Solver
- Data Import/Export
- › Optimization
- › Diagnostics
- Hardware Implementation
- Model Referencing
- Simulation Target
- › Code Generation
- › HDL Code Generation

Load from workspace
- ☑ Input: [t, u] — Connect Input
- ☐ Initial state: xInitial

Save to workspace or file
- ☑ Time: tout
- ☐ States: xout — Format: Dataset
- ☑ Output: yout
- ☐ Final states: xFinal — ☐ Save complete SimState in final state

Figure 5-27. *Configuration parameters changed to load an input signal from the workspace and export an output signal back to the workspace*

In addition, we can add the Scope block to the model by clicking on the signal going to the Out1 block and using the right mouse button options (Create & Connect Viewer --> Simulink --> Scope), as shown in Figure 5-28, a. After selecting Scope, the scope sign shows up on top of the signal going to the Out1 block, as in Figure 5-28, b.

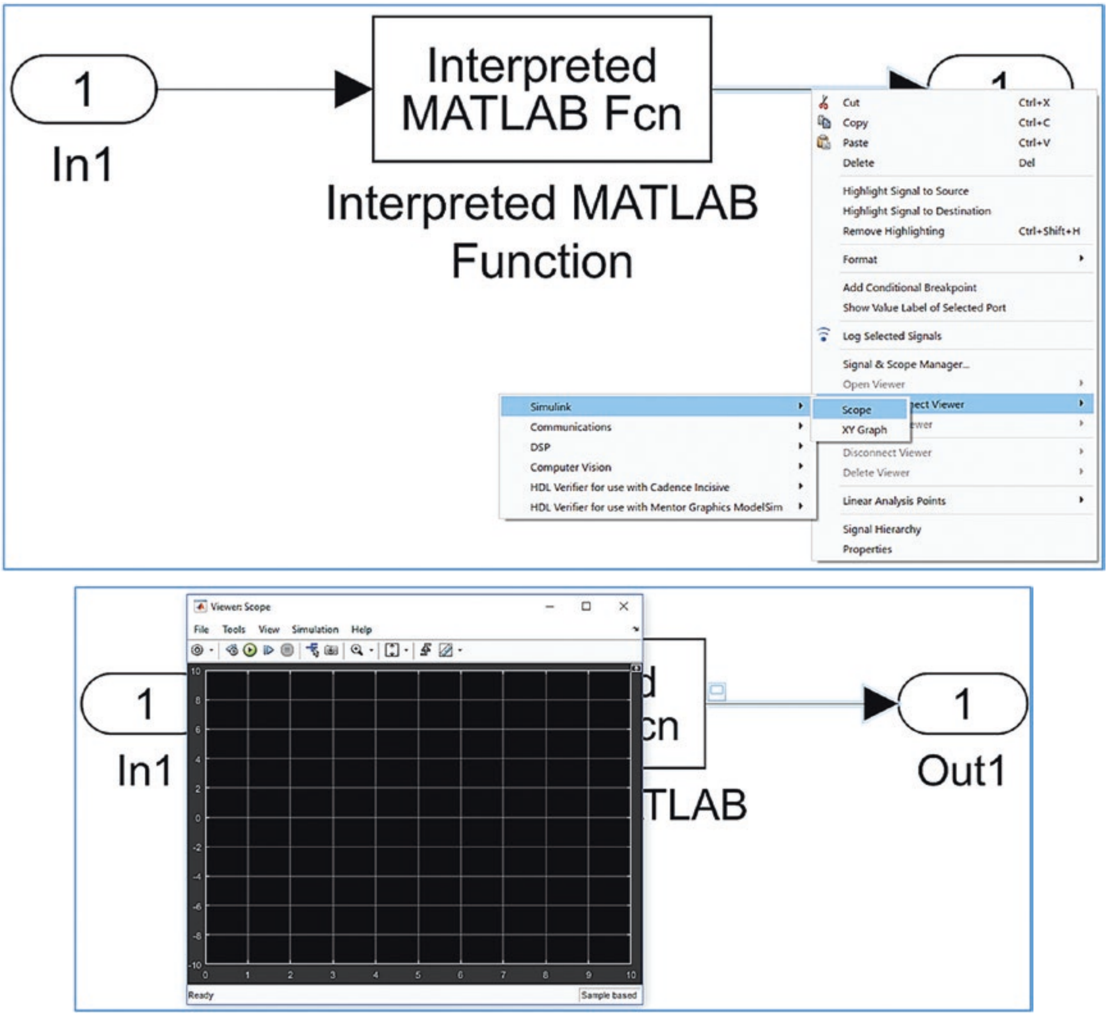

1
In1
Interpreted MATLAB Fcn
Interpreted MATLAB Function
Cut Ctrl+X
Copy Ctrl+C
Paste Ctrl+V
Delete Del
Highlight Signal to Source
Highlight Signal to Destination
Remove Highlighting Ctrl+Shift+H
Format
Add Conditional Breakpoint
Show Value Label of Selected Port
Log Selected Signals
Signal & Scope Manager...
Open Viewer
Create & Connect Viewer
Disconnect Viewer
Delete Viewer
Linear Analysis Points
Signal Hierarchy
Properties
Simulink
Communications
DSP
Computer Vision
HDL Verifier for use with Cadence Incisive
HDL Verifier for use with Mentor Graphics ModelSim
Scope
XY Graph
Viewer: Scope
File Tools View Simulation Help
Ready
Sample based
Out1

Figure 5-28. Adding a Scope block to a signal

In addition, make one important adjustment (Time display offset: -3*pi) in the Scope block to make it display the whole simulation results completely, as demonstrated in the example and shown in Figure 5-21. Now, you save the model (Ex3__Function_compute_In_Out.slx) and simulate it after entering this in the MATLAB workspace:

```
>> t=[-3*pi:pi/3000:3*pi]'; u=t;
```

After simulating the three alternative models Ex3__Function_compute.slx, Ex3_Function_Compute_Simple.slx, and Ex3_Function_Compute_In_Out.slx, the simulation results are identical. Via these examples, you have seen how easily one block can be substituted for another, how easily subsystems can be created from the existing models by associating parts of interconnected blocks, and how models can be simplified by reducing the number of blocks used in them.

Simulating a Mechanical System

Let's consider a mechanical spring-mass-damper system with Newtonian friction that is formulated by the following differential equation: $m\ddot{x}(t)+bsign(\dot{x})\dot{x}^2(t)+kx(t)=f(t)$.

Let's treat the given model of the system as continuous and discrete systems in order to demonstrate how to model and simulate such system in Simulink. Note that solving and simulating differential equations via Simulink modeling is explained more in Chapter 8. Here, we put more emphasis on model building, adjusting block parameters, and interacting Simulink with MATLAB. Moreover, we address the issues of modeling continuous and discrete systems and of creating subsystems.

Given: $m=0.52\,kg;\ b=0.00525\frac{kg}{m};\ k=165.5\frac{N}{m};\ f(t)=A\cos(\omega t);\ \omega=131\frac{rad}{\sec}$; A = 2.3 and all initial conditions are "zero". Sampling time: t_s = 0.01. The parameters of the system are m for mass, b for the damping coefficient, k for stiffness, f(t) for the input force, A for magnitude, and ω for frequency.

1. Collect all the necessary blocks from the Simulink Library by dragging and dropping them in a blank model window. Figure 5-29 shows the required blocks, taken from Simulink/ Math Operations, Continuous, Discrete, Sources, Signal Routing and Sinks.

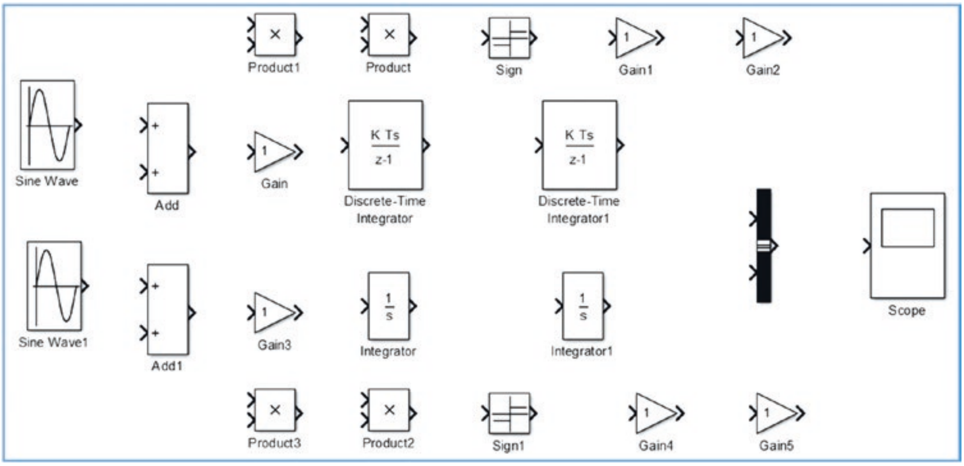


Figure 5-29. *Necessary the blocks for continuous and discrete systems*

2. Adjust the blocks in a more readable order by moving them around and rotating some of them by 180 degrees. The Product, Sign, and Gain blocks need to be rotated. You can do that by selecting a block and pressing the Ctrl+R keys from the keyboard or using the right mouse button's Rotate & Flip options. See Figure 5-30.

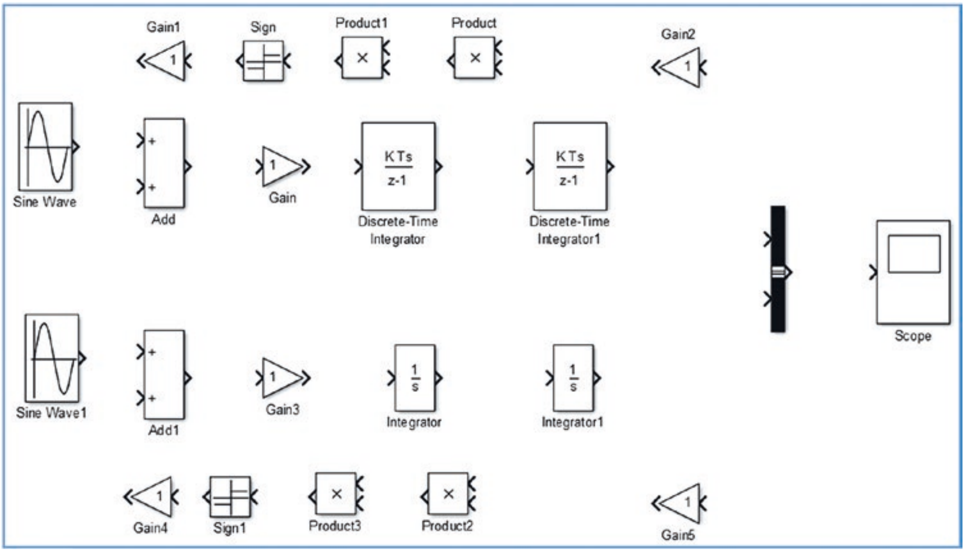
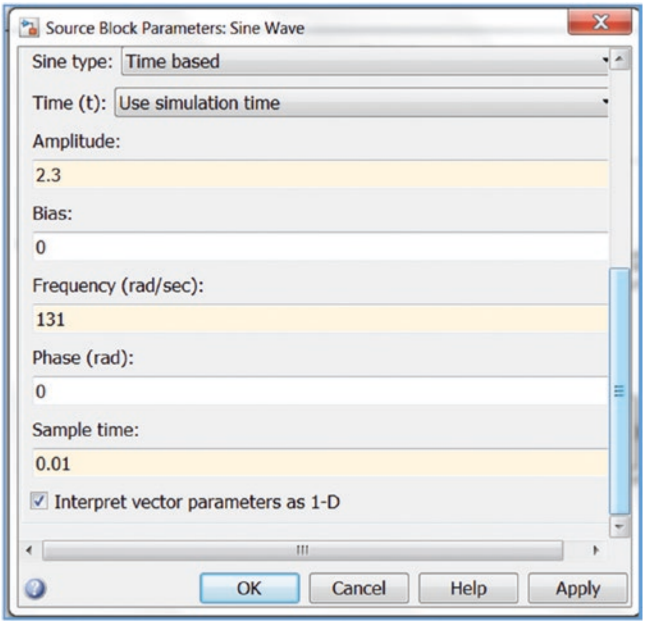


Figure 5-30. *Adjusted blocks*

3. Adjust the parameters of the blocks. First adjust amplitude, frequency, and sampling time for the Sine Wave and Sine Wave1 blocks, which are input signals (see Figure 5-31). Note that for Sine Wave1, the sampling time is set to 0 as by default it is used for continuous system modeling.



Source Block Parameters: Sine Wave

Sine type: Time based

Time (t): Use simulation time

Amplitude: 2.3

Bias: 0

Frequency (rad/sec): 131

Phase (rad): 0

Sample time: 0.01

☑ Interpret vector parameters as 1-D

***Figure 5-31.** Adjusting Sine Wave block's parameters*

4. Note how the sine wave sign in the Sine Wave block has changed from a smooth curve to a stairs curve when Sample time is set to be 0.01. Change the Gain values for Gain and Gain3to 1/m, and for Gain1 and Gain4 to b, and for Gain2 and Gain5 to k. Also, change the signs in the Add and Add1 blocks to -+-.

5. All blocks are connected and complete models are attained. In addition, two notations are added: Discrete System and Continuous System; see Figure 5-32. The complete model is saved (`Ex4_Discrete_Continuous_Sys.slx`).

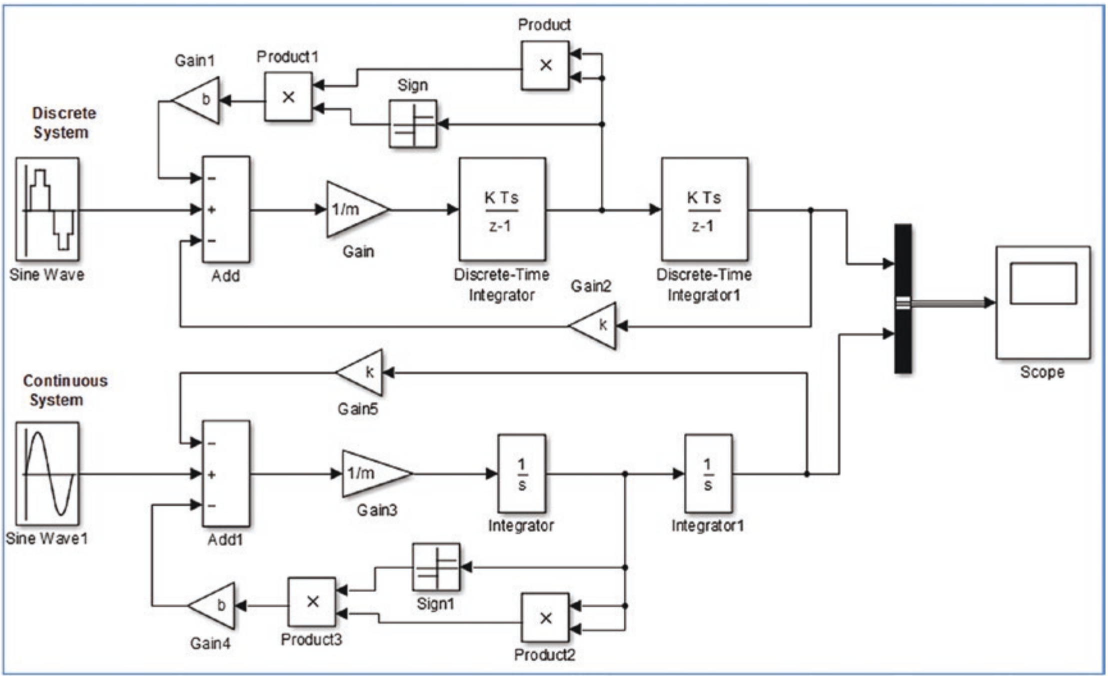


Figure 5-32. Complete models of discrete and continuous systems

6. You can execute these models by pressing the Ctrl+T keys from the keyboard or pressing the Run button.

Now the completed models (`Ex4_Discrete_Continuous_Sys.slx`) seem to be ready for simulation. However, if we execute them, error message windows will be prompted and no results will be attained. The reason for that is the values of three parameters—*m*, *k*, *b*—are not defined yet. You can define them in several different ways, one of which is to specify the values of the parameters in each block or in the MATLAB workspace. You can also specify this information in the Model properties/Callback/InitFcn, which can be accessed via File ➤ Model Properties. Enter in the Command Window: `>> m=0.52; k=155; b=.00144`.

Subsequently, click the Run button in the Simulink model window. You can change the Scope block's Style settings (via drop-down options --> Style) and obtain the results of the simulation displayed in the Scope block; see Figure 5-33.

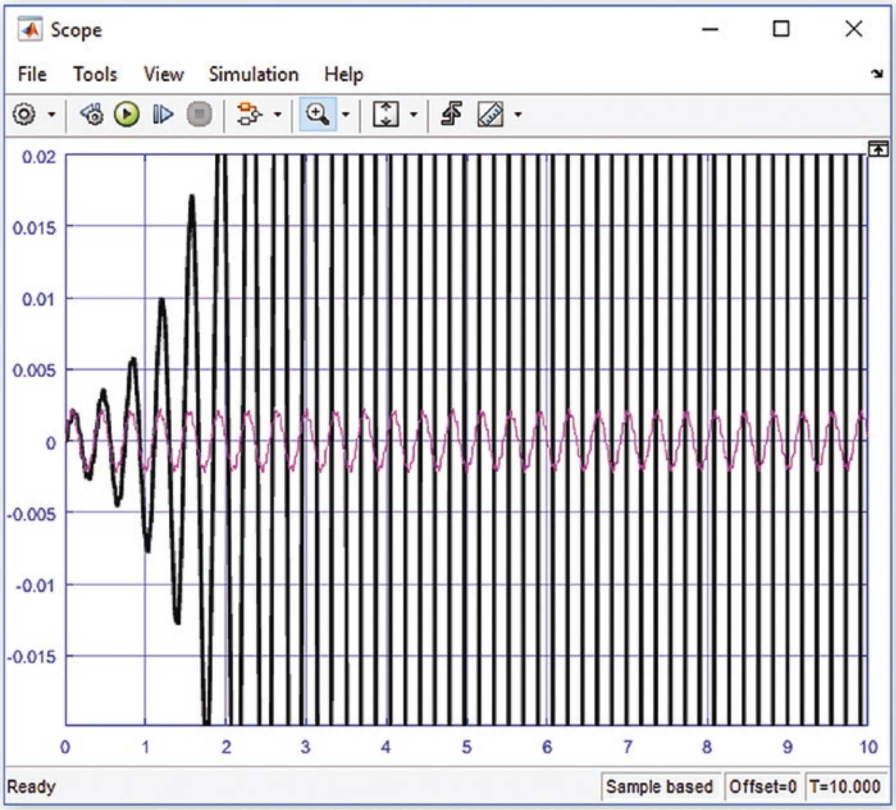
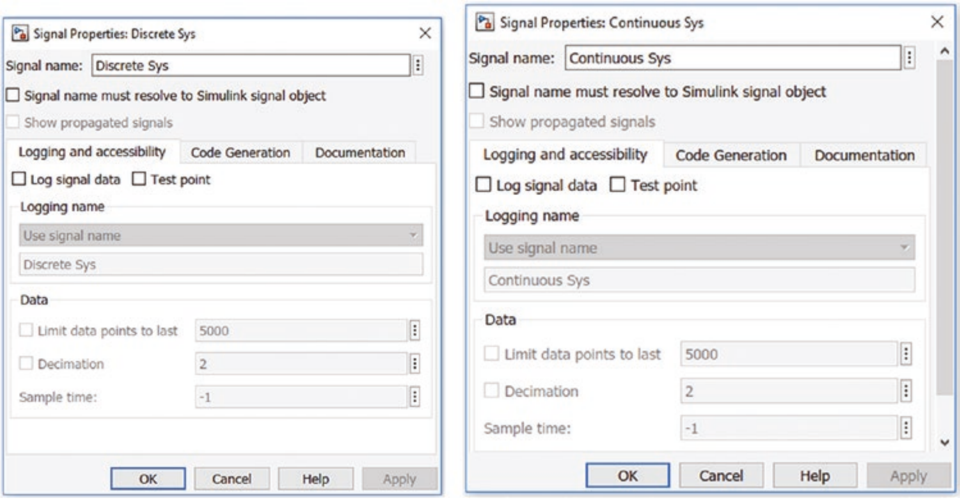


Figure 5-33. *Simulation results*

From the simulation results, you can see that one of the systems (the discrete one) is not stable and should be fixed. The problem with this discrete system modeling resides in the sampling time, which has to be adjusted. That can easily be fixed from the input signal bock; that is, the Sine Wave. In this block's sample, the time value is changed from 0.01 to 0.0001, which is 100 times smaller than initially set.

Note If the sample time is set to -1 in a discrete integrator block, that makes the sample time be inherited automatically from an input signal source.

In addition to making the plot in the Scope more readable with legends displayed for the discrete and continuous system models, let’s check the Legends option of the Scope block’s properties . Second, click the signal going from Discrete-Time Integrator1 to Bus Creator and use the right mouse button’s option to access Properties (Signal Properties). There, you specify the Signal Name to be Discrete Sys and the click OK. Similarly, change the signal name for the signal going from Integrator1 to Bus Creator and name the signal Continuous Sys. Then click OK, as shown in Figure 5-34.



Signal Properties: Discrete Sys

Signal name: Discrete Sys

□ Signal name must resolve to Simulink signal object

□ Show propagated signals

□ Log signal data □ Test point

Logging name: Use signal name

Discrete Sys

Data

□ Limit data points to last 5000

□ Decimation 2

Sample time: -1

Signal Properties: Continuous Sys

Signal name: Continuous Sys

□ Signal name must resolve to Simulink signal object

□ Show propagated signals

□ Log signal data □ Test point

Logging name: Use signal name

Continuous Sys

Data

□ Limit data points to last 5000

□ Decimation 2

Sample time: -1

Figure 5-34. *Signal name change*

After these three changes, the whole model runs and the next result is obtained, as shown in Figure 5-35. Note that it is a zoomed-in view along the horizontal axis.

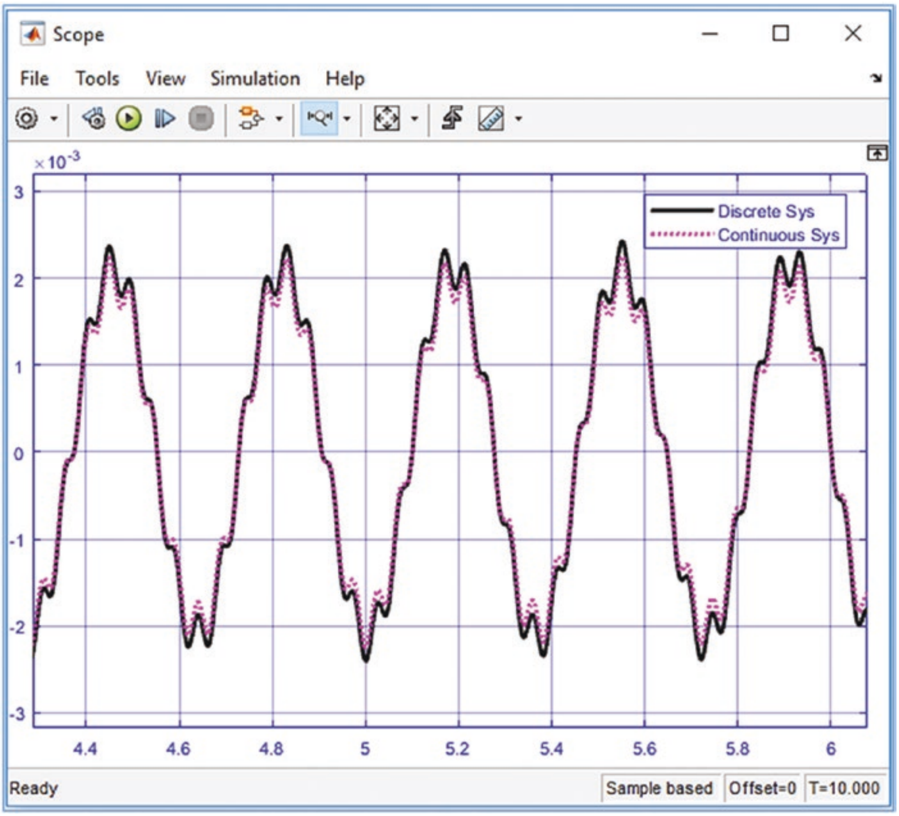


***Figure 5-35.** Simulation results are displayed in Scope with its adjusted properties*

You have completed and verified the system models that can be simplified by using a subsystem option, as shown in the previous example (see Figure 5-25; `Ex3_Function_Compute_Simple.slx`). Any system containing more than two blocks can be simplified or rather substituted by employing a subsystem block. There are several ways to create subsystems from system models. The easiest way is to select model blocks meant to be under one subsystem and then use the right mouse button options of Create Subsystem from Selection or press Ctrl+G from the keyboard. You create the two subsystems (Subsystem and Subsystem1) from the discrete and continuous system models (see Figure 5-36).

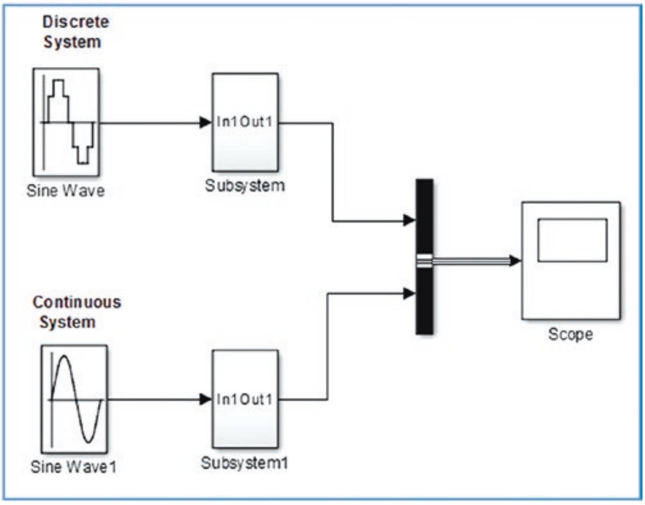


***Figure 5-36.** The model containing two subsystem models*

A model under any subsystem can be accessed by double-clicking on it. The subsystem representing the discrete system contains the model shown in Figure 5-37.

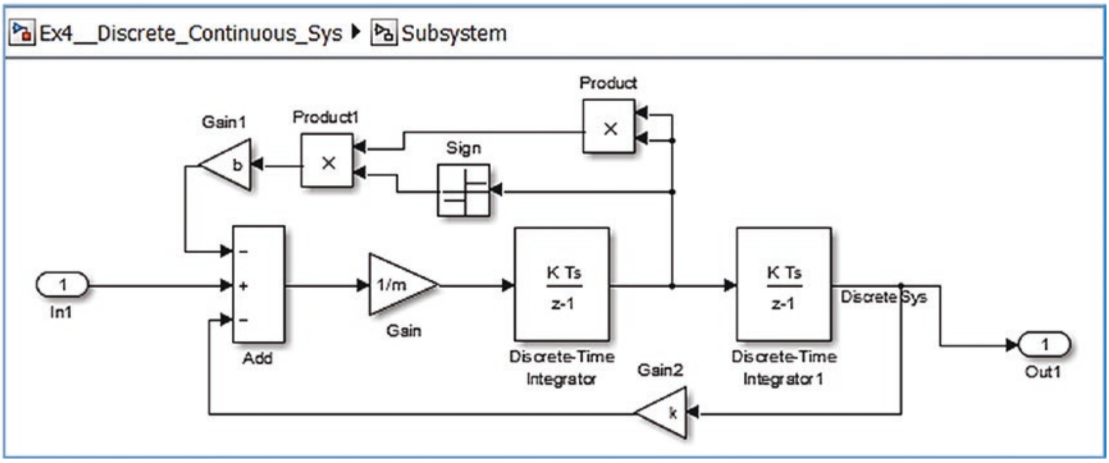


***Figure 5-37.** Subsystem is composed of this model*

Also, it is possible to reverse the process of subsystems by clicking on a subsystem block and using the right mouse button option of Subsystem & Model Reference ➤ Expand Subsystem. You can also do this by pressing the Ctrl+Shift+G buttons after clicking on a subsystem block.

In this exercise, you learned how to build discrete and continuous systems and how Simulink handles such composition of systems, but you have not made any adjustments to solver parameters. Accuracy and efficiency of simulation processes can be improved by adjusting solver type (variable step solver selected by default or fixed step solver) and parameter settings (solver, error tolerances, solver algorithm, step size if fixed step solver chosen, zero-crossings, and so forth). More discussions and explanations of these salient issues are highlighted in Chapter 8 on differential equations.

Working with a Second Order Differential Equation

We'll build a Simulink model of the given system expressed by a second order differential equation: $A\ddot{q}(t)+B\ddot{q}(t)+Cq(t)=F(t)$, where *F*(*t*) is applied force (input signal to the system) associated with the MATLAB's function file. *F*(*t*) is a rectangular pulse approximated by the Fourier series: $F(t)=\frac{Amp}{\pi}\sum_{n=1}^{N}(1-\cos(n\pi))\sin(n\pi t)$, which is implemented in MATLAB via the `function_pulse.m` function file:

```
function F=function_pulse(t, Amp, n)
% HELP. Two input arguments, viz. t, Amp, and n are needed for
% simulation, where t is time vector, Amp is amplitude of a pulse and n %
number of approximation terms in Fourier series.

F(1,:)=(Amp/pi)*(1-cos(pi))*sin(pi*t);
for ii=2:n
    F=F+(Amp/(ii*pi))*(1-cos(ii*pi))*sin(ii*pi*t);
end
return
```

Let's build a Simulink model associated with the function file `function_pulse.m`. Moreover, you'll employ Simulink blocks—Repeating Sequence and Signal Generator—to generate a rectangular pulse signal. You'll explore the options with MATLAB associated function file and Simulink blocks for input signal generation. In addition, you'll explore a few key Simulink modeling tools and aides, such as model explorer, model advisor, code generation in C/C++, report generation, and so forth. For numerical simulations, the following values are taken: *A* = 2; *B* = 4; *C* = 200; *Amp* = 10; *n* = 25.

The complete model of this exercise is shown in Figure 5-38. It associates the given function file called `function_pulse.m` via the Interpreted MATLAB Fcn block. The input signal F(t) rectangular pulses are generated via two ways—one Interpreted MATLAB Fcn with three input variables (Amp, n, t) and a signal generator with two inputs (amp and frequency). You can compare the simulation results from the two input signal sources, i.e. the MATLAB associated input signal generation (`pulse_function.m` embedded via Interpreted MATLAB Fcn) vs. Simulink Library block (the signal generator). Moreover, you'll see how to use Simulink's Model properties to enter the model parameters, such as Amp, n, A, B, C, and how to use Multiport Switch block.

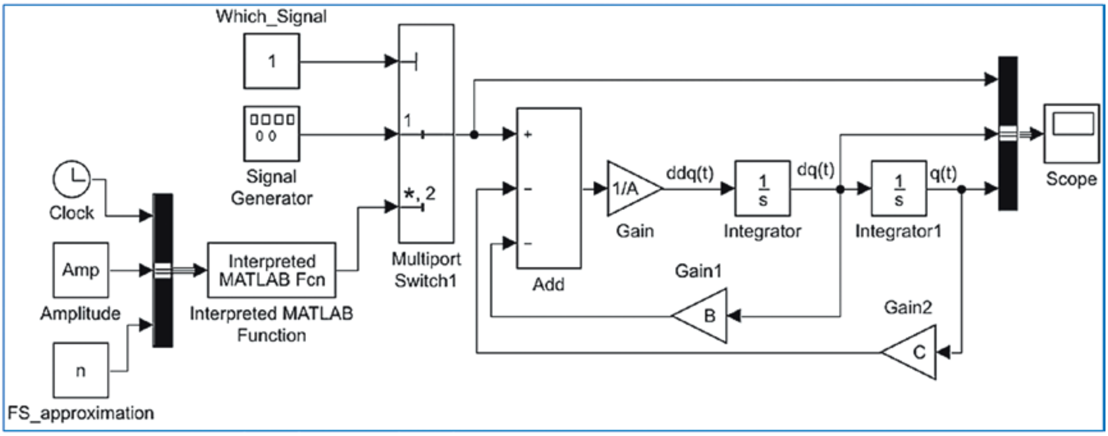


Figure 5-38. *Complete model of the second order ODE:* $A\ddot{q}(t)+B\ddot{q}(t)+Cq(t)=F(t)$

The necessary blocks to model the given exercise are Add, Gain, Integrator, Bus Creator, and Scope, which you can drag and drop to a blank model window. The blocks are connected, as shown in Figure 5-38. Note that the three signals connected with Bus Creator and Scope blocks are called Input, dq(t), and q(t), and they represent input signal, pulse, velocity, and displacement. This is displayed via legends in the Scope block plot.

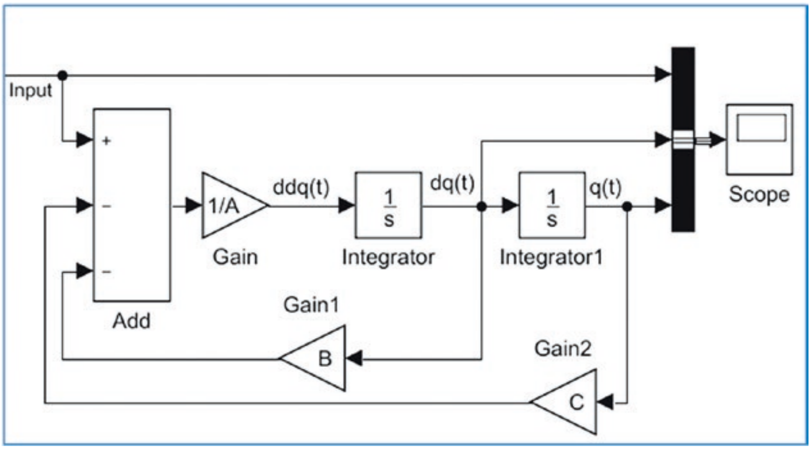


***Figure 5-39.** Simulink model of the given second order differential equation without Input force signal generators*

The function file (called `function_pulse.m`) is associated via Interpreted MATLAB Fcn block with the Simulink model (see Figure 5-38). Interpreted MATLAB Fcn is modified to call the MATLAB function: `function_pulse(u(1), u(2), u(3))`, where `u(1)` calls the time signal, `u(2)` calls the amplitude Amp, and `u(3)` calls `n` number of the Fourier series approximation. Thus, the Interpreted MATLAB Fcn block requires three input signals simultaneously. That can be done via the Bus Creator block, as shown in Figure 5-40. Note that you can change block names by clicking on the name of each block and typing the new name. Note that block tags are not considered during the model simulation and thus, they have only an informative character for the user/programmer.

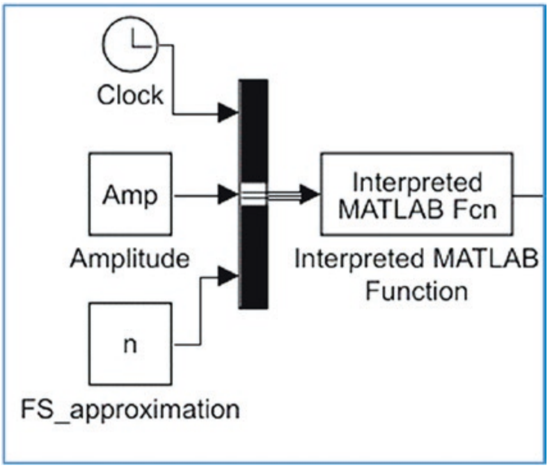
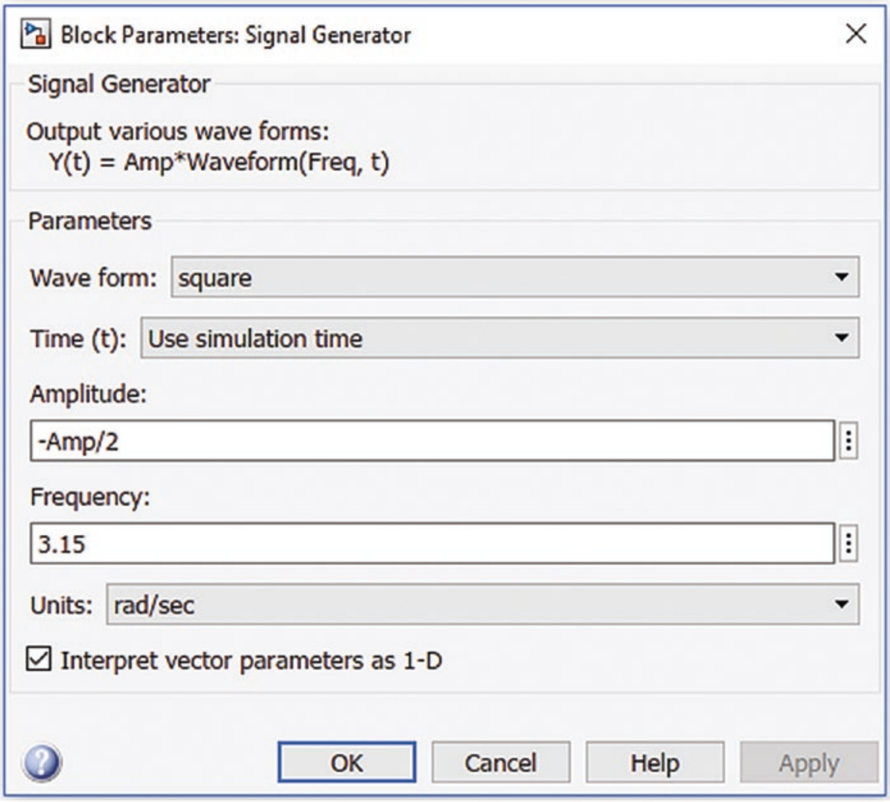


***Figure 5-40.** MATLAB Fcn block with three inputs*

Subsequently, one Simulink block, called Signal Generator, is added to the model and its parameters to generate pulses (Amplitude = -Amp/2 and Frequency = 3.15 [rad/sec]). It's then adjusted according to the given pulse parameters, as shown in Figure 5-41.


Block Parameters: Signal Generator

Signal Generator

Output various wave forms:
Y(t) = Amp*Waveform(Freq, t)

Parameters

Wave form: square

Time (t): Use simulation time

Amplitude: -Amp/2

Frequency: 3.15

Units: rad/sec

☑ Interpret vector parameters as 1-D

Figure 5-41. *Adjusted parameters of the Signal Generator block*

To connect two input signal sources (Signal Generator and MATLAB Interpreted Fcn), add the Multiport Switch block and connect it to another Constant block to specify a source signal block for selection. Finally, name the three signals going to the Scope block via Bus selector block. Moreover, adjust the Scope block's Style options to make the output signals readable. A completed model is shown in Figure 5-42.

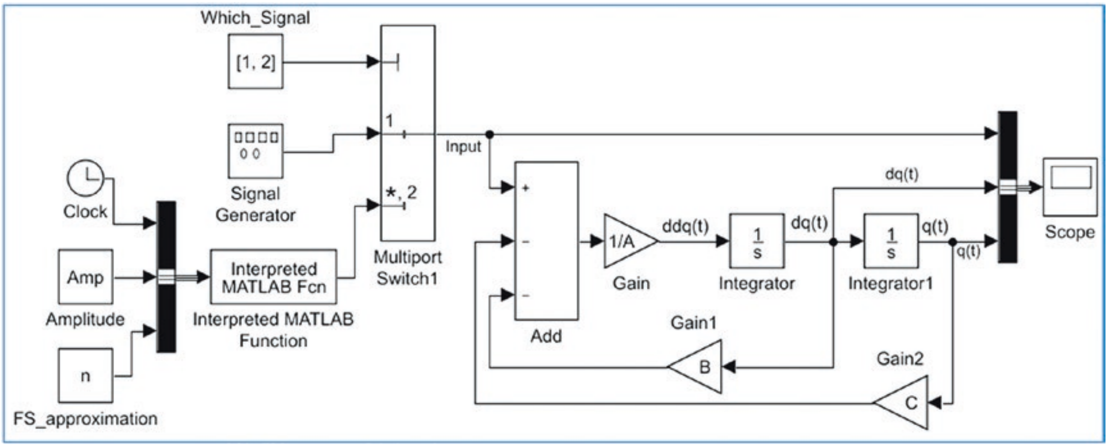


Figure 5-42. *Complete Simulink model called Ex5_Function_PULSE.slx*

Before you start the simulation, you have to specify the values for A, B, C, Amp, and n. You can do that via the Callbacks option, from File ➤ Model Properties ➤ Model Properties ➤ Callbacks (Model callbacks) ➤ InitFcn. In the Model initialization function window (or alternatively, in the Command Window), type `A=2; B=4; C=200; n=25; Amp=10` and click OK. When you execute the model, both input signals are taken in the order of first and second with respect to the Constant block (called `Which_Signal`) values 1, 2. They correspond to Input 1 – Signal Generator and MATLAB Interpreted Fcn. The simulation results in Figure 5-43 show that the two Input signals and two pairs of Output signals (dq(t), q(t), which represent velocity $\dot{q}$ and displacement q, respectively, are well converged. Note that Input: 1 is Signal Generator and Input: 2 is MATLAB Interpreted Fcn.

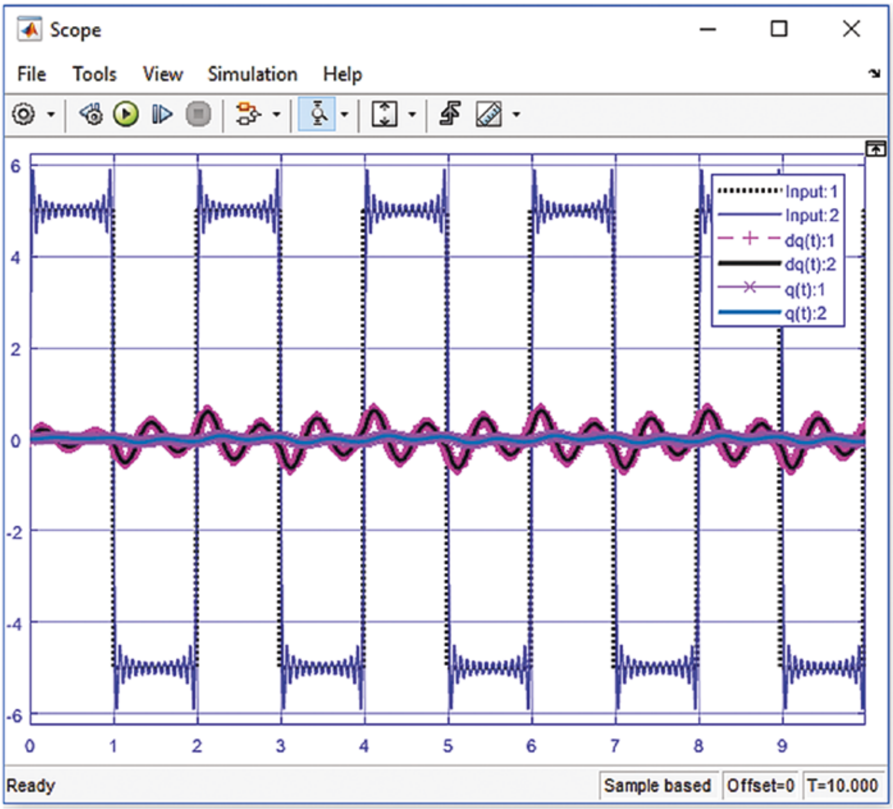


Figure 5-43. *Simulation results: a) from Signal Generator, b) from Repeating Sequence, and c) from Interpreted MATLAB Fcn*

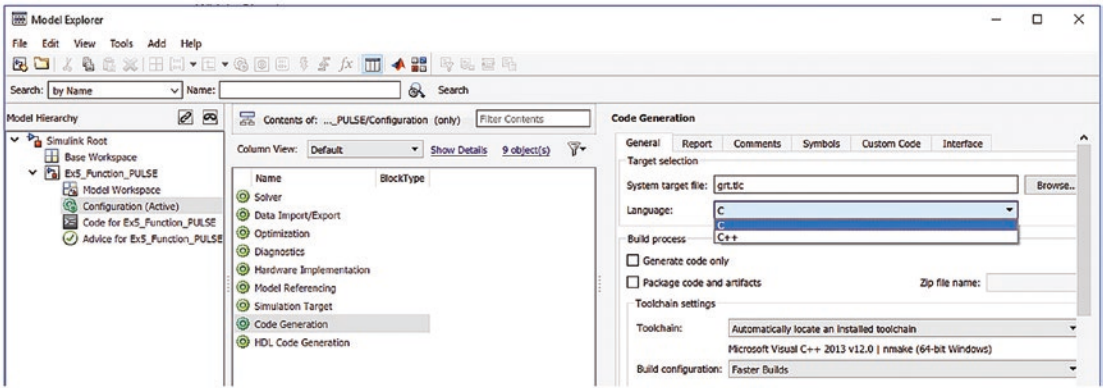
It is clear from the simulation results displayed in Figure 5-43, that all three blocks generating pulse input signals have resulted in approximately the same excitation in the system. Simulink model blocks can be associated with MATLAB files if they are correctly modeled and adjusted. It should be noted that the second input signal generation approach via Interpreted MATLAB Fcn is less accurate. It approximates the Fourier Series from the M-file `function_pulse.m`. Moreover, it is slower since it calls an external M-file to generate the signal.

Simulink Model Analysis and Diagnostics

Simulink Model Analysis and Diagnostics tools provide good assistance to programmers for improving their models in terms of simulation speed, efficiency, and elimination of inaccurate and inefficient simulations. Therefore, it is recommended to perform analysis and diagnostics for efficiency and adequacy of employed blocks and combinations, chosen solver type, and many other options. All of these options can be explored via the Model Explorer and the Model Advisor tools. Via the Model Explorer tools, you can generate C/C++code of a Simulink model, obtain a profile report of a model, start a model advisor, reset the configuration parameters of solver, input/output, optimization, and code generation, and much more. Let's look at some of the tools within the Model Explorer, considering the previous example.

Code Generation

Code generation (see Figure 5-44) can be accessed via the Model Explorer or Model Configuration Parameters buttons.



Search: by Name

Name:

Contents of: ..._PULSE/Configuration (only)

Filter Contents

Column View: Default

System target file: grt.tlc

Language: C

□ Generate code only

□ Package code and artifacts

Zip file name:

Toolchain: Automatically locate an installed toolchain

Microsoft Visual C++ 2013 v12.0 | nmake (64-bit Windows)

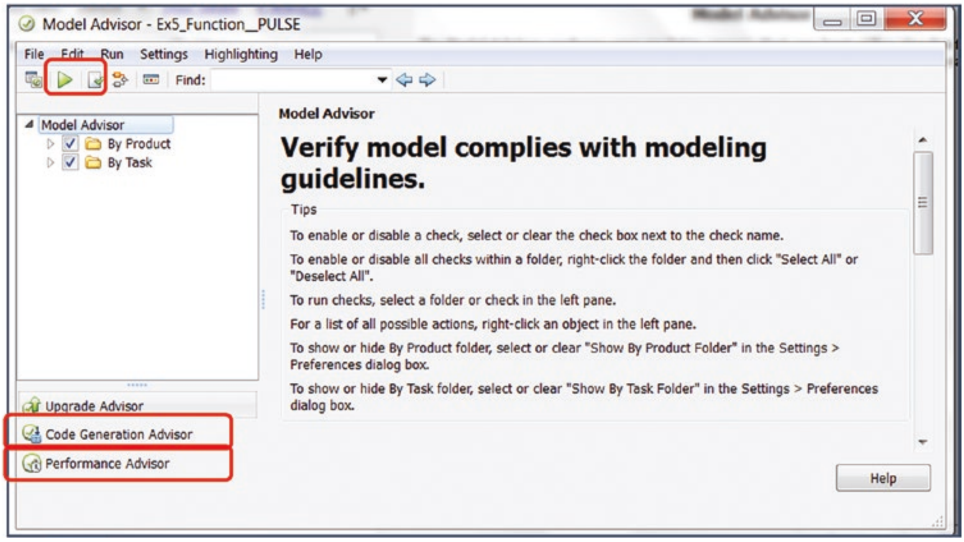
Build configuration: Faster Builds

Figure 5-44. *Model Explorer tools*

After clicking Generate Code Only and Package Code and Artifacts, click the Generate Code button. Subsequently, the C code (C is the chosen language) will be generated. Note that there are some constraints in code generation; for instance, a chosen solver has to be a fixed step and not all blocks used are compatible with code generation in C/C++. If these or other such requirements are not satisfied, the C/C++ code cannot be generated. Also, the code generation process depends on the installed compiler type and version.

Model Advisor

Model Advisor ✓ tools (see Figure 5-45) can be helpful in identifying where problems have occurred within a model and where optimization is required. It identifies problems with code generation and model performance by product, by task types, or both.



Model Advisor

Verify model complies with modeling guidelines.

Tips

To enable or disable a check, select or clear the check box next to the check name.

To enable or disable all checks within a folder, right-click the folder and then click "Select All" or "Deselect All".

To run checks, select a folder or check in the left pane.

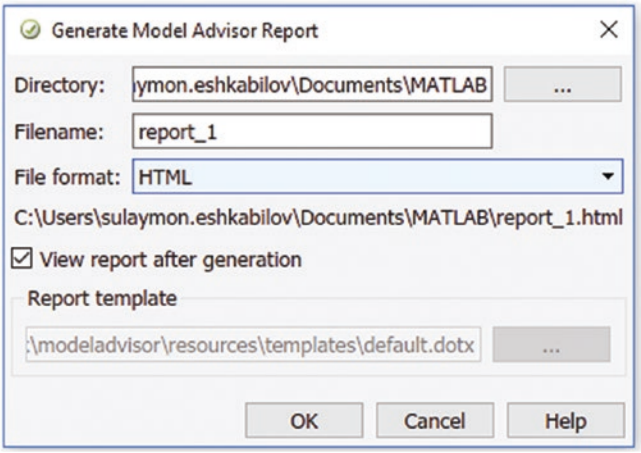
For a list of all possible actions, right-click an object in the left pane.

To show or hide By Product folder, select or clear "Show By Product Folder" in the Settings > Preferences dialog box.

To show or hide By Task folder, select or clear "Show By Task Folder" in the Settings > Preferences dialog box.

Figure 5-45. *Model Advisor tools*

You first choose which process to get help/advice from the Model Advisor and then click the Run ▷ button. In this example, we chose the Model Advisor with By Product and By Task. Once the Model Advisor is launched, all diagnostic checks of the model (`Ex5_Function_PULSE.slx`) are run and the report of all passed, failed, warning, and not run points is prepared. You can view the report by clicking the Generate Report button in the Model Advisor window. The Generate Report button prompts the Generate Model Advisor Report window, from which you can select the directory (where to save the generated report), filename, file format (HTML by default, PDF, or Word), and tickmark option to view the report after it's generated . See Figure 5-46. We chose HTML, which is a default report format of the Model Advisor.



Generate Model Advisor Report

Directory: ymon.eshkabilov\Documents\MATLAB

Filename: report_1

File format: HTML

C:\Users\sulaymon.eshkabilov\Documents\MATLAB\report_1.html

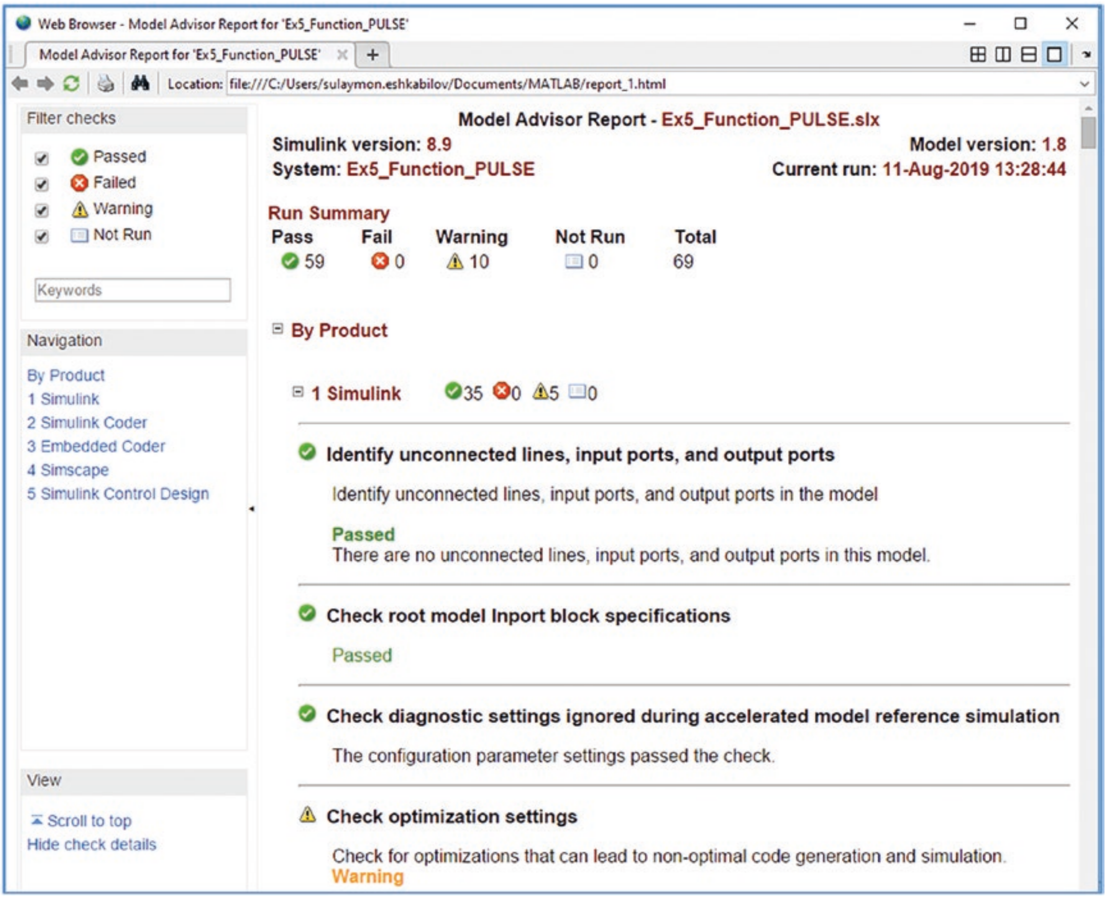
☑ View report after generation

Report template

:\modeladvisor\resources\templates\default.dotx

Figure 5-46. *The Generate Model Advisor Report window*

Moreover, there are warnings concerning double precision operations used by the blocks of the model. The blocks (Interpreted MATLAB Function) are not supported by code generation. In addition, the Clock, Integrator, Integrator1, Signal Generator, and Interpreted MATLAB Function blocks are not recommended for C/C++ production deployment.


Web Browser - Model Advisor Report for 'Ex5_Function_PULSE'
Model Advisor Report for 'Ex5_Function_PULSE'
Location: file:///C:/Users/sulaymon.eshkabilov/Documents/MATLAB/report_1.html
Filter checks
☑ Passed
☑ Failed
☑ Warning
☑ Not Run
Keywords
Navigation
By Product
1 Simulink
2 Simulink Coder
3 Embedded Coder
4 Simscape
5 Simulink Control Design
View
Scroll to top
Hide check details

Model Advisor Report - Ex5_Function_PULSE.slx

Simulink version: 8.9 Model version: 1.8

System: Ex5_Function_PULSE Current run: 11-Aug-2019 13:28:44

Run Summary

Pass	Fail	Warning	Not Run	Total
✓ 59	✗ 0	⚠ 10	□ 0	69

⊟ By Product

⊟ 1 Simulink ✓35 ✗0 ⚠5 □0

✓ Identify unconnected lines, input ports, and output ports

Identify unconnected lines, input ports, and output ports in the model

Passed
There are no unconnected lines, input ports, and output ports in this model.

✓ Check root model Inport block specifications

Passed

✓ Check diagnostic settings ignored during accelerated model reference simulation

The configuration parameter settings passed the check.

⚠ Check optimization settings

Check for optimizations that can lead to non-optimal code generation and simulation.
Warning

Figure 5-47. *Model Advisor Report*

Figure 5-47 shows part of the Model Advisor Report for `Ex5_Function_PULSE.slx`. The report is created in HTML format and shows 59 Pass, 0 Fail, 10 Warning, 0 Not Run, and in total of 69 Run. By scrolling down the report, you can see where the model passed and where it had some warning issues, such as optimization settings—it's recommended to set the parameter of "Remove Code from Floating-Point to Integer Conversions That Warms Out-Of-Range Values (`EfficientFloat2IntCast`)" to on. Another recommendation is to set the parameter "Inline invariant signals (InlineInvariantSignals)" to on. Furthermore, another warning is Check Data Store Memory blocks for multitasking, strong typing, and shadowing issues. Duplicate data store names checking is not set to error. Duplicate usage of data store names can lead to unintended shadowing of data stores of higher model scope. For this reason, consider changing the duplicate data store names setting to error.

Another interesting warning was linked to bus signals. The warning says: Check bus signals treated as vectors. Bus signal treated as vector by the Simulink software. Identify bus signals in the model that are treated as vectors by the Simulink software.

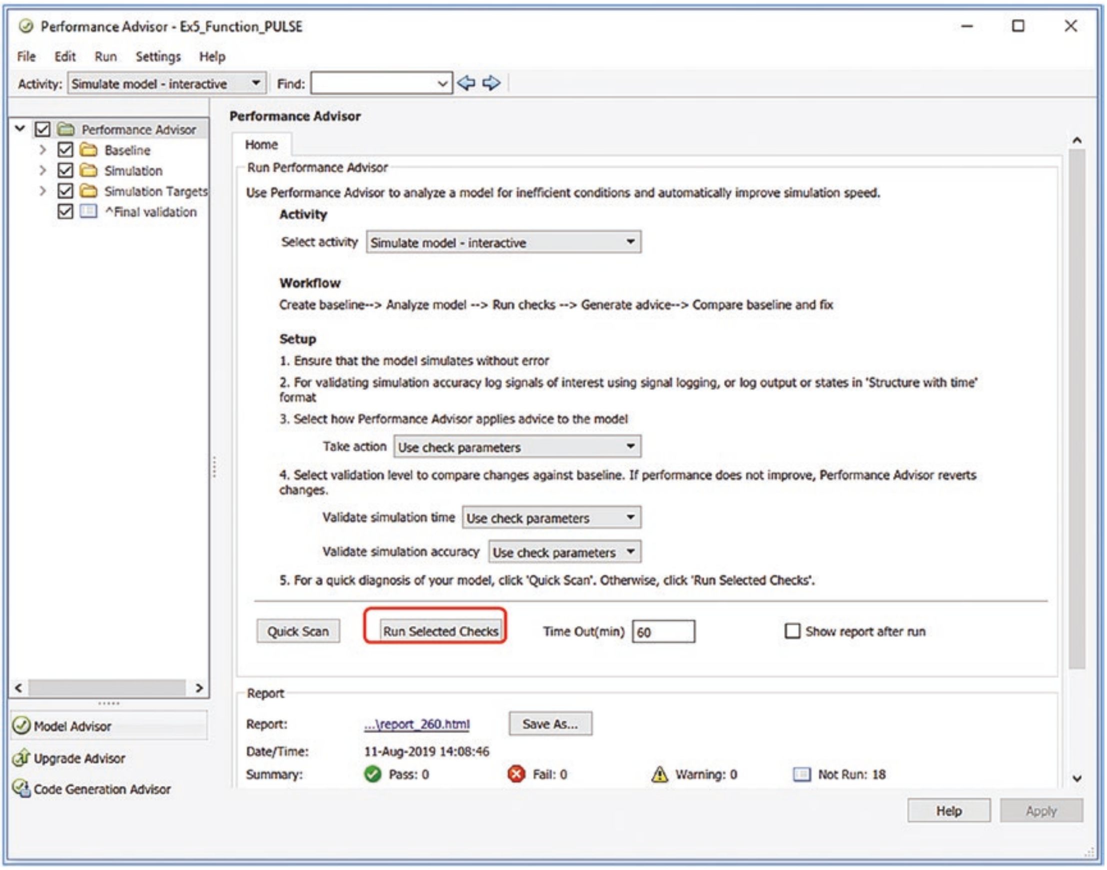
Bus signal feeding input port 1 of the block: `Ex5_Function_PULSE`/Interpreted MATLAB Function. Bus signal feeding input port 1 of the block in `Ex5_Function_PULSE/Scope`.

Recommended Action: The model contains bus signals that the Simulink software implicitly converts to vectors. However, the model is not configured to explicitly convert these signals to vectors. To fix this issue, insert Bus To Vector blocks at the imports of the blocks listed above.

You can do this automatically, by either pressing the modify button below or running the `Simulink.BlockDiagram.addBusToVector` function. You can do this manually, using the Simulink --> Signal Attributes library.

By studying the Model Advisor's reports, you can improve your model by removing bugs, simulation bottlenecks, and unwanted warnings, and substituting some of the inefficient blocks in the model.

In addition to the Model Advisor, you can also employ Optimization tools under Model Explorer or Configuration parameters to optimize parameters and blocks in our model. In addition, to locate bugs or bottlenecks, we can use debugging tools. They can be accessed via the menu bar: Simulation ➤ Debug ➤ Debug Model. Another way to learn about the model's performance is from the menu bar: Analysis ➤ Performance Tools ➤ Show Profile Report (select) and then ➤ Performance Tools ➤ Performance Advisor. The options are displayed in Figure 5-48.



Activity: Simulate model - interactive

Performance Advisor

Run Performance Advisor

Use Performance Advisor to analyze a model for inefficient conditions and automatically improve simulation speed.

Activity

Select activity: Simulate model - interactive

Workflow

Create baseline--> Analyze model --> Run checks --> Generate advice--> Compare baseline and fix

Setup

1. Ensure that the model simulates without error
2. For validating simulation accuracy log signals of interest using signal logging, or log output or states in 'Structure with time' format
3. Select how Performance Advisor applies advice to the model

   Take action: Use check parameters

4. Select validation level to compare changes against baseline. If performance does not improve, Performance Advisor reverts changes.

   Validate simulation time: Use check parameters

   Validate simulation accuracy: Use check parameters

5. For a quick diagnosis of your model, click 'Quick Scan'. Otherwise, click 'Run Selected Checks'.

Time Out(min): 60 □ Show report after run

Report

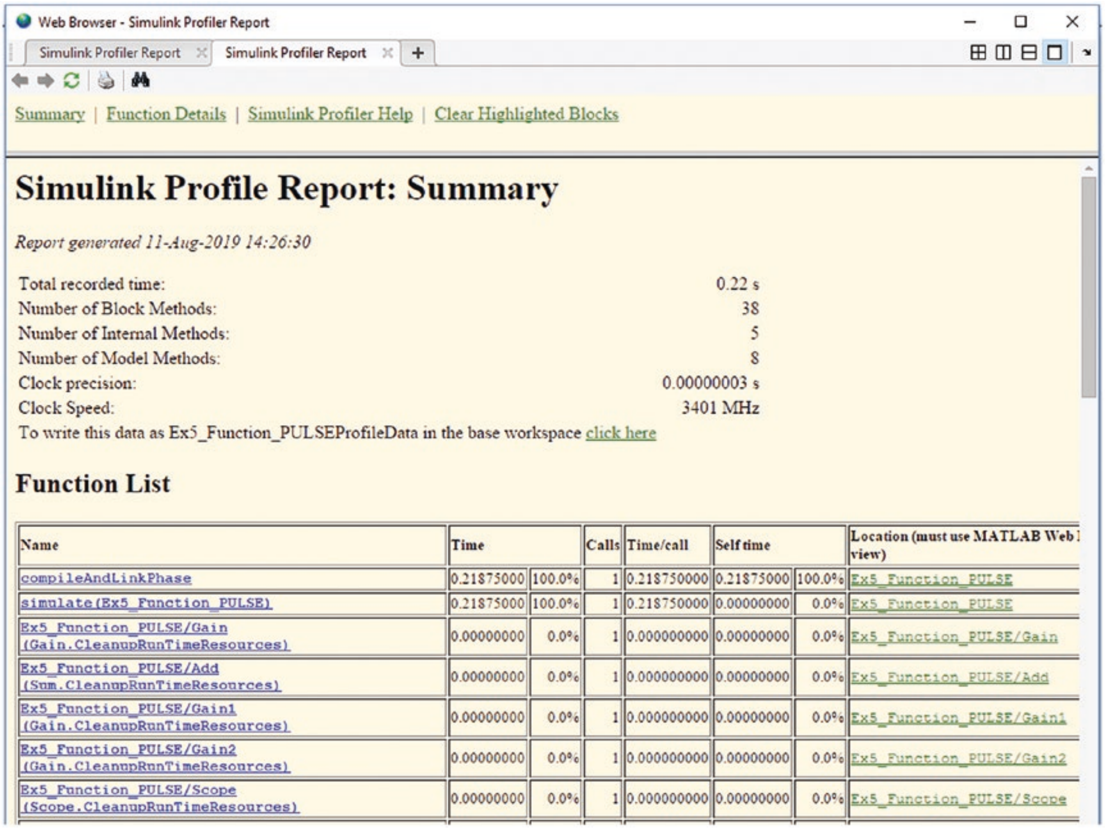
Report: ...\report_260.html

Date/Time: 11-Aug-2019 14:08:46

Summary: ✓ Pass: 0 ☒ Fail: 0 ⚠ Warning: 0 □ Not Run: 18

Figure 5-48. *Performance Advisor options*

After clicking Run Selected Checks (see Figure 5-48), the Simulink Profile Report is displayed. It's composed of the Summary and Simulink Performance Advisor Report, and it displays a complete picture of the model and its simulation processes.


Web Browser - Simulink Profiler Report
Simulink Profiler Report
Simulink Profiler Report
Summary | Function Details | Simulink Profiler Help | Clear Highlighted Blocks

Simulink Profile Report: Summary

Report generated 11-Aug-2019 14:26:30

Total recorded time:	0.22 s
Number of Block Methods:	38
Number of Internal Methods:	5
Number of Model Methods:	8
Clock precision:	0.00000003 s
Clock Speed:	3401 MHz

To write this data as Ex5_Function_PULSEProfileData in the base workspace click here

Function List

Name	Time		Calls	Time/call	Self time		Location (must use MATLAB Web I view)
compileAndLinkPhase	0.21875000	100.0%	1	0.218750000	0.21875000	100.0%	Ex5_Function_PULSE
simulate(Ex5_Function_PULSE)	0.21875000	100.0%	1	0.218750000	0.00000000	0.0%	Ex5_Function_PULSE
Ex5_Function_PULSE/Gain (Gain.CleanupRunTimeResources)	0.00000000	0.0%	1	0.000000000	0.00000000	0.0%	Ex5_Function_PULSE/Gain
Ex5_Function_PULSE/Add (Sum.CleanupRunTimeResources)	0.00000000	0.0%	1	0.000000000	0.00000000	0.0%	Ex5_Function_PULSE/Add
Ex5_Function_PULSE/Gain1 (Gain.CleanupRunTimeResources)	0.00000000	0.0%	1	0.000000000	0.00000000	0.0%	Ex5_Function_PULSE/Gain1
Ex5_Function_PULSE/Gain2 (Gain.CleanupRunTimeResources)	0.00000000	0.0%	1	0.000000000	0.00000000	0.0%	Ex5_Function_PULSE/Gain2
Ex5_Function_PULSE/Scope (Scope.CleanupRunTimeResources)	0.00000000	0.0%	1	0.000000000	0.00000000	0.0%	Ex5_Function_PULSE/Scope

Figure 5-49. *Simulink Model Profile Report Summary*

Thus, it is recommended that you run Model Advisor and Performance Advisor options in order to obtain the many help hints to improve a your model's performance. Also, the profile report generator can be recalled and executed using commands in order to locate inefficient operations and blocks of the model.

```
>> profile on; sim('Ex5_Function__PULSE.mdl'); profile viewer
```

The profile report generator works well with all M-files and Simulink models and provides comprehensive reports including bottlenecks within a code/script/model in terms of computation and execution time spent on each command and operation.

Summary

In this chapter, we covered most of the essential graphical programming tools and some common blocks in the Simulink package, including signal sources, matrix operations, integration, visualization, signal routing, and C code generation. Moreover, the chapter highlighted and demonstrated, via numerical simulation examples, a few salient points on how to adjust parameters, use solver tools, set error tolerances, and improve the performance of Simulink models. You learned how to create subsystems from existing models and how to associate Simulink models with MATLAB scripts and function files. You worked with the Simulink Model Analysis and Diagnostics tools and learned how to obtain Model Advisor and Performance Reports.

Exercises for Self-Testing

Exercise 1

Write an M-file using a `while, end` loop control statement to compute values of the cosine function $g(\theta) = \cos(\theta)$ for $\theta = 0\ldots\pi$ with 2,000 incremental steps. Stop computation when the value of the function $g(\theta) \approx 0.99999$. Also, display after how many steps the computation process is halted and plot your simulation results. Also display the end value of 0.99999 in the same plot.

Exercise 2

Compute an area of a circle, square, and rectangle with respect to these user entries:

W = input(‘Width of a rectangle: ’)

L = input(‘Length of a rectangle: ’)

R = input(‘Radius of a circle: ’)

S = input(‘Side length of a square: ’)

There are several scenarios to consider in your script: (1) if a user enters two dimensions for width (W) and length (L) of a rectangle, your code has to compute the area of a rectangle and display it with comments; (2) if a user enters two dimensions, such as the radius (R) and side (S) of a square, it has to compute the area of a circle and square, respectively, and display it with comments; (3) if a user enters all the required entries, it has to compute three areas and display it with adequate comments; (4) if a user doesn't enter all the entries or misses any of the required entries (W, L, R, S), your code has to display `"You need to ENTER all dimensions!"` The `input()`, `isempty()`, `exist()`, `length()`, `numel()`, `size()`, and `fprintf()` commands can be employed.

Exercise 3

Write a script to display student grades based on their earned points. Student grades have to be defined according to the following scales:

F → 0…65

D → 66 … 70

C → 71 … 81

B → 82 … 87

A → 88 … 100

Use `input()`, `disp()`, `fprintf()` and `if, elseif, elseif, elseif, else ... end`.

Test your script with these example points: 70, 82, 35, 90, 99, 56, 81, 89, 66, 87, 88, 83, 71, 69, and 55.

Exercise 4

These exercises cover `while, end` and `for, end` loop control operators:

1. Compute the series: $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$ using `while, end` and `for, end` loop control statements. Take $n = 101$ and plot error values over the iterations.

2. Compute the sum of the series: $1^2, 2^2, 3^2, \ldots n^2$. Take $n = 500$ and compute the total sum $\sum_{n=1}^{500} n^2$ using `while, end` and `for, end` loop control statements. Plot the sum as a function of `n`.

3. Compute the sum of this series: 1 * 2 + 2 * 3 + 3 * 4 + 4 * 5 + ... This can be r-written as a sum $\sum_{k=1}^{N} \mathrm{k}(\mathrm{k}+1)$. To compute the total sum, use `while, end` and `for, end` loop control statements. Plot the sum of as a function of `k`.

4. Compute this series (discovered by Euler in 1735): $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n^2}$ using `while, end` and `for, end` loop control statements. Take $n = 1\ldots99$ and plot the error values over iterations. Run exhausted computation and halt the computation process when the error is smaller than 10^{-13}.

5. Compute the total sum of this series of odd numbers: 1, 3, 5, 7, ... They can be written in the form of this sequence: $2n + 1$. Take $N = 111111$ and compute the total sum $\sum_{n=0}^{N} (2\mathrm{n}+1)$ using `while, end` and `for, end` loop control statements. Halt the iteration process when the sum is larger than 30869136. Display the final number as an integer. Plot the summing iteration process.

Exercise 5

Write a script file with the conditional statements `if`, `elseif`, `elseif`, `else`, `end` and `function handle (@)` to compute the values of this function:

$$g(a,b,t) = \begin{cases} e^{-\mathrm{a}t} + \mathrm{b}t & t \le -13 \\ \cos(\mathrm{a}t) - \sin(\mathrm{b}t) & -3 \ge t \ge -13 \\ \mathrm{a}t + e^{bt} & -3 \le t \le 0 \\ 1 & t > 0 \end{cases}$$

$$a = 3,\ b = 2$$

Exercise 6

These exercises cover `while, end` and `for, end` loop control operators:

1. Compute this series: $f(x)= \sum_{n=1}^{\infty}\frac{b_n sin\left(\left(2n-1\right)\pi x\right)}{2L}$ using `while ... end` and `for ... end` loop control statements. Take $n = 11$, $b_n = [-5...5]$, $x = [0...L]$ and $L = 10$. For x take a step-size of $\Delta x=\frac{L}{100}$. Plot all the computation results.
2. Compute $f(t) = \cos(20t) - \sin(10t)$ for $t \in [-\pi, \ \pi]$ with $\Delta t = \pi/50$ using `while ... end` and `for ... end` loop.

Exercise 7

These exercises cover `while ... end` and `for ... end` loop operators:

1. Compute the total sum of this series: $\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}...=\sum_{m=3}^{55}\frac{m}{m+1}$. Compute the total sum using `while, end` and `for, end` loop control statements. Plot the sum as a function of `m`.
2. Compute the total sum of this series: $10+30+90+270+810+2430+...=\sum_{m=0}^{65}\left(10*3^{\mathrm{k}}\right)$. Compute the total sum using `while, end` and `for, end` loop control statements, and halt the computation process when the sum is larger than 18446744073709551615. Display the final sum as an integer. Plot the sum as a function of `k`.
3. Compute the following sequence 4, 1, 0.5, 0.25, ... until it is smaller than 0.000005 by using `while, end` and `for, end` loop control statements. Note that the sequence can be formulated as $4 * (0.5)^{n-1}$. Find the `n` that makes the sequence smaller than 0.000005.
4. Compute the total sum of this series: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+...$ until it is equal to or larger than 0.999 999 999. Note that the sum of this series can be expressed with $\sum_{\mathrm{k}=0}^{\mathrm{n}}\frac{1}{2}\left(\frac{1}{2}\right)^{\mathrm{k}}$.

5. Compute the total sum of $\sum_{n=0}^{m}\frac{(-1)^{n} x^{2n+1}}{(2n+1)!}$ for $m = 202$ and $x = 2.5$ by using `while, end` and `for, end` loop control statements.

6. Compute the total sum of $\sum_{n=1}^{m}\frac{(-1)^{n+1}(x-1)^{n}}{n}$ for $m = 1001$ and $x = 2$ by using `while, end` and `for, end` loop control statements.

Exercise 8

Fix the two errors in the following script:

```
%% Find out if the entry is a Scalar or NOT.
% Prepare your entry data that MUST be in array
% or matrix format of any size: 1-by-1, 2-by-2, 2-by-3, etc, etc!
% Your entry can be also any standard array generating functions!
ABC=input('Enter ANY numerical entry of any size surrounded with square
brackets [  ]:   ');
if isnumeric(ABC) && isscalar(ABC)
    fprintf('This is a scalar: %20g \n', ABC);
else
    format short
    fprintf('Your entry is not scalar, but an array \n', ABC);
    fprintf(ABC);
end
```

1. After fixing the errors, the code has to produce the following output in the Command Window with the user entry of `[1, 2; 3,-1]`.

```
Enter ANY numerical entry of any size within [  ]:   [1, 2; 3,-1]
Your entry is not scalar, but an array
     1     2
     3    -1
```

2. The code has to produce the following output in the Command Window with the user entry of `['ab' 'bc'; 'cd' 'ef']`.

```
Enter ANY numerical entry of any size within [  ]:   ['ab' 'bc';
'cd'  'ef']
Your entry is not scalar, but an array
abbc
cdef
```

3. The code has to produce the following output in the Command Window with the user entry of `13.12`.

```
Enter ANY numerical entry of any size within [  ]:   13.12
This is a scalar:                13.12
```

Exercise 9

Fix the three errors in the following script:

```
%% Find out whether the array is square and if it is, show its size.
% Prepare your entry data that MUST be in array
% or matrix format of any size: 1-by-1, 2-by-2, 2-by-3, etc, etc!
% Your entry can be also any standard array generating functions!!!
ABC=input('Enter ANY numerical entry of any size within [  ]:   ');
[Rows, Cols]=size(ABC);
if isnumeric(ABC) && Rows==Rows
    fprintf('This is a square ARRAY! ');
    fprintf('Your entry is of %5g -by- %5g  square ARRAY \n', Cols, Rows);
else
    format short
    fprintf('Your entry is NOT a square array \n')
    fprintf('BUT an ARRAY of size %5g - by - %5g \n', Cols, Cols);
end
```

After fixing the errors, the code should produce the following output in the Command Window, given these entries:

1. Entry: `[magic(5)]`

```
Enter ANY numerical entry of any size within [  ]:   [magic(5)]
This is a square ARRAY!
Your entry is of     5 -by-     5  square ARRAY
```

2. Entry: [rand(3,5)]

```
Enter ANY numerical entry of any size within [  ]:   [rand(3,5)]
Your entry is NOT a square array
BUT an ARRAY of size     3 - by -     5
```

Exercise 10

Fix the two errors in the following script:

```
%% Find out: the user entry is scalar or not. If it is, display it.
% otherwise, show the variable type.
ABC=input('Enter ANY numerical entry of any size within [  ]:   ');
if isnumeric(ABC)
    fprintf('This is a Scalar! \n');
    fprintf('Your entry is a scalar:  %5g  \n', ABC);
else
    class(ABC, 1)
end
```

After fixing the two errors, the following output should be obtained with the respective inputs.

For entry 1:

```
Enter ANY numerical entry of any size within [  ]:   123
This is a Scalar!
Your entry is a scalar:    123
```

For entry 2:

```
Enter ANY numerical entry of any size within [  ]:   '1001011'
ans =
    'char'
```

For entry 3:

```
Enter ANY numerical entry of any size within [  ]:   [1 3 -2]
ans =
    'double'
```

Exercise 11

Fix the two errors in the following script:

```
%% Find out: the array is real and square. If it is, display it;
% otherwise, show its size and type.
% NB: size(), display(), class() can be used.
% Prepare your entry data that MUST be in array
% or matrix format of any size: 1-by-1, 2-by-2, 2-by-3, etc.
% Your entry can be also any standard array generating functions!

ABC=input('Enter ANY numerical entry of any size surrounded with [  ]:   ');
[Rs, Cs]=size(ABC);
if ischar(ABC) && Rs==Cs
    fprintf('This is a square array! \n');
    disp(ABC);
elseif
format short
fprintf('This is not a square array & its size: %5g-by-%5g \n', Rs, Cs);
disp(num2str(ABC));
end
```

After fixing the two errors, the following output should be obtained with the respective inputs.

Entry 1:

```
Enter ANY numerical entry of any size within [  ]:   magic(3)
This is a square array!
     8     1     6
     3     5     7
     4     9     2
```

Entry 2:

```
Prepare your entry data that MUST be in array or matrix format of any size
1-by-1, 2-by-2, 2-by-3, etc., etc.! with real value elements
Your entry can be also any standard array generating functions!!!
Enter ANY numerical entry of any size within [  ]:   [1, 2/0; 0/0, 1]
```

```
This is a square array!
      1   Inf
   NaN     1
```

Entry 3:

```
Your entry can be also any standard array generating functions!!!
Enter ANY numerical entry of any size within [  ]:   [1, 2/0; 0/0, 1; 1, 0]
This is not a square array and its size:     3 - by -     2
  1  Inf
NaN    1
   0
```

Exercise 12

Fix the five errors in the following script:

```
%% Q7. Computing area of a circle, square and rectangle w.r.t the user
entries:
W = input('Width of a rectangle:          ', 's');
L = input('Length of a rectangle:         ', 's');
R = input('Radius of a circle:            ', 's');
S = input('Side length of a square:       ', 's');
if isempty(R) && isempty(S)
    A1=W*L;
    fprintf('Area of a rectangle:  A1 =  %5g \n', A1);
elseif isempty(W) && isempty(L) && exist('R','var') && exist('S', 'var')
    A2 = pi*R^2; A3 = S^2;
    fprintf('Area of a circle:  A2 =  %5g \n', A2);
    fprintf('Area of a square:  A3 =  %5g \n', A3);
elseif isempty(W) && isempty(L) && isempty(R)
    A3 = S^2;
    fprintf('Area of a square:  A3 =  %5g \n', A3);
elseif isempty(S) && isempty(W) && isempty(L)
    A2 = pi*R^2;
    fprintf('Area of a circle:  A2 =  %5g \n', A2);
```

```
else exist('W','var') && exist('L','var') && exist('R','var') &&
exist('S','var')
    A1=W*L; A2 = pi*R^2; A3 = S^2;
    fprintf('Area of a rectangle:  A1 =  %5g \n', A1);
    fprintf('Area of a circle:  A2 =  %5g \n', A2);
    fprintf('Area of a square:  A3 =  %5g \n', A3);
else
    fprintf('You need to ENTER some dimensions! \n')
end
```

After fixing the errors, the following entries should be displayed in the Command Window:

```
Width of a rectangle:         1
Length of a rectangle:        1
Radius of a circle:           1
Side length of a square:      1
Area of a rectangle:  A1 =    1
Area of a circle:  A2 =  3.14159
Area of a square:  A3 =       1
```

Exercise 13

Fix the five errors in the following script:

```
%% Assessing the student performances
clc; clearvars
SP =input('Enter the student grade:  ');
if SP <65
    disp('Student Grade is  F ')
elseif SP>=66 && SP<=71
disp('Student Grade is  D ')
elseif SP>71 && SP<=81
    disp('Student Grade is  C ')
elseif SP>82 && SP<87
    disp('Student Grade is  B ')
```

```
else
    disp('Student Grade is  A ')
end
```

After fixing the errors and running the script with the inputs 65, 71, 81, 82, 87, and 87.5 in sequential order, the following output should be shown in the Command Window:

```
Enter the student grade:  65
Student Grade is  F
Enter the student grade:  71
Student Grade is  C
Enter the student grade:  81
Student Grade is  C
Enter the student grade:  82
Student Grade is  B
Enter the student grade:  87
Student Grade is  B
Enter the student grade:  87.5
Student Grade is  A
```

Exercise 14

Write a script (program) that computes all solutions:

1. But displays only real solutions of these third order polynomial equations for any values of a, b, c, *and* f: i) $x^3 + bx^2 + cx = 0$; ii) $ax^3 + f = 0$; iii) $ax^3 + cx = 0$; iv) $x^2 + bx + c = 0$.
2. But displays only complex solutions of these equations for any values of a, b, c, and f: i) $ax^2 + c = 0$; ii) $ax^3 + f = 0$; iii) $ax^2 + bx + c = 0$

Exercise 15

Write a script (program) that computes the volume and weight of the model that may have a form of a cube, cylinder, and rectangular prism. Users need to enter (via an input prompt) the necessary geometric dimensions of the model and enter or select material properties (such as density) from given data (aluminum, copper, and steel) in your script. Your script has to write all the computed and user input data sets into an external file called `RESULTS.txt` with explanatory comments in it along with the numerical data.

Exercise 16

Edit and correct the following script in order to display the current date and time correctly in the Command Window.

```
format short e
T=clock;
fprintf('This year is: %n4 \n', T(1))
if T(2)==1
 sprintf('It is: %f4 -st month of the year:  %n4 \n', T(2),T(1))
elseif T(2)==2
 sprintf('It is: %f4 -nd month of the year:  %n4 \n', T(2),T(1))
elseif T(3)==3
 sprintf('It is: %f4 -rd month of the year:  %n4 \n', T(2),T(1))
else
 sprintf('It is: %f4 -th month of the year:  %n4 \n', T(2),T(1))
end
sprintf('current time is: %lo o"clock %lO min \n', T(4), T(5))
sprintf('and %s secs \n', (T(6)))
```

Correct this script in order to obtain the correct date and time in the following format:

```
It is: 11 - day of the 6-th month of the year:  2014
current time is: 15 o"clock 3 min  and 17.136 secs
```

Exercise 17

Given that $y(x, t) = y_0 \sin(kx - \omega t)$ is the solution of the wave equation, where, $k = \frac{2\pi}{\lambda}$ is the wave number, $\omega = 2\pi f$ is the angular frequency, and λ is the wave length.

1. Write an anonymous function with a function handle to compute the values of $y(x, t)$ with the input arguments `k`, ω,`x`, and `t`.

2. Write an inline function to compute the values of $y(x, t)$ with the input arguments `k`, ω,`x`, and `t`.

3. Write a function file to compute the values of $y(x, t)$ with the input arguments `k`, ω,`x`, and `t`.

Exercise 18

The equation for a power factor of a series RC (resistor-capacitor) circuit with no inductance is $cos\,\delta = \frac{RC\omega}{\sqrt{1+(RC\omega)^2}}$.

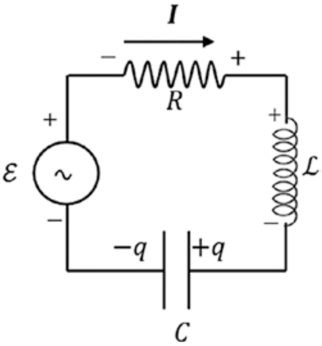
1. Write an anonymous function with the function handle to compute the values of δ with the input arguments `R`, `C`, and ω.

2. Write an inline function to compute the values of δ with the input arguments `R`, `C`, and ω.

3. Write a function file to compute the values of δ with the input arguments `R`, `C`, and ω and plot δ vs. ω.

Exercise 19

The equation for charge in an RLC (Resistor-Inductance-Capacitor) circuit in a series is determined by Kirchhoff's law: $L\ddot{q}+R\dot{q}+\frac{q}{C}=\mathcal{E}_{max}\cos\omega t$.



1. Write an anonymous function with a function handle for solving the given second order differential equation for $q(t)$ with the input arguments of R, L, C, ω, and t.

2. Write an inline function to solve the given second order differential equation for $q(t)$ with the input arguments o*f* R, L, C, *ω*, and *t*.

3. Write a function file to solve the given second order differential equation for $q(t)$ with the input arguments R, *L*, C, *ω*, and *t*.

4. Create a Simulink model to simulate the given RLC system.

Exercise 20

The acceleration of a skydiver is determined by:

$$a = g\left(1 - \frac{v^2}{3600}\right)$$

Where $g = 9.81\ \ m/\text{sec}^2$.

1. Write an anonymous function to compute a as a function of speed v.

2. Write a function file to compute a as a function of speed v and plot a vs. v.

3. Create a Simulink model to simulate acceleration of a skydiver.

4. Compute terminal speed for a skydiver.

Exercise 21

A truck of mass *m* accelerates from rest at $t = 0$ with constant power *P* along a level road. The speed of the truck as a function of time is given by: $v(t) = \left(\frac{2P}{m}\right)^{\frac{1}{2}}\sqrt{t}$

If $x = 0$ at time $t = 0$, the position function $x(t)$ is given by: $x(t) = \left(\frac{8P}{9m}\right)^{\frac{1}{2}}\sqrt{t^3}$, where $P = 550\ kW$ and m = 15000 kg.

1. Write a function handle to compute the speed of the truck $v(t)$ as a function of time *t*.

2. Write an inline function to compute the position of the truck from the function $x(t)$ as a function of time *t*.

3. Create a Simulink model to obtain numerical values of $v(t)$, x(t) as a function of t in the MATLAB workspace and compare the results with the ones obtained from the function handle and inline function.

4. Build plots of $x(t)$ *vs*. t, $v(t)$ *vs*. t in two separate plot figures.

Exercise 22

In a crash test, a car travelling 100 km/h (28 m/sec) hits an immovable concrete wall. We can treat this problem in general by approximating that car the body is one piece, despite the fact that different parts of the car will accelerate differently upon impact. In fact, upon impact, the center of the car moves forward less than half of its length. Let's assume that the stopping distance of the car upon hitting the wall until full stop is 0.72 m. The time to full stop will be equal to $\Delta t = \frac{\Delta x}{v_{ave}} = \frac{0.72}{0.5v_0 + 0.5v} = \frac{0.72}{0.5v} = \frac{0.72}{14} = 0.051\,sec$

The average acceleration of the car until full rest is equal to

$$a = \frac{\Delta v}{\Delta t} = \frac{v_0 - v}{\Delta t} = -\frac{v}{\Delta t} = -544.44\,m/\text{sec}^2$$

Note that this is about 55g, which means very high acceleration (deceleration) takes over a very large amount of energy from inertia forces and converts it into heat. Write a function file to compute `a` as a function of Δx (for different front bumper types) and v (for all cases, take $v_0 = 0$) for a crash test of any type of cars with different traveling speeds of v.

Exercise 23

Create a function file (called `Ex23.m`). It should have one input argument (the planted year of the maple tree, which is a four-digit integer: 0 … 2018) and two output variables: `AGE` (calculated age of the maple tree) and `NOTE` (the age of the maple tree), as well as warning/message boxes with notes.

1. If the planted year of the tree is before 1000, the outputs are: `NOTE = "CANNOT be TRUE", AGE = [ ]` (empty) and also display an error box with a note `"Check Your Entry"`.

2. If the planted year of a tree is in between 1000 and 1918, the outputs are: `NOTE = "NEED to be PROTECTED", AGE = [ ∀ ]` (computed age with respect to the current year) and a warning dialog box with a note: `"Maple Tree is to be Under Protection"`.

3. If the planted year of a tree is in between 1919 and 2000, the outputs are: `NOTE = "WELL fit", AGE = [ ∀ ]` (computed age with respect to the current year) and a message box with a note: `"GOOD one for a timber"`.

4. If the planted year of a tree is in between 2001 and 2018, the outputs are: `NOTE = "TOO young", AGE = [ ∀ ]` (computed age with respect to the current year) and display a message box with a note `"TOO young for timber"`.

Exercise 24

The Spiral of Archimedes (also called the Archimedean Spiral) is a spiral curve (named after the third-century Greek mathematician Archimedes). The equation of the spiral is written in the polar coordinate system using the following equation: $r = a + b\theta$. Where r is the distance from the origin and θ is the angle of that point in radians with respect to the origin. The parameters a, b in the equation are real numbers that control the spiral and the distance between successive spiral turnings, respectively.

1. Compute the Spiral of Archimedes for $\theta = 0:\frac{\pi}{10000}:13\pi,\ a = -1.25,\ b = -2.25$ with a vectorization method.

2. Compute the Spiral of Archimedes for $\theta = 0:\frac{\pi}{10000}:13\pi,\ a = -1.25, b = -2.25$ with `for ... end` and `while ... end` loops without memory allocation.

3. Compute the Spiral of Archimedes for $\theta = 0:\frac{\pi}{10000}:13\pi,\ a = -1.25,\ b = -2.25$ with `for ... end` and `while ... end` loops with memory allocation.

4. Compare the computation efficiencies in Steps 2 and 3 against Step 1 using `tic ... toc`.

Exercise 25

Create a function file (called `Ex25.m`) that takes these four input variables: `n`, b_n, Δx, and `L` and these three output variables: $f(x)$, ALL_cell, ALL_struct.

Compute this series: $f(x) = \sum_{n=1}^{\infty} \frac{b_n sin\left((2n-1)\pi x\right)}{2L}$ using `for, end` loop control statements. Take $n = 1 : 11$, $b_n = [-5 \ldots 5]$, $x = [0 \ldots L]$, $L = 10$. $\Delta x = \frac{L}{2000}$.

ALL_cell is a cell array composed of $f(x)$, x, b_n.

ALL_struct is a structure array composed of $f(x)$, x, b_n, ALL_cell.

Exercise 26

Create a function file (called `Ex26.m`) that takes one input variable (`k`) and one output variable (`S`).

Compute a total sum of this series: $10 + 35 + 65 + 110 + \ldots = \sum_{m=0}^{55}\left(10 * 2.5^k\right)$. Compute the total sum using `while, end` and `for, end` loop control statements.

1. Halt the computation process when the sum is larger than $2.5 * 10^{18}$ and display in the Command Window at the iteration step where the computation halted and the computed sum of series.

2. Display the difference between the computed sum of series and $2.5 * 10^{18}$.

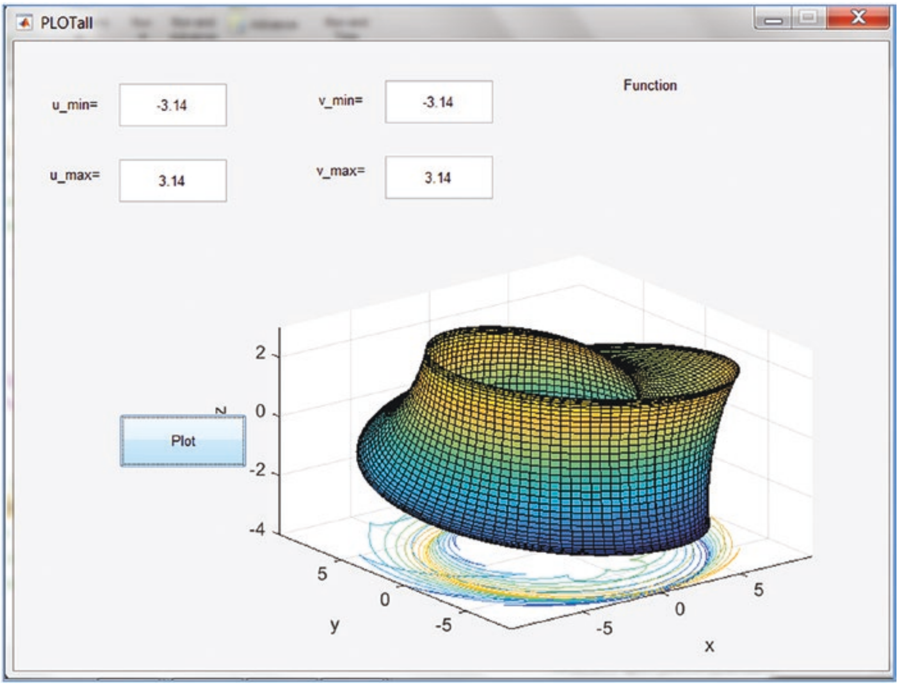
3. Display the final sum and the iteration number as integers, but collect all of the sums `S` from every step.

Exercise 27

Write a script file to compute solutions of the third order polynomial $a_3x^3 + a_2x^2 + a_1x = 0$ for any given values of its coefficients a_1, a_2, a_3 with a user prompt input and by using conditional statements `if`, `elseif`, `else`, and `end`. Include in your script to print out solutions of the polynomial in the external `*.dat` file in the correct data formats, including complex values.

Exercise 28

Design/model and rewrite an M-file in the next GUI model to compute a torus function and plot its 3D plot (shown in the next GUI model figure called `PLOTall`) by taking user inputs. (See Chapter 6 for how to build 3D plots.)



u_min= -3.14

v_min= -3.14

u_max= 3.14

v_max= 3.14

Use the umbilic torus function defined by:

$$x = \sin u\left(7 + cos\left(\frac{u}{3} - 2v\right) + 2cos\left(\frac{u}{3} + v\right)\right)$$

$$y = \cos u\left(7 + cos\left(\frac{u}{3} - 2v\right) + 2cos\left(\frac{u}{3} + v\right)\right)$$

$$z = sin\left(\frac{u}{3} - 2v\right) + 2sin\left(\frac{u}{3} + v\right)$$

For $-\pi \le u \le \pi$, $-\pi \le v \le \pi$.

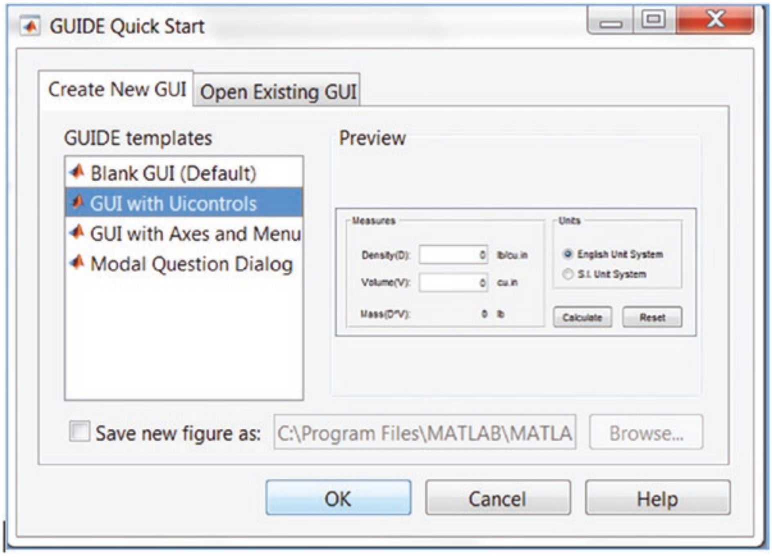
1. Create a standalone application of the GUI model plotting an umbilic torus function.
2. Associate the GUI with another M-file to display the existing (external) image in `*.jpg` format, for instance.
3. Create a function file called `umbilic_torus.m` to compute *x*, *y*, *z* for inputs of *u* and *v* and create an `.mex` file from it.

Exercise 29

Use MATLAB's example of a GUI with UIControls. See the next figure.

Create a GUI model to compute the volume of a cylinder and rectangular prism with the necessary user entries. For a cylinder, use radius and height; for a rectangular prism, use length, width, and height. This should all be in the SI Unit System only. Also, perform the following tasks:

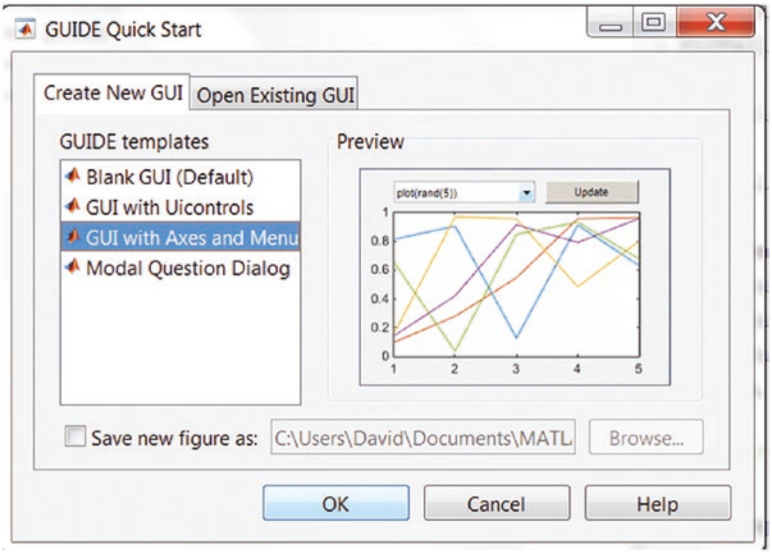
1. Create a standalone application of the GUI model.
2. Associate the GUI model with another M-file that computes the values of $f(t) = e^{sin(250t) + 2.5\cos(750t)}$ for $t = -\pi \ldots \pi$ with $\Delta t = 1/10000$ and plays a sound of $f(t)$ with a sampling frequency of $f_s = 10000\ Hz$ by employing MATLAB's `sound()` function.



□ Save new figure as: C:\Program Files\MATLAB\MATLA

Exercise 30

Open MATLAB guide's GUI with Axes and Menu and use it to build a new GUI model.



□ Save new figure as: C:\Users\David\Documents\MATL

The new GUI model should have one drop-down option (Pop-up Menu button) to select a function type ($\sin\left(\frac{2\theta}{3}\right)$, $\cos\left(\frac{5\varphi}{3}\right)$ and quadratic polynomial $x^2 + 6x + 3$). These are computed by separate M-files for plotting. Include an Edit Text block so users can specify value ranges for the arguments: θ, φ, x. Also, add a pushbutton to close the GUI model with a message dialog box, with a message "All Completed!" and a sound of rectangular pulses, e.g., `rectpuls()`.

Exercise 31

1. Write a function file (called `Ex31_a.m`) that generates of a linear system $Ax_i = b$ with `N` unknowns of x_i. `A` is a square matrix of size N-by-N and `b` is a column matrix of size N-by-1. `A` and `b` must be generated with the random integer `randi()`. (For more information on matrix algebra, see Chapter 7, "Linear Algebra"). Note that there is one input argument, called `N`.
2. Write a function file (called `Ex31_b.m`) that takes input arguments for `A` (e.g. ,1000-by-1000) square matrix and `b` (e.g., 1000-by-1) column matrix generated by the function file (`Ex31_a.m`) from Step 1. Solve the system solutions in three different ways: `x=linsolve(A,b), x=inv(A)*b; x=A\b`.
3. Create a GUI model that recalls two the M-files (`Ex31_a.m` and `Ex31_b.m`) from Steps 1 and 2, computes solutions of the system, and displays them in a plot of *1:N* vs. x_i.
4. Create a standalone application from Step 2 and find out which functions (not supported in C/C++) create errors while generating the executable application's project files.

Exercise 32

Write C code called `Say_HELLO.c` to display these words:

```
Hello World!
This is a Great DAY!
Live and Learn...
```

Create a `.mex` file from the C code called `Say_HELLO.c` and test it in the MATLAB workspace.

Exercise 33

Perform the following matrix operations in Simulink:

$$M_1 = \begin{bmatrix} -3 & -j & 2j \\ 2 & 5 & 0 \\ 1 & 2j & 4j \end{bmatrix}; \; M_2 = \begin{bmatrix} 2.5 & 1 & i \\ -2 & 3i & 3i \\ 3 & 0 & 4j \end{bmatrix}; \; v = \begin{pmatrix} 2 \\ -3j \\ 1 \end{pmatrix};$$

$$M_1 * v + \sqrt{M_2{}^{T}} * \frac{\sqrt[3]{2.5}}{1.15} * \sqrt{v} - \left(M_2 * M_1{}^{-1}\right) * \sqrt[3]{v} = ?$$

Exercise 34

To compute values of the following two math functions by modeling in Simulink, save the computation results in a separate `*.mat` file and MATLAB workspace simultaneously. Display them in a Scope block's plot figure with new settings for legend, color of plotted lines of data points, and plotting all data points displayed. Given: $F_1(t) = e^{2t^2-13t+25}, F_2(t) = 2t^2 - 13t + 25, \; t = -2\pi \ldots 2\pi, \;\; \Delta t = \pi/5000.$

Exercise 35

Create a Simulink model to compute numerical solutions of the following first order differential equations.

$$\begin{cases} \dot{y}_1(t) = -3y_1 - 2y_2, & y_1(0) = 1 \\ \dot{y}_2(t) = 4y_1 + 2y_2, & y_2(0) = 1 \end{cases}$$

Hint: For more information about ordinary differential equations, see Chapter 8.

CHAPTER 6

Plots and Data Visualization

The MATLAB package has numerous built-in functions used to visualize numerical data via plot, graphs, charts, and animations. There are two-dimensional (2D) and three-dimensional (3D) plots, charts, maps, etc. In addition, some of the MATLAB toolboxes have specific plot and visualization functions. Building plot figures is very straightforward and can be done in two different ways, one of which is using commands and writing scripts and the other is using GUI tools. In this chapter, we discuss and demonstrate some of the most essential tools and techniques used to build line, bar, pie, surface, mesh plots, graphs, and animated plots via examples.

Basics of Plot Building

Under the PLOTS tab of the main menu (see Figure 6-1), there are all available plots in the current MATLAB package, including its installed toolboxes.

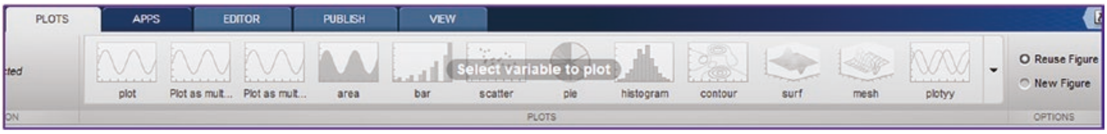


Figure 6-1. *PLOTS tab of the MATLAB Desktop menu*

If any data (variable) residing in the workspace is chosen, the applicable plot commands will become active. For instance, if the selected variable size is 20 by 1, then the plots shown in Figure 6-2 will become active, indicating that these plots can be used.

S. Eshkabilov, *Beginning MATLAB and Simulink*, https://doi.org/10.1007/978-1-4842-5061-7_6

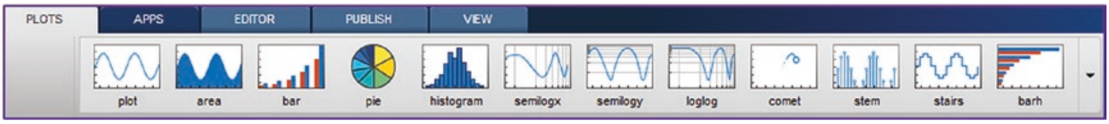


Figure 6-2. Possible plots in the PLOTS tab for the selected data

If the chosen data (variable) is 10 by 100, then the plots shown in Figure 6-3 will become active instead.

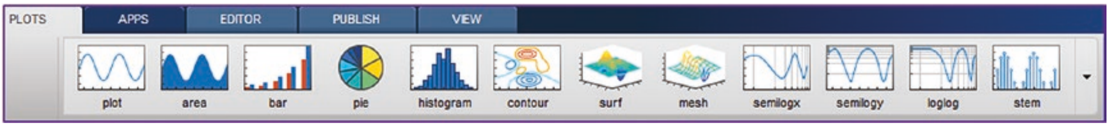


Figure 6-3. Possible plots in the PLOTS tab for the selected data

PLOT()

Let's look at a few examples that create 2D line plots using the `plot()` command and GUI tools.

Example 1: Plotting Two Rows of Data

Given two sets of data: A=[1, 3 4.5 5 6.7 8.1 9.5 10.3] and B=[-2.32, 0.23 , 2.14, 2.22, 3.92, 6.67, 7.41, 6.43].

Let's plot them using the `plot()` command.

```
>> A=[1, 3 4.5 5 6.7 8.1 9.5 10.3]; B=[-2.32, 0.23 , 2.14, 2.22, 3.92,
6.67, 7.41, 6.43]; plot(A, B)
```

The plot figure is shown in Figure 6-4.

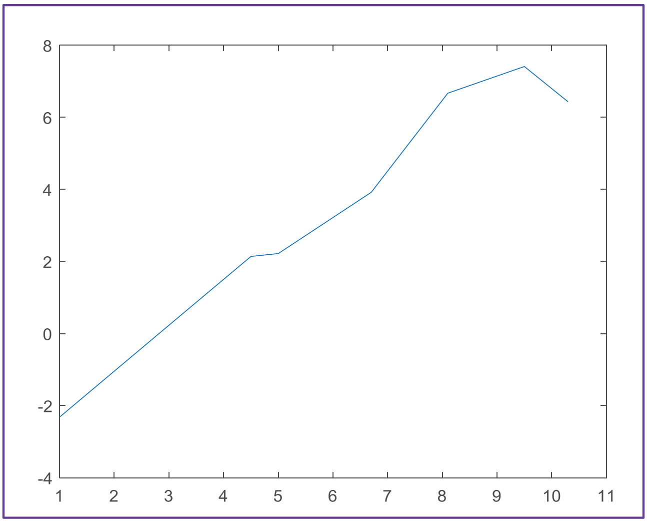


Figure 6-4. *Possible plots in the PLOTS tab for the selected data*

The same plot can be generated by using GUI tools and selecting the variables A and B in the workspace. Hold down Ctrl from the keyboard and use right mouse button (RMB) clicks to select multiple variables. Then with the left mouse button (LMB) options, you can choose plot(A, B) or click on the PLOTS tab and choose Plot: plot.

Moreover, the A and B data can be plotted as a scatter, pie, or histogram plot.

Let's look at another example that plots computed data points.

Example 2: Plotting Function Values

Given: $y(t) = 1.2\ sin(2t + 10),\ t = [-2\pi, 2\pi]$.

By using the following commands in the command window, we get a plot of $y(t)$ vs. t, as shown here.

```
>> t=-2*pi:2*pi;  % value ranges for t are assigned
>> y=sin(2*t+10); % y values are computed
>> plot(t, y)     % t vs. y is plotted
```

These commands produce the plot figure shown in Figure 6-5.

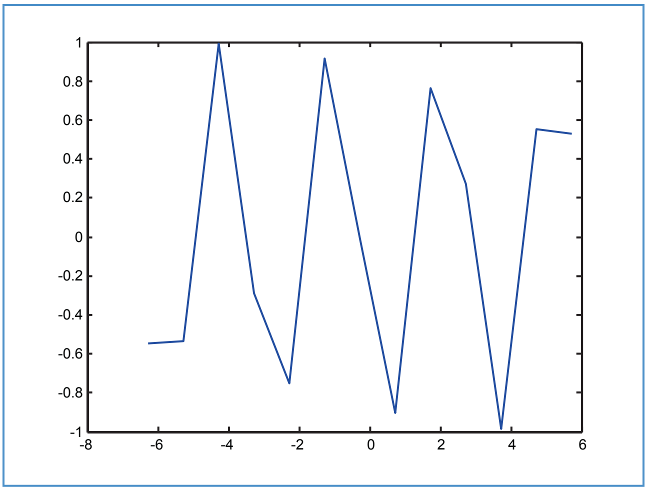


Figure 6-5. *Plot of y(t) = 1.2 sin(2t + 10), t = [−2π, 2π].*

The plot, shown in Figure 6-5, is not smooth even if it looks like a sine wave. So, where is the flaw?

The flaw is in the very first command: >> t=-2∗pi:2∗pi. It creates an array of *t* with a step size of *Δt* = 1, which is a default setting in the linear space definition. In this case, function values are computed at *t* = *-6.28, -5.28, -4, 28 … 5.28,* and *6.28*. In order to fix this problem, the step size must be specified and taken smaller than 1.0; for instance, $\Delta t = \frac{\pi}{20}$.

```
>> t=-2*pi:pi/20:2*pi; y=sin(2*t+10); plot(t, y)
```

After taking $\Delta t = \frac{\pi}{20}$, the plot of *y*(*t*) becomes smooth and periodic, as shown in Figure 6-6.

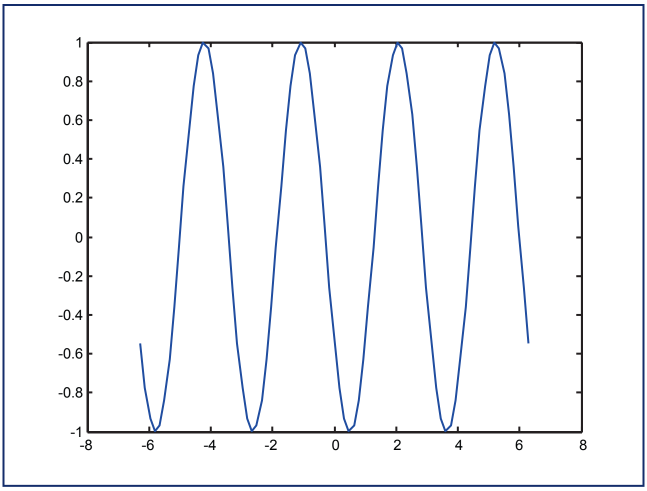


Figure 6-6. *Plot of* $y(t)=sin(2t+10)\,vs.\;\; t=-2\pi\ldots2\pi,\; \Delta t=\frac{\pi}{20}$

Example 3: Building a Histogram

You are given 1,000 data points generated by a normally distributed pseudorandom number generator function called `randn()`. Let's build a histogram of the data points. See Figure 6-7.

```
>> F=randn(1,1000);
>> hist(F)
```

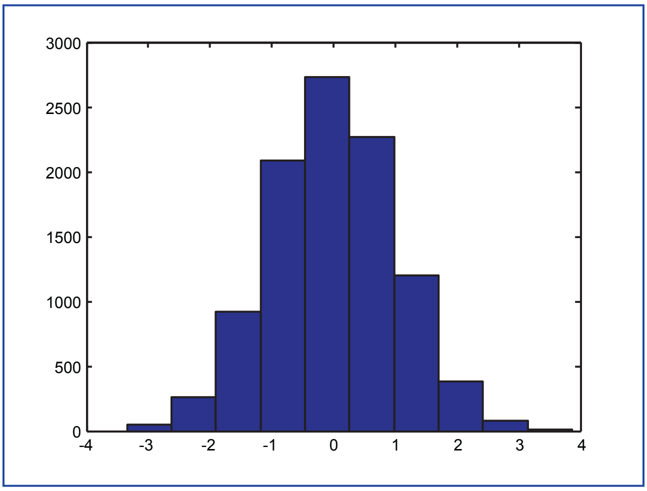


Figure 6-7. *Histogram of 1,000 normally distributed random numbers with the hist() command*

TITLE, XLABEL, YLABEL, AXIS, GRID, and LEGEND

Plot titles, axis labels and scales, grid settings, and legend options can be implemented in plot figures by employing the appropriate commands, such as `title()`, `xlabel()`, `ylabel()`, `axis()`, `grid`, and `legend()`. Let's demonstrate how to use them via examples.

TITLE()

The `title()` command is used to set the plot title name tag. For example, `title('Unit circle')` gives the current plot figure a title of `Unit circle`.

Alternative syntax of this command is `title 'Unit circle'`. That is compatible with later versions of MATLAB.

XLABEL, YLABEL, and ZLABEL

The `xlabel`, `ylabel`, and `zlabel` commands are used to assign the axis labels or titles for the x, y, or z axis, respectively. The syntax is `xlabel('t, [s] '), ylabel('f(t)')`. An alternative syntax for these commands is `xlabel 't, [s] ', ylabel ' f(t)'`, which assigns the labels for the x and y axes to `t, [s]` and `f(t)`, respectively.

AXIS

By default, MATLAB scales the axes of a plot to fit the data. The `axis` command is used to set the scaling factor for the plot figure. General syntax of the `axis` command is as following.

For 2D plots:

```
xlim([xmin xmax])           % Min and Max limits set for x axis
ylim([ymin ymax])           % Min and Max limits set for y axis
axis([xmin xmax ymin ymax]) % Min and Max limits set for x & y
```

For 3D plots:

```
axis([xmin,xmax,ymin,ymax,zmin,zmax]) % Min and Max limits
```

In addition, axes can be set up in a few different ways by these following commands:

```
axis equal  % Both or all 3 axes have equal tick marks
axis square % Current axis box square in size
axis normal % Restores current axis box to full size
axis off    % Removes all axis labels, tick marks, background
axis on     % Restores all axis labels, tick marks, background
axis tight  % Sets axis limits to the range of data
axis ij     % Puts MATLAB into its "MATRIX" axis mode
```

GRID

The `grid on` command is used to set grids on the plot figure to make it more legible, e.g. use `grid on` to set major grids or `grid minor` to set minor grids.

Let's use these additional plot commands.

Example 4: Plotting a Unit Circle with Plot Tools

Let's create a unit circle defined by x = sin(t), y = cos(t), $t = [0, 2\pi]$. (see Figure 6-8).

```
>>t=0:pi/100:2*pi; x=sin(t); y=cos(t); scatter(x, y);
>>title('Unit circle defined by x=sin(t) vs. y=cos(t)'); axis tight;
>>grid on; axis tight; xlabel('sin(t)'), ylabel('cos(t)')
```

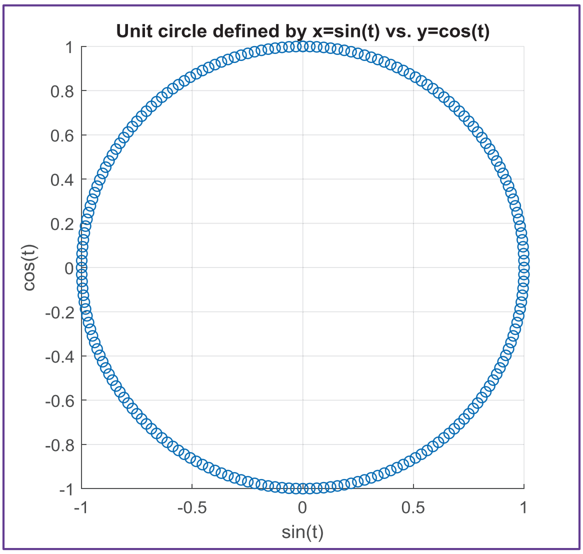


Figure 6-8. *Plot of a unit circle with plot tools*

There is an alternative solution in the polar coordinate system, using the `polar()` command, as shown in Figure 6-9.

```
>> t=0:pi/100:2*pi;x=sin(t);y=cos(t);r=sqrt(x.^2+y.^2);polar(t,r)
>> title('Unit circle defined by x=sin(t) vs. y=cos(t)')
```

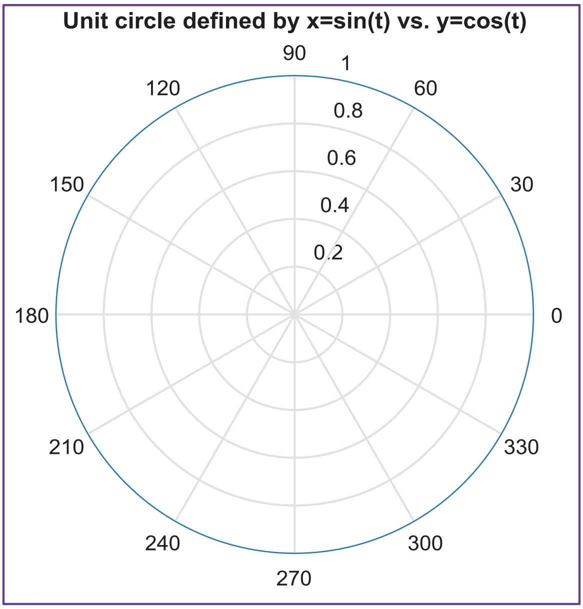


Figure 6-9. *Plot of a unit circle in the polar coordinate system*

Note While plotting x vs. y, the data size (i.e., the number of elements in x and y) has to be equal; otherwise, MATLAB will return an error. This is a very common mistake.

LINE and MARKER Specifiers

Line and marker specifiers are useful for distinguishing a few data sets plotted in a single plot area. The specifiers can be line colors, types, and markers. Line specification (line specifier), marker specification (marker specifier), and color specification (color specifier) are explained in Tables 6-1, 6-2, and 6-3.

***Table 6-1.** Line Style Specifiers*

Specifier	Line Style
-	Solid line (by default)
--	Dashed line
:	Dotted line (colon)
-.	Dash-dot line

***Table 6-2.** Marker Specifiers*

Specifier	Marker Type
+	Plus sign
o	Circle
*	Asterisk
.	Point
x	Cross
'square' or s	Square
'diamond' or d	Diamond
^	Upward-pointing triangle
v	Downward-pointing triangle
>	Right-pointing triangle
<	Left-pointing triangle
'pentagram' or p	Five-pointed star (pentagram)
'hexagram' or h	Six-pointed star (hexagram)

Table 6-3. *Color Specifiers*

Specifier	Color
R	Red
G	Green
B	Blue
C	Cyan
M	Magenta
Y	Yellow
K	Black
W	White

All plot functions (except for the family of easy plots, i.e. `ezplot`, `ezsurf`, and `ezmesh`, and implicit function plot, `fimplicit`, `fimplicit3`, and `fsurf`) accept a line specification argument that defines three components to specify lines with:

- Color
- Marker symbol
- Line style

For example: `>> plot(x, f, ':pb')`
Command plots *x vs. f* using a colon (:) place pentagram markers (p) at the data points, and color a line and a marker in blue (b). We can specify the components (in any order) as a quoted string after the data arguments. Note that line specifications are single strings, not property-value pairs, for example:

```
>> plot(x, h, 'd')
```

Command plots *x vs. h* with a marker diamond without any line. Note if the color of a plot line or marker is not specified, then the color by default is blue.

Moreover, while using the `plot` and `plot3` functions, we can specify other characteristics of lines using graphics properties:

> `LineWidth`—Specifies the width (in points) of the line, e.g., `'LineWidth', 2`
>
> `MarkerSize`—Specifies the size of the marker in points, e.g., `'MarkerSize', 5`
>
> `MarkerEdgeColor`—Specifies the color of the marker or the edge color for filled markers, e.g., (circle `'o'`, square `'s'`, diamond `'d'`, pentagram `'p'`, hexagram `'h'`, and the four triangles `'<', '>', '^', 'v'`)
>
> `MarkerFaceColor`—Specifies the color of the face of filled markers.

In addition, we can specify the `LineStyle`, `Color`, and `Marker` properties instead of using the symbol string. This is useful when you want to specify a color that is not in the list. You can use the red-green-blue (RGB) values, e.g., to plot the line specifications:

```
plot(t,y, 'LineWidth', 1/25, 'LineStyle', '--','Color', 'magenta',...
'Marker', 'v')
```

This plots *t vs. y* data with a line width of 1/25 inch, a line style of dashed lines, magenta dashed lines, and data points marked with triangle markers looking downward.

Let's look at another example.

Example 5: Plotting Sine Function Values with Plot Tools

Plot this function: $y(\alpha) = sin(2\alpha + 10), \alpha = [-2\pi, 2\pi]$.

Here is the solution script (`plot_ex5.m`) containing the commands to compute $y(\alpha)$ with respect to $\alpha = -2\pi : \frac{\pi}{20} : 2\pi$. It has the plot commands specifying line color (`'b'` is blue), line type (`':'` is colon), line width (`2`), marker type (`'d'` for diamond), marker size (`7`), and marker face color (`'m'` for magenta). Moreover, it displays the plot title text (`'Plot of function y = sin(2a+10)'`) with the font size of 13 and similarly, x- and y-axis label text (`'a', 'y(a)'`) with the font size of 13.

```
%% plot_ex5.m
a=-2*pi:pi/20:2*pi; y=sin(2*a+10);
plot(a,y,'bd:','LineWidth',2,'MarkerSize',7,'MarkerFaceColor','m')
grid on; axis([-2*pi 0.5*pi -1 1]);
title('Plot of function y = sin(2a+10)', 'FontSize', 13)
xlabel('a'); ylabel('y(a)','FontSize',13)
```

After executing the code (`plot_ex5.m`), the plot figure shown in Figure 6-10 is created.

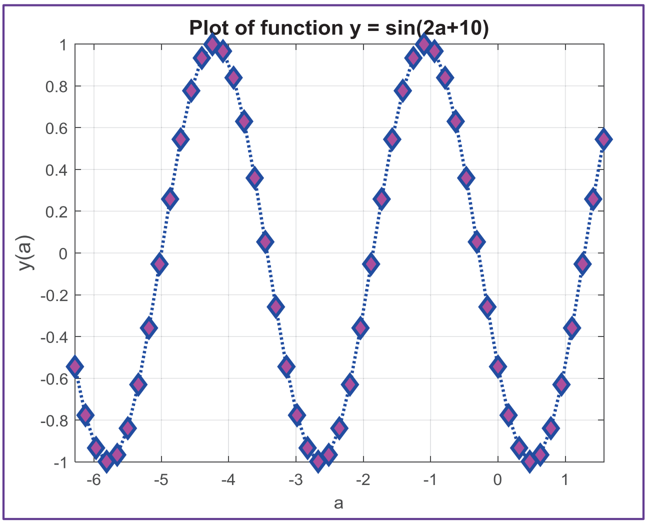


Figure 6-10. *Plot of* $y(\alpha) = \sin(2\alpha + 10), \quad \alpha = [-2\pi, 2\pi]$

Special Characters

MATLAB can recognize various special characters by using TeX markup and LaTEX code syntax. It is very handy to employ LaTEX in plots as well. The following examples display special characters and Greek letters in the plot figure in the title, axis label, plot text, or notation. You write special characters, such as the Greek letters, by using the backslash operator (\). Here are some examples:

- `title('\alpha')` displays α on the plot title
- `xlabel('\beta')` displays β on the x–axis label
- `text(0, 1, '\rightarrow')` displays ➤ in the plot area at the coordinate points of (0, 1)

- `title('e^{at}')` displays e^{at} in the plot title
- `ylabel('\fontsize{13} y(x)')` displays $y(x)$ on the y-axis with the font size of 13
- `title('\it y(\alpha) vs. \alpha'` displays $y(\alpha)$ *vs.* α in italic in the plot title
- `xlabel('\bf f, [s^{-1}]')` displays **f**, [$\mathbf{s}^{-1}$] in bold font on the x-axis label
- `title('\copyright by SE')` displays © by SE in the plot title
- `text(2,1,'\Gamma \approx \pi/2')` displays $\Gamma \approx \pi/2$ in the plot area at the coordinate points of (2,1)

For more extensive information and help on TeX and LaTEX, type the following in the command window:

```
>> help tex; help latex
```

Example 6: Plotting Sine Function Values with Plot Tools

Plot this function: $y(\alpha) = sin(2\alpha + 10), \alpha = [-2\pi, 2\pi]$.

The answer script (`plot_ex6.m`) has plot commands with line and marker specifiers, such as line type (`'-'` is a solid line), line width (`2`), line color (`'b'` for blue), marker type (`'o'`), marker size (`7`), and marker face color (`'y'` for yellow). Moreover, it includes commands for the title text, containing the Greek letter (α) in italic and a font size of 13. Axis labels include the Greek letter (α) with a font size of 13. It includes the text note $\leftarrow \alpha = 0^0$ displayed on the plot area, as shown in Figure 6-11.

```
a=-2*pi:pi/20:2*pi; y=sin(2*a+10);
plot(a,y,'b-o','LineWidth',2,'MarkerSize',7,'MarkerFaceColor','y');
grid on; axis([-2*pi 0.5*pi -1 1]);
title('\it \fontsize{13} Plot of function y = sin(2\alpha+10)')
ylabel('\fontsize{13} y(\alpha)');  xlabel('\fontsize{13} \alpha')
text(0,0, '\leftarrow \fontsize{13} \alpha = 0^0');
```

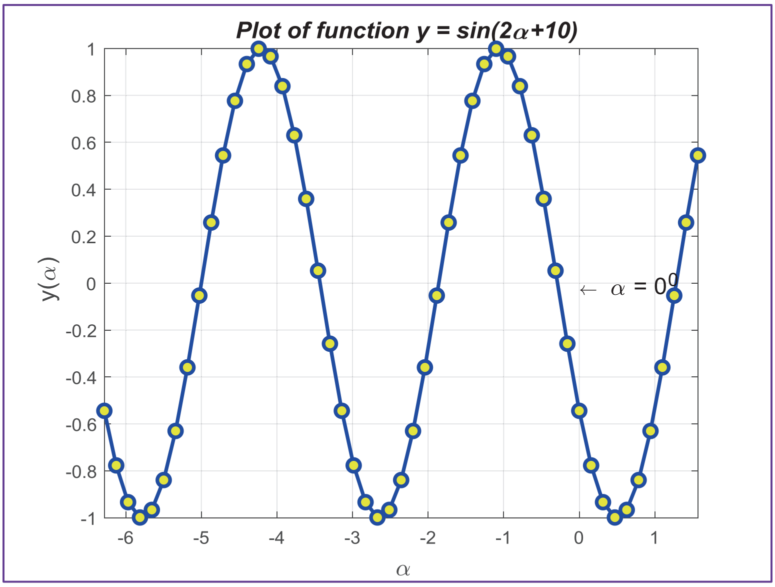


Figure 6-11. *Plot of y(α) = sin(2α + 10), α = [−2π, 2π]*

Plot Two Data Sets in Two Y–Y Axes

Sometimes we may need to plot two separate data sets or function values in one plot figure in two vertical (y-y) axes. This type of plot is relatively easy to implement and there are two approaches to it. One approach is applicable to older versions of MATLAB and the other works with more recent versions.

Let's look at the following example.

Example 7: Plotting Two Function Values on Y-Y Axes

Given:

- $y(\gamma) = 2.725e^{-0.1\gamma} cos(2.725\,\gamma + 25);$ $\quad \gamma = 0\ldots15\,\pi;$
- $z(\gamma) = 0.725\,sin(0.725\gamma + 25)\; ln\,(0.5\gamma);$ $\quad \gamma = 0\ldots15\,\pi;$

The following code shows two solutions (`plot_ex7.m`). Version 1 is for older versions of MATLAB and Version 2 is for recent versions. The main difference between the two versions are the commands `plotyy` and `yyaxis` (`left` and `right`). MATLAB is forward compatible in most cases, so the scripts and command syntax written for older versions are valid with newer versions.

```
% Version 1. For older versions of MATLAB
clf          % Clean up an open figure
gamma=0:pi/40:15*pi;
y=2.725*cos(2.725*gamma+25).*exp(-.1*gamma);
z=0.725*sin(0.725*gamma+25).*log(0.5*gamma);
% gamma vs. y and gamma vs. z plotted for each value of gamma.
[AX,Y1,Y2] = plotyy(gamma, y, gamma, z, 'plot');
% Title of the plot is in LaTEX.
title(['\fontsize{9} y=2.725*cos(2.725*\gamma+25)*e^{-0.1\gamma}',...
'& z=0.725*sin(0.725*\gamma+25)*log(0.5*\gamma)'])
xlabel('\fontsize{15} \gamma');
set(get(AX(1),'Ylabel'),'String',...
'y=2.725*cos(2.725*\gamma+25).*e^{-^0.1\gamma}')
set(get(AX(2),'Ylabel'),'String',...
'z=0.725*sin(0.725*\gamma+25)*log(0.5*\gamma)')
grid on; axis tight
```

```
%% Version 2. For recent versions of MATLAB
gamma=0:pi/40:15*pi;
y=2.725*cos(2.725*gamma+25).*exp(-.1*gamma);
z=0.725*sin(0.725*gamma+25).*log(0.5*gamma);
yyaxis left  % Select the left y-axis to plot the data
plot(gamma, y, 'r-o'),
ylabel('y=2.725*cos(2.725*\gamma+25).*e^{-^0.1\gamma}')
yyaxis right % Select the right y-axis to plot the data
plot(gamma, z, 'b--x');
ylabel('z=0.725*sin(0.725*\gamma+25)*log(0.5*\gamma)'),
grid minor,
title(['\fontsize{9} y=2.725*cos(2.725*\gamma+25)*e^{-0.1\gamma}',...
'& z=0.725*sin(0.725*\gamma+25)*log(0.5*\gamma)']), axis tight
```

The obtained plots are different as well. In Version 1, there are some limitations, such as plot line, marker, and color specifications, which cannot be adjusted. Only the default colors are used (see Figure 6-12).

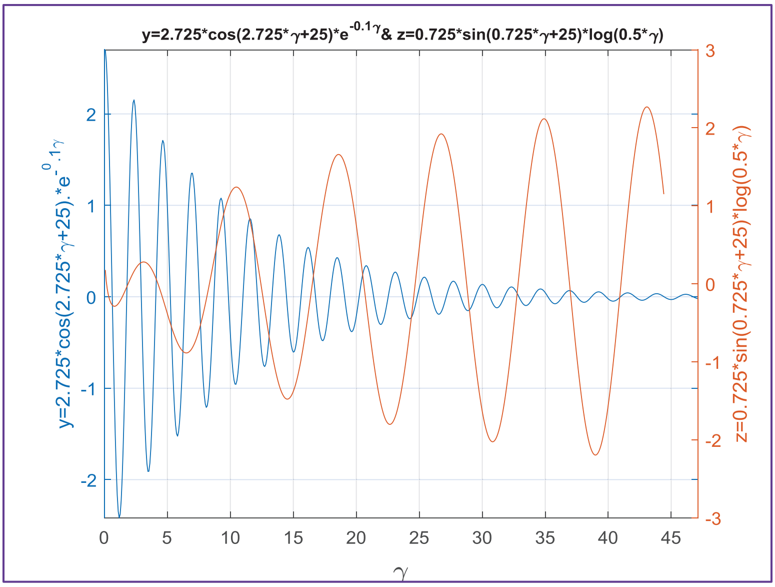


Figure 6-12. *Plot of two function values on y-y axes (Version 1)*

The Version 2 solution shown in Figure 6-13, for more recent versions of MATLAB, is more flexible. It includes adjustable plot specifiers, such as marker type, line type, color, size, etc.

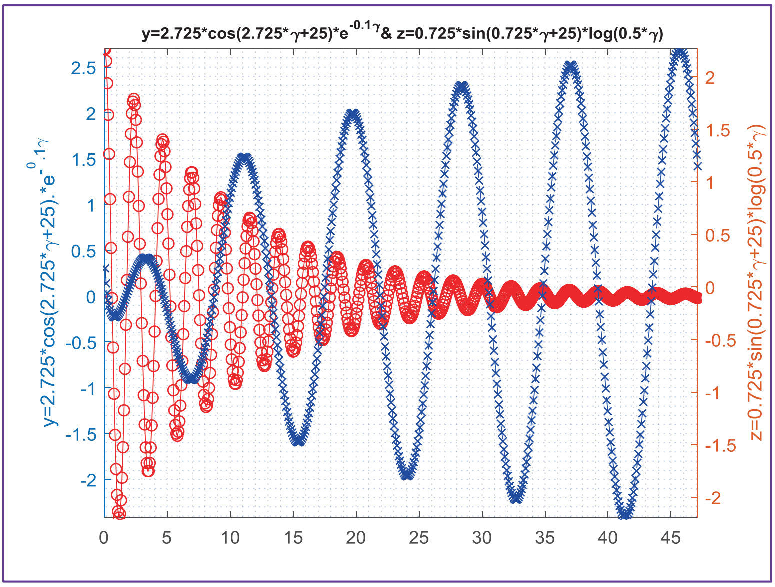


Figure 6-13. *Plot of two function values on y-y axes (Version 2)*

Sub-plots

Several plots can be plotted in a plot figure as sub-plots by employing the `subplot()` command.

Example 8: Building Sub-Plots of Functions

Given these four functions:

$$z(\gamma)=0.725\,sin(0.725\gamma+25)ln(0.5\gamma)$$

$$y(\gamma)=2.725\,e^{-0.1\gamma}cos(2.725\,\gamma+25)$$

$$x(t)=e^{2.725\,cos(2.725t+25)}$$

$$w(t)=0.725\,sin(0.725\,ln(0.5t)+25).$$

$$\gamma = 0\ \ldots\ 15\pi;\ t = -5\pi\ \ldots\ 5\pi;$$

Let's plot them as sub-plots in a single plot figure. The solution script (`plot_ex8.m`) computes and plots all four functions on one plot as four separate sub-plots (see Figure 6-14).

```
%% plot_ex8.m
gamma=0:pi/40:15*pi;
y=2.725*cos(2.725*gamma+25).*exp(-.1*gamma);
z=0.725*sin(0.725*gamma+25).*log(0.5*gamma);
t=-5*pi:pi/40:5*pi;
x=exp(2.725*cos(2.725*t+25));
w=0.725*sin(0.725*log(0.5*t)+25);
figure % Creates a blank figure
subplot(2,2,1); plot(gamma, y, 'r-'); grid on
title('y=2.725*cos(2.725*\gamma+25)*e^{-0.1\gamma}')
subplot(2,2,2); plot(gamma, z, 'b-'); grid on
title(‚z=0.725*sin(0.725*\gamma+25).*log(0.5*\gamma)')
subplot (2,2,3)
semilogx(t,x,'r-')  % x axis in log. Scale for demo purposes
grid on
axis([0.05 25 0 15.5]); % Assign min and max values for x & y axes
```

```
title(,x=e^{2.725cos(2.725t+25)}')
subplot(2,2,4)
semilogy(t, w, 'b-')      % y axis in log. Scale
grid on; axis([-17 17 0.025 5.5]),
title('w=0.725*sin(0.725*log(0.5*t)+25)')
```

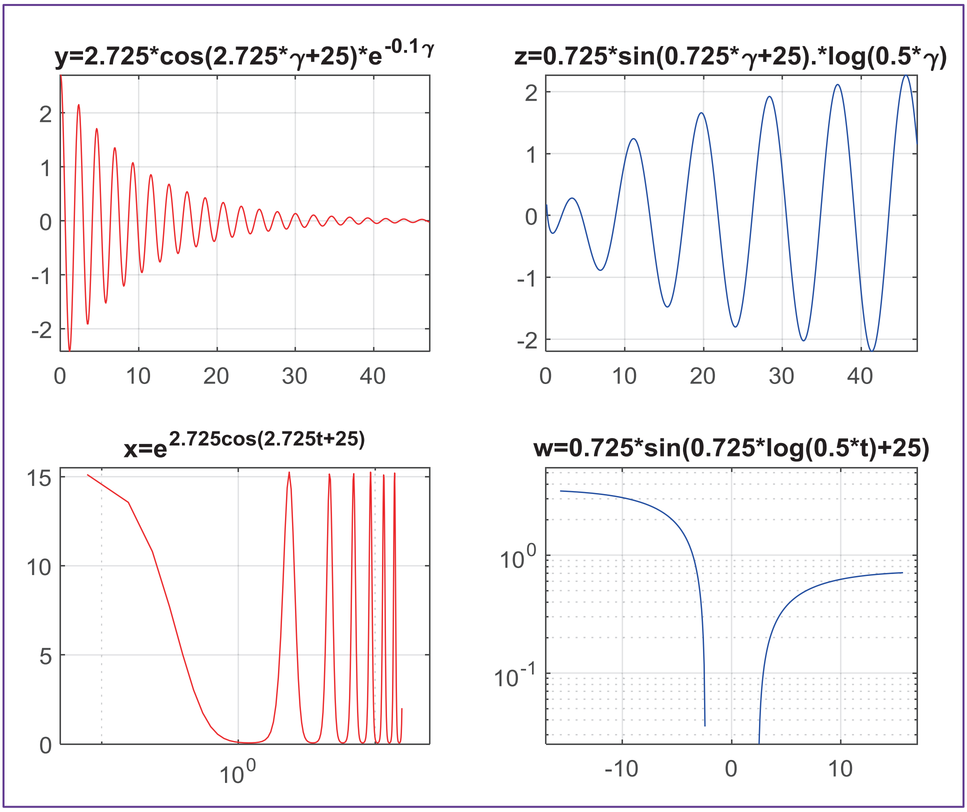


Figure 6-14. *Plot of four function values as sub-plots in one plot figure*

In addition, when all the plots are built, the following warning messages are displayed in the Command Window:

```
Warning: Negative data ignored
Warning: Imaginary parts of complex X and/or Y arguments ignored
Warning: Negative data ignored
```

Imaginary parts of computed values are omitted when plotting them as complex numbers.

LEGEND

The legend command is used to specify legends of plotted data sets to recognize the data, e.g. legend('y(t)', 'z(t)', 'h(t)'). Any special notations of TeX and LaTEX can be implemented with the legend command. For example, legend('x(\alpha)', 'y(\xi)') displays $x(\alpha)$ and $y(\xi)$ in the plot area.

HOLD

To hold and plot several plots in one plot figure, the hold on command is used. Once all the data sets are plotted, the held plot needs to be activated with the hold off command. Alternatively, when plotting sets of data in loops, you can use the hold all command to assign a unique color to each set of data.

Example 9: Plotting a Few Function Values in One Plot

Given: $y_1 = \frac{\sec(x)}{x}; y_2 = \frac{100\sin x}{x}; y_3 = \frac{10\cos x}{x}; y_4 = \frac{10\tan x}{x}; y_5 = \frac{e^x}{x}; x = [-3\pi, 3\pi].$

Let's compute and plot numerical values of these five functions together in a single plot figure.

```
%% plot_ex9.m
close all
x = linspace(-3*pi, 3*pi, 3333);
y=[sec(x)./x; 100*sin(x)./x;10*cos(x)./x;10*tan(x)./x; exp(x)./(x);];
plot( x, y(1,☺' 'g'o'); hold on
plot(x, y(2,:',–'-', x, y(3,:)' ":', x, y(4,:','b'." 'LineWid'h',1.5)
plot(x, y(5,:',"-','LineWid'h',1.5)
titl'('[y_1, y_2, y_3, y_4, y_5] vs.'x')
xlabe'('\it'x'); ylabe'('\it y function valu's')
legen'('sec(x)'x" '100*sin(x)/' ','10*cos(x)'x','10*tan(x)'x" 'e^x'x')
axis([-3*pi, 3*pi, -100, 100]); hold off; grid on
shg % Show a plot figure
```

After simulating the script, Figure 6-15 shows the result.

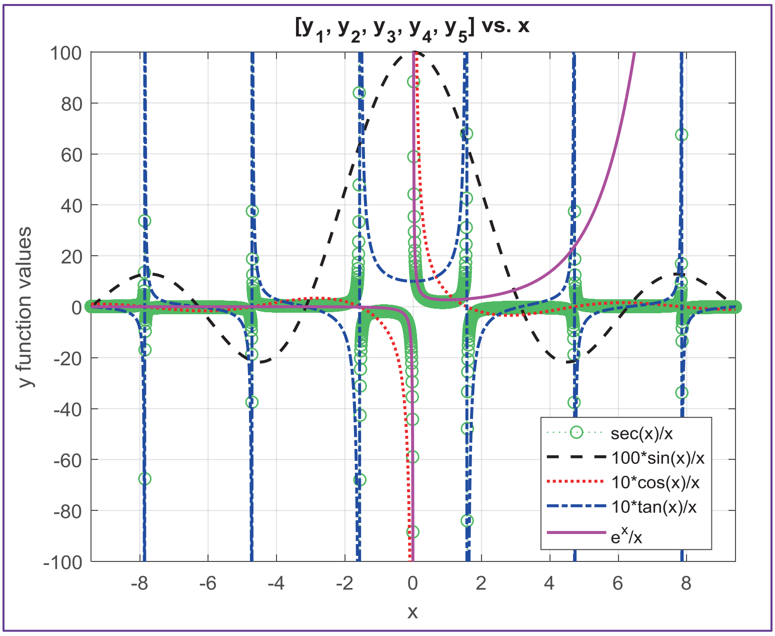


Figure 6-15. *Plotting several function values in one plot figure*

If there are different sets of data to be plotted in a single plot figure, loop control operators can be employed to handle line color, type, marker specifications, and legends. A line type, color, and marker type can be pre-specified as a string and used later. Let's consider the following example.

Example 10: Plotting Function Values with Different Line Markers and Colors

Given $F(t)=\sin(2t)*\sqrt{\sin(\mu t\sqrt{t+t^2+t^3})},\ t=0\cdots 3\pi,\ \mu=1\cdots 7.$

If we take the stepwise approach that we used in the previous examples, our script becomes considerably long and untidy. Thus, we take a different approach to create a set of line and marker specifiers first and then use the loop control statements to choose from our pre-defined specifiers. You do this for every iteration while plotting the data sets with respect to every value of μ separately. The solution script (`plot_ex10.m`) is rather compact. See Figure 6-16.

```
%% plot_ex10.m
close all; clearvars
t=0:.001*pi:1*pi;
% Line color, type and marker specifiers are defined:
Labelit  = {};
Colorit  = 'bgrcmkgrckmbgrygr';
Lineit   = '--:-:--:-:--:----:----:--';
Markit  = 'odxsh+*^v<p>.xsh+od+*^v';
for ii=1:7
% Line and marker specifier are taken for each iteration:
    Stylo = [Colorit(ii) Lineit(ii) Markit(ii)];
    F     = sqrt(sin(ii*t.*sqrt(t+t.^2+t.^3))).*sin(2*t);
    plot(t, F, Stylo), hold on
    Labelit{ii} = ['\mu = ' num2str(ii)];
    legend(Labelit{:})
end
```

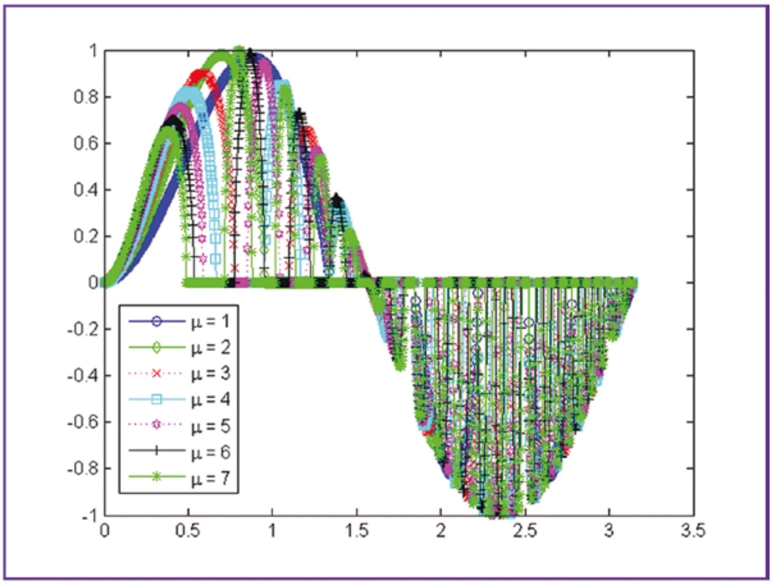


***Figure 6-16.** Plotting function values with different line markers and colors*

EZPLOT, FPLOT, and FIMPLICIT with Function Handles (@)

`EZPLOT` and `FPLOT` are used to plot mathematical expressions with the function handle (`@`) and implicitly defined argument values. For example:

```
fplot(@(t)([sin(t), cos(t), exp(cos(t))]));
ezplot(@(x)(sin(x)));
fimplicit(@(x,y) (25-(x.^2+y.^2)));
```

Or with a range of given argument values. For example:

```
fplot(@(t)([sin(t),cos(t),exp(cos(t))]),[-pi,pi]); ezplot(@(x)
(sin(x)),[0,2*pi]);
fimplicit(@(x,y)(25-(x.^2+y.^2)),[-5,5]);
```

The function handle (`@`) can be defined within `fplot,` `ezplot` and `fimplicit` or outside:

```
G=@(t)([t.^2+2*t-13,sqrt(t.^3+3*t),(t.^5+t^(3/4))^(3/2)]);
fplot(G, [-2.5, 2]);
```

Example 11: Plotting a Mathematical Expression with ezplot()

Given $f\left(x\right)=0.5\cos\frac{\omega\pi x}{180}+0.75\sin\frac{\omega\pi x}{180}$; ω = 00200, with an argument of *x*, the function *f*(*x*) is summation of the cosine and sine functions within the frequency range of $0\ s\ s\frac{rad}{sec}$ with the increment of $\Delta\omega = 20\frac{rad}{sec}$. Here is the solution script (`plot_ex11.m`) with a loop control statement (see Figure 6-17).

```
%% plot_ex11.m to demonstrate EZPLOT and FPLOT
close all
for N=0:20:200
    FH=@(x)(0.5*cos(N*pi*x/180)+0.75*sin(N*pi*x/180));
    ezplot(FH, [-6.25, 6.25]); hold all % or fplot(FH, [-6.25, 6.25])
end
hold off
```

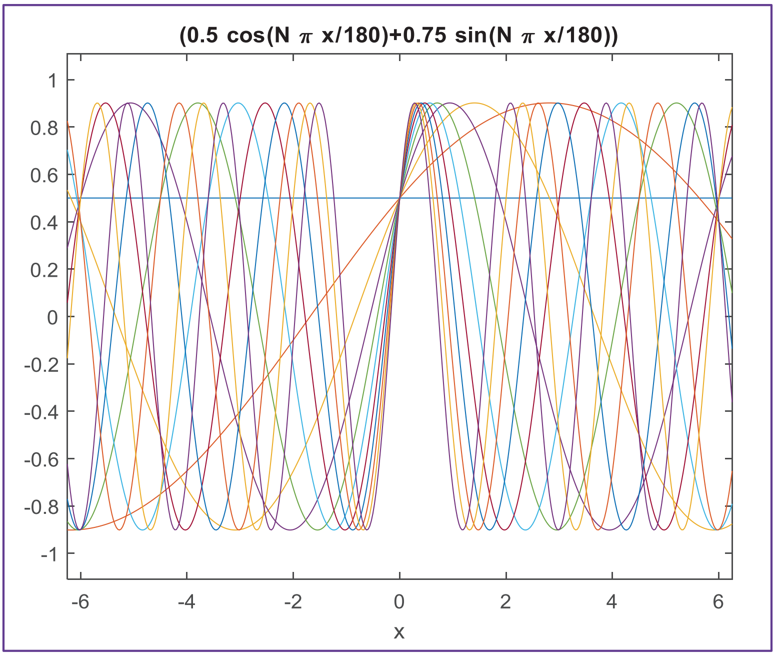


***Figure 6-17.** Plotting a mathematical expression with ezplot()*

`fplot`, `fimplicit`, and `ezplot` are easy to employ with mathematical expressions that have one or two arguments to plot, with or without specifying their ranges. An important feature of `fplot` is that it can compute the values of the expression without plotting it, as follows:

```
[X, Y] = fplot(@(x)(0.5*cos(N*pi*x/180)+0.75*sin(N*pi***x/180)));
```

GTEXT, TEXT, and GINPUT

The GTEXT and TEXT commands are used to add annotations and text strings to the plot figure. GTEXT is used to place text with a mouse. TEXT is used to add text descriptions to data points. The general syntax of the GTEXT command is as follows:

```
gtext('Text', 'PropertyName', PropertyValue,...)
gtext({'First line','Second line'},'FontName','Times','Fontsize',12)
```

The general syntax of the TEXT command is:

```
text(x, y, 'Text Message')
```

Here are two examples of employing text and gtext to display text or notes on plot figures. The gtext and text graphical annotation commands are used:

```
>> text(0,0, '\alpha  = \pi/2')
>> gtext(['\alpha  = ' num2str(13) ' \pi'])
```

The text command displays $\alpha = \frac{\pi}{2}$ in the current plot figure surface at [0, 0] and gtext displays $\alpha = 13\ \pi$ where a user clicks on with the cursor.

GINPUT is a graphical input from a mouse. It is a very handy tool to locate the points of our interest in a plot figure using a mouse pointer. The general syntax of this command is:

```
[X,Y] = ginput(N);  % N is number of points to select
```

This command returns N points of X and Y coordinates positioned/chosen using a mouse. The following command selects an unlimited number of points until the Enter key is pressed from the keyboard.

```
[X,Y] = ginput();
```

To locate points of interest on the plot or the local or global minima of the plotted data, the ginput is a good graphical tool:

```
>> [x, y] = ginput(3);
```

This command brings the cursor automatically to the surface of the current figure. It selects three points, of which the x and y coordinates will be measured and saved in the workspace under the variable names x and y. If there is no open figure, ginput opens a blank plot figure. Let's look at an example of employing these commands in combination.

Example 12: Locate and Display Minimum Values of a Function Plot in a Plot Figure

Given a function $f(x) = \sin(1.313^{-1.7*x}) - \cos(1.313^{1.7*x})$

Let's compute and plot $f(x)$, and then find its minimum values within a region of $x = -6.5\ldots6.5$. In addition, we display the found value on the plot figure by employing text and gtext. We use two different ways to locate a minimum of a given function with ginput.

The solution script (plot_ex12.m) uses a mouse cursor with ginput.

```
% plot_ex12.m
fun_fun=@(x)sin(1.313^(-1.7*x))-cos(1.313^(1.7*x));
fplot(fun_fun, [-6.5, 6.5]); grid on; hold on
[xm, ym]=ginput(3);    % Use mouse cursor carefully to locate minima
TXT1=['1st Local Min (square) @x= ', num2str(xm(1))];
gtext(TXT1, 'fontsize', 11)
text(-2,0,['2nd Local Min (circle) @x= ', num2str(xm(2))]);
text(-2, -0.5, ['3rd Local Min (star)   @x= ', num2str(xm(3))]);
plot(xm(1),ym(1),'rs', xm(2),ym(2), 'bo', xm(3),ym(3), 'kp', ...
'markersize',13,'markerfacecolor', 'm'); hold off
title('sin(1.313^{-1.7*x})-cos(1.313^{1.7x}')
xlabel('\it x'), ylabel('\it f(x)')
```

Within this script, the text and gtext commands are used to insert the three local minimum values of f(x) with the cursor clicks activated by ginput. After executing the script, we select the local minima of the plotted data from left to right and print the local minima on the plot figure. The plot figure shown in Figure 6-18 is the result.

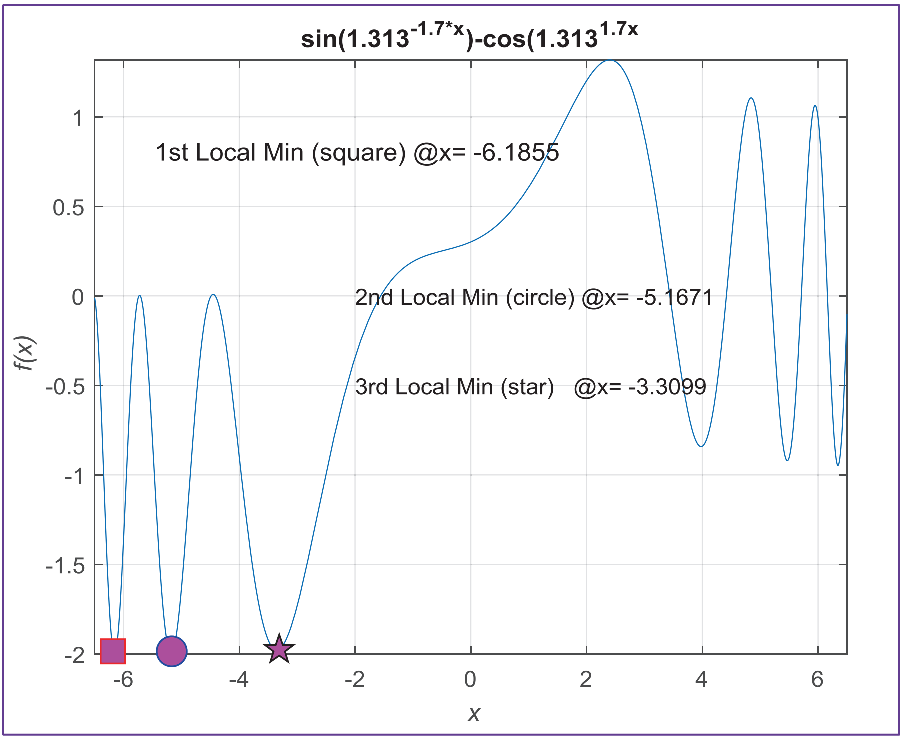


***Figure 6-18.** Plot figure with located minimum values displayed*

Note that in Figure 6-18, the first local min (the square) @x = -6.1855 is displayed with the gtext command the other two local minima (the second and third minima) are displayed with the text command.

The GINPUT function is particularly useful when we need to locate several points in a plot with their corresponding coordinate points, but accuracy depends on how well or how precisely we can place a mouse cursor over the points of interest.

Axis Ticks and Tick Labels

In some cases, it might be necessary to display specific values of tick marks on the plot axis. In those cases, the xticks, xticklabels, yticks, yticklabels, zticks, and zticklabels commands will be employed respectively for the x, y, and z axes.

Example 13: Display X-Axis Tick Labels

Given $G = [\sin(2t), \cos(2t), e\hat{}(\sin(2t) + \cos(2t))]$, $t = [0, 2\pi]$. Put the tick marks on the x-axis at $\left[0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\right]$.

Here is the solution script (plot_ex13.m).

```
%% plot_ex13.m to demonstrate Xticks and Xticklabels
H=@(t)([sin(2*t), cos(2*t), exp(sin(2*t)+cos(2*t))]);
fplot(H, [0, 2*pi], '-*')
xticks([0:pi/2:3*pi]); xticklabels({'0', '\pi/2', '\pi', '3\pi/2',
'2\pi'});
xlabel('\it t'), ylabel('\it H'),
title('\it \fontsize{9} Xticks & Xticklabels'), grid on
```

This solution script (plot_ex13.m) results in a plot with the x-axis tick labels displayed, as shown in Figure 6-19.

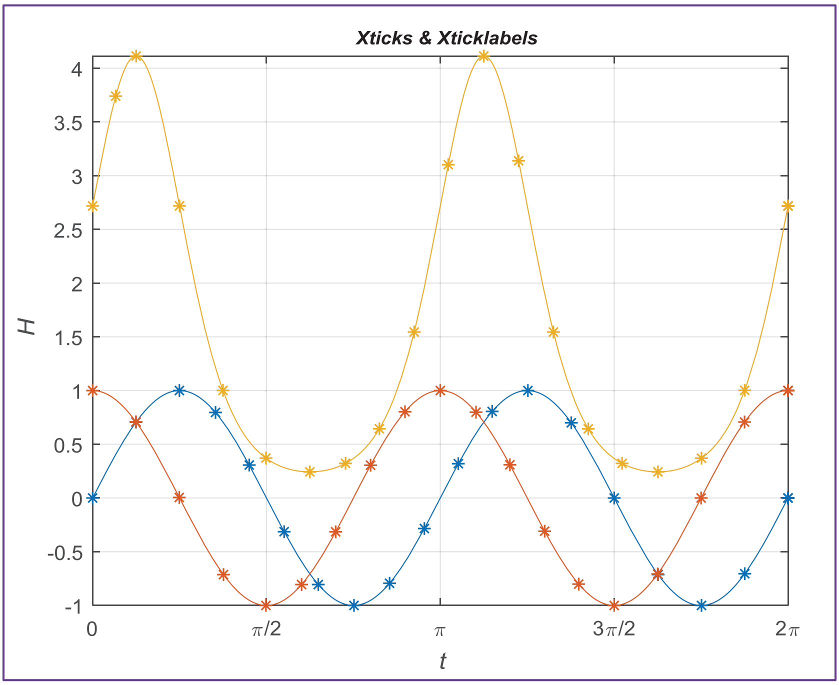


***Figure 6-19.** Display X-axis tick labels*

Figure Handles

The plot figure properties and plot line and marker specifiers (such as line style, color, width, marker type, size, and face color) can be set and managed via a figure handle, e.g., `FH = plot(x, y)`. The handle FH will contain all the figure properties of the plot. Let's look at the following example to see how to manage plot figure properties.

Example 14: Working with Figure Handles

Given $G = [\sin(2t), \cos(2t), e\hat{}(\sin(2t) + \cos(2t))]$, $t = [0, 2\pi]$.

Here is the solution script (`plot_ex14.m`).

```
%% plot_ex14.m - Plot Handle.
% Part 1. Plot figure and get a handle: HG
clf                                 % Clear current figure
t = 0:pi/30:2*pi;
GH = [sin(2*t); cos(2*t); exp(sin(2*t)+cos(2*t));];
HG = plot(t, GH); shg
%% Part 2. Change plot properties
% Set Line Color
HG(1).Color='r';                    % HG(1).color = [1 0 0]; % Alternative
HG(2).Color='g';                    % HG(2).color = [0 1 0]  % Alternative
HG(3).Color='b';                    % HG(3).color = [0 0 1]  % Alternative
% Set Line Type
HG(1).LineStyle = '-';
HG(2).LineStyle = '--';
HG(3).LineStyle = '-.';
% Set Line Width
HG(1).LineWidth = HG(1).LineWidth+0.5;
HG(2).LineWidth = HG(2).LineWidth+0.5;
HG(3).LineWidth = HG(3).LineWidth+0.5;
% Set Marker type
set(HG(1), 'marker', 'p')           % Marker is a Pentagon
set(HG(2), 'marker', 'd')           % Marker is a Diamond
set(HG(3), 'marker', 'o')           % Marker is a Circle
% Set Marker size
```

```
set(HG(1), 'MarkerSize', 5)
set(HG(2), 'MarkerSize', 7)
set(HG(3), 'MarkerSize', 9)
% Set Marker Face color
set(HG(1), 'markerfacecolor', [1 0 1])
set(HG(2), 'markerfacecolor', [0 1 1])
set(HG(3), 'markerfacecolor', [1 1 0])
title('\fontsize{11} G(\alpha) vs. \alpha. [\alpha = 0:\pi/30:2\pi]')
xlabel('\alpha'), ylabel 'F(\alpha)', grid on, axis tight
legend('sin(2\alpha)', 'cos(2\alpha)',...
    'e^{sin(2\alpha)+cos(2\alpha)}')
```

Part 1 of the script produces the plot figure shown in Figure 6-20 and Part 2 of the script changes the properties (the line and marker specifiers), as shown in Figure 6-21.

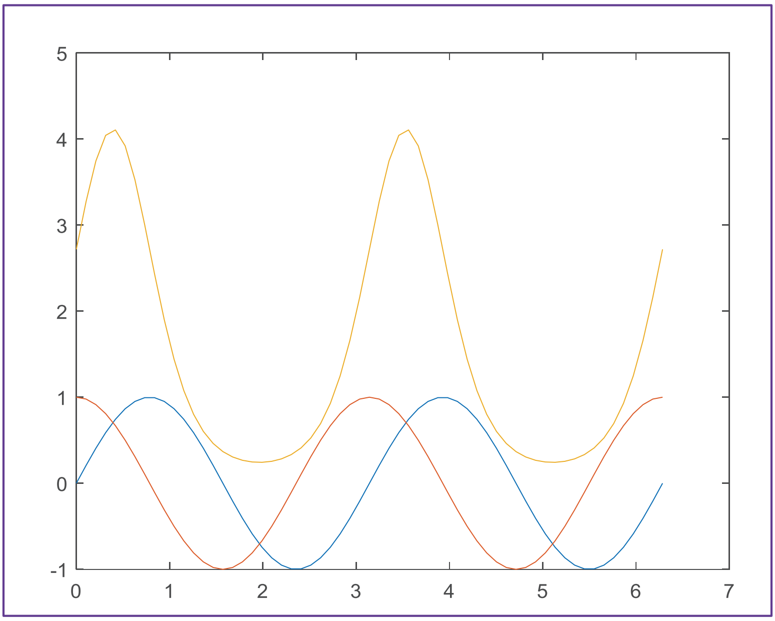


Figure 6-20. *Plot figure of three function values with default values of the plot() command*

The plot figure in Figure 6-21 shows all the changed properties (line and maker specifiers) and the added title, axis labels, and legends.

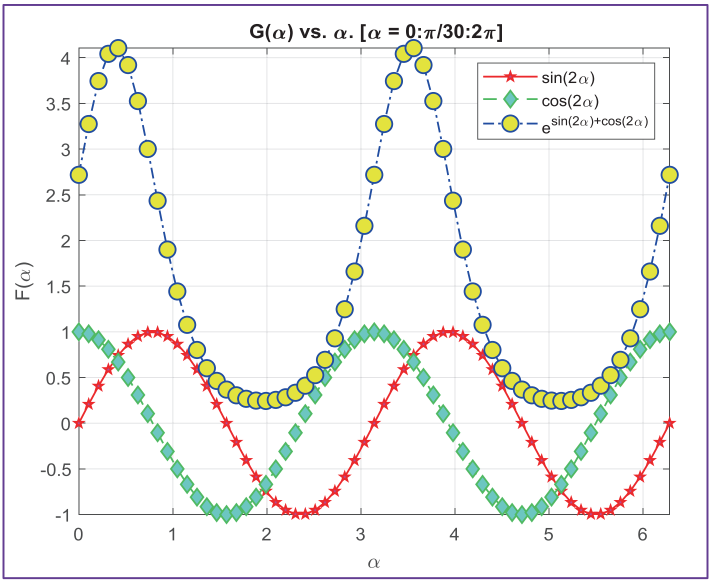


Figure 6-21. *Plot figure of three function values*

All of the 2D plot properties can be also altered using GUI tools (via the Property Inspector), which are accessed by activating the Plot Edit option with a single click on ↖ and then double-clicking on the plot area.

3D Surface Plots

To create and edit 3D bars, charts, and plots, you use the `bar3h`, `pie3`, `ezsurf`, `surf`, `surfc`, `mesh`, `contour3`, `ribbon`, `waterfall`, and `plot3` functions. Almost all of commands, tools, and functions that you use to build 2D plots are also applicable in building 3D plots. This includes `title`, `axis label`, `axis scale`, `legend`, and so forth.

Example 15: Creating a 3D Pie Plot with pie()

Given A =10; B = 15; C = 20; D = 17; E = 28; F = 10; H = [A, B, C, D, E, F], let's build a pie chart of the data set (see Figure 6-22).

```
pie3(1:6,H, {'A', 'B', 'C', 'D', 'E','F'}), title('Share Holders')
```

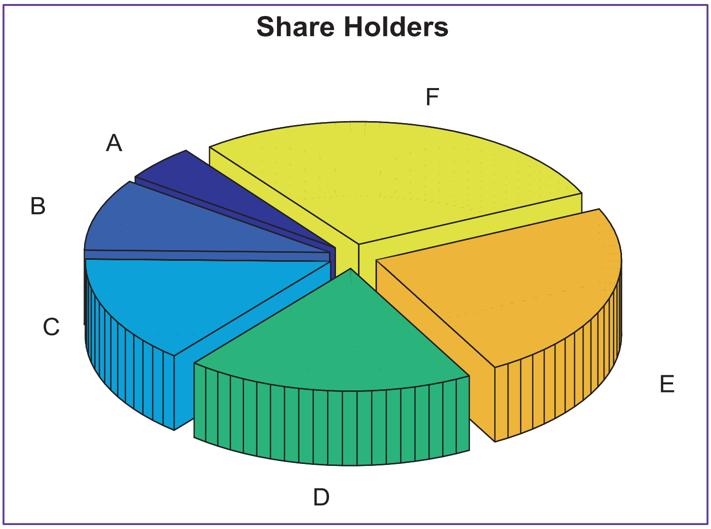


Figure 6-22. *3D pie plot*

Example 16: Creating a 3D Surface Plot with ezsurf()

Given $h(z, \beta) = \ln(0.01 * z) * cot(2.5 * \beta)$.

For this example, we define a function handle using the `ezsurf` command (see Figure 6-23):

```
ezsurf(@(z, beta)log(0.01*z)*(1./tan(2.5*beta)))
```

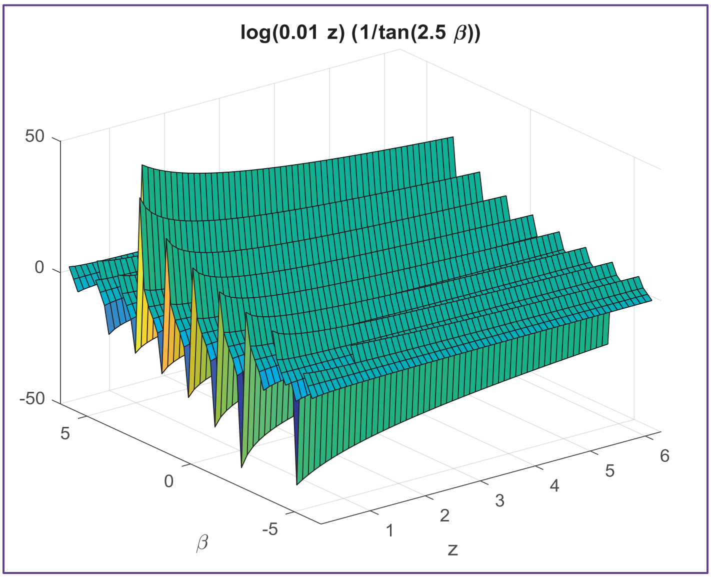


Figure 6-23. *3D surface plot*

Example 17: Creating a 3D Mesh Plot with ezmesh()

Given this function $h(\alpha, x) = cos(2.1\alpha)e^{0.1x}$

In this exercise, we create a symbolic math function of $h(\alpha, x)$ by using the inline function with a surface plot command: `ezsurf()` (see Figure 6-24).

```
ezmesh(inline('cos(2.1*alpha)*exp(0.1*x)'))
```

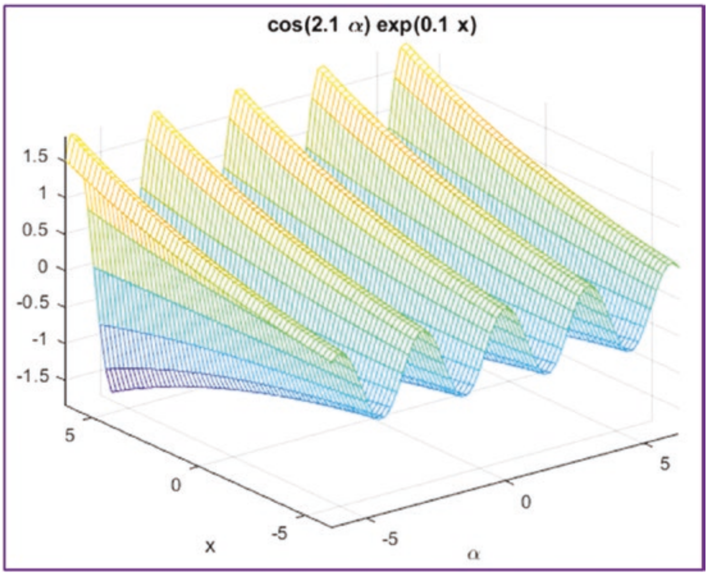


Figure 6-24. *3D mesh plot*

Example 18: Creating a 3D Surface-Contour Plot with ezsurfc()

Given $f(x,y) = 0.5\cos\dfrac{\omega\pi x}{180} + 0.75\sin\dfrac{\omega\pi y}{180}$; $\omega = 50 \ldots 500$, $x = [-\pi, \pi]$, $y = [-\pi, \pi]$, with two arguments x and y, let's build a 3D plot $f(x, y)$ for certain frequency ranges with an animated simulation.

```
close all
for omega=50:5:150
 handle=@(x,y)(0.5*cos(omega*pi*x/180)+0.75*sin(omega*y*pi/180));
    ezsurfc(handle, [-pi, pi], [-pi, pi]);
end
```

This script builds a 3D plot (see Figure 6-25) of the given function $f(x, y)$ with an animated simulation with respect to the frequency ranges $\omega = 50 \dots 150$.

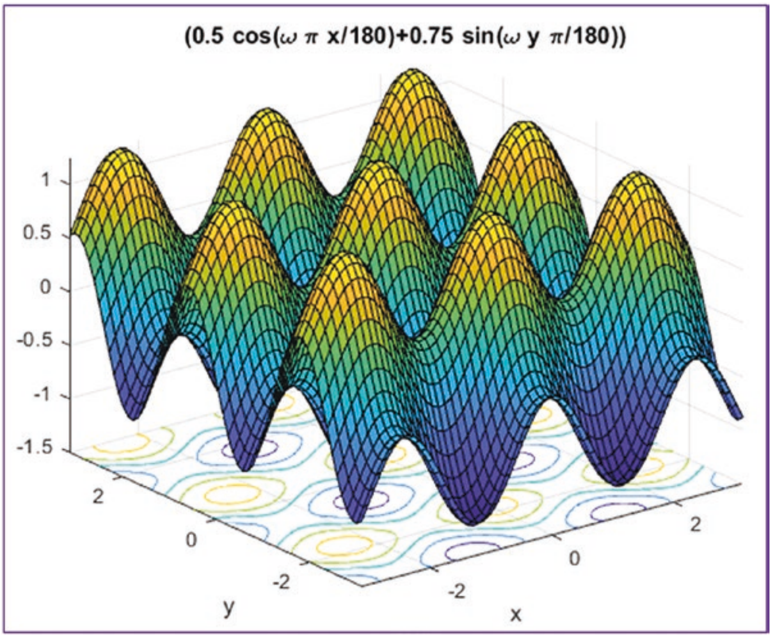


Figure 6-25. *3D surface-contour plot with surfc()*

Example 19: Creating a 3D Plot of an Electric Potential Field

The electric potential field V at a point, due to two charged particles, is given by

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

Where q_1 and q_2 are charges of the particles in coulombs (C), r_1 and r_2 are the distances of the charges from the point (in meters), and ϵ_0 is the permittivity of free space, whose value is

$$\epsilon_0 = 8.854e10^{-12}\frac{C^2}{Nm^2}$$

Suppose the charges are $q_1 = 2p10^{-10}C$ and $q_2 = 4n10^{-10}C$. Their respective locations in the *xy* plane are (0.3, 0) and (−0.3, 0). You will plot the electric potential field on a 3D surface plot, *V* plotted on the *z* axis over the ranges of *x* and *y*, defined by $-0.25\ nx \leq 0.25$ and $-0.25\ ny \leq 0.25$. You can create a 3D plot in two ways:

- By using the `surf` function
- By using the `mesh` function.

Here is the script (`plot_ex19.m`) that computes the given electric potential field problem and plots its results.

```
%% plot_ex19.m
% q1, q2 are charges of the particles in coulombs (C)
% r1, r2 are distances of the charges from the point in meters
% epsilon is permittivity of free space
% r1,r2, V(r1,r2) are coordinate systems for plotting
close all
[r1, r2] = meshgrid(-0.25:0.01:.25);
epsilon  = 8.854e-12;q1 = 2e-10; q2=4e-10;
coeff    = (1./(4*pi*epsilon));
V = coeff.*(q1./r1 + q2./r2);
figure(1) % Surface plot
surface(r1,r2,V); xlabel('r1'); ylabel('r2'); zlabel('V(r1,r2)')
title(['Fig. 1. Electric potential field', ...
'of two charged particles with surface plot'])
grid on, view(-15,15), axis tight, colormap Jet
figure(2) % Meshed plot with contour
mesh(r1,r2,V); xlabel('r1'); ylabel('r2'); zlabel('V');
title(['Fig. 2. Electric potential field', ...
'of two charged particles with mesh plot']);
axis vis3d; colormap hsv
```

After running this script, the 3D plots shown in Figure 6-26 and Figure 6-27 are obtained.

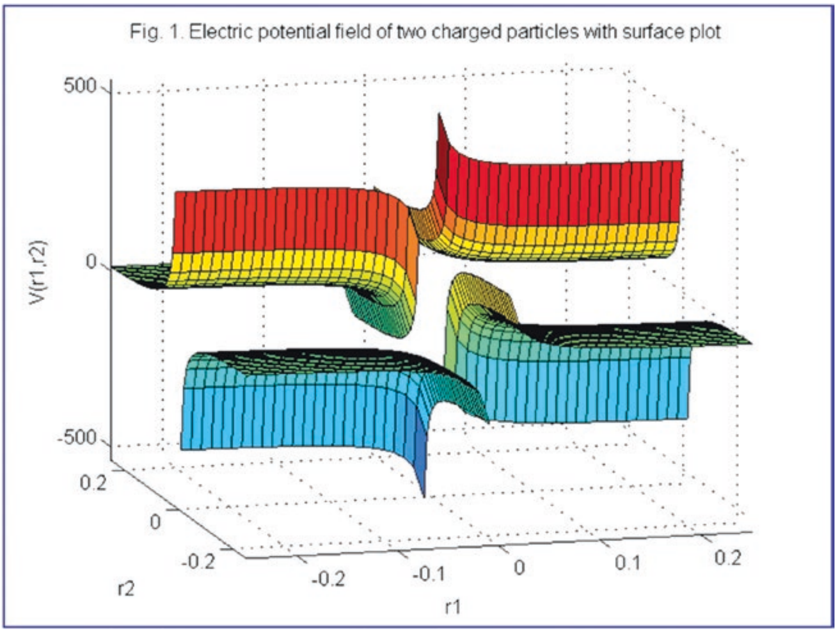


***Figure 6-26.** 3D plot surface plot of the potential field of the two charged particles*

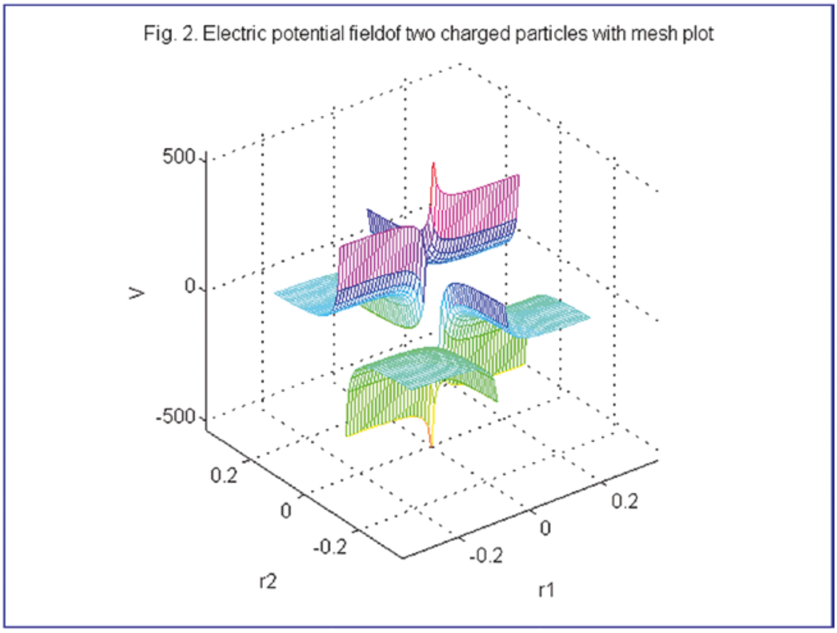


***Figure 6-27.** Mesh plot of the electric potential field of two charged particles*

Example 20: Creating 3D Plots with waterfall() and ribbon()

Given $G(t,\theta) = \ln(|\cos(0.5t + 5\theta) + \cosh(5t + 0.5\theta)|)$, $t = -0.2\pi \ldots 0.2\pi$; $\theta = -0.2\pi \ldots 0.2\pi$.

We define function variables as linearly spaced vectors (vector space) or arrays and then compute their function values according to the defined arrays. After that, you can build plots of the computed data. Here is the complete solution script (plot_ex20.m).

```
% plot_ex20.m

[t, theta]=meshgrid(linspace(-0.2*pi, 0.2*pi, 50));
G=log(abs(cos(0.5*t+theta*5)+sinh(5*t+0.5*theta)));
figure(1), waterfall(t,theta,G);
xlabel('t-axis'); ylabel('\theta-axis');
zlabel('G(t,\theta) function values');
title(['3D surface-waterfall plot of',... 'G(t,\theta)=ln(abs((cos(0.5*t+\
theta*5)+sinh(5*t+0.5*\theta)))']);
figure(2)
ribbon(G), title('3D ribbon plot of the function G(t, \theta)');
xlabel('t-axis'); ylabel('\theta-axis');
zlabel('G(t,\theta) function'); axis vis3d
```

By executing the script (plot_ex20.m), you get the plots of the $G(t,\theta)$ function shown in Figure 6-28 and Figure 6-29.

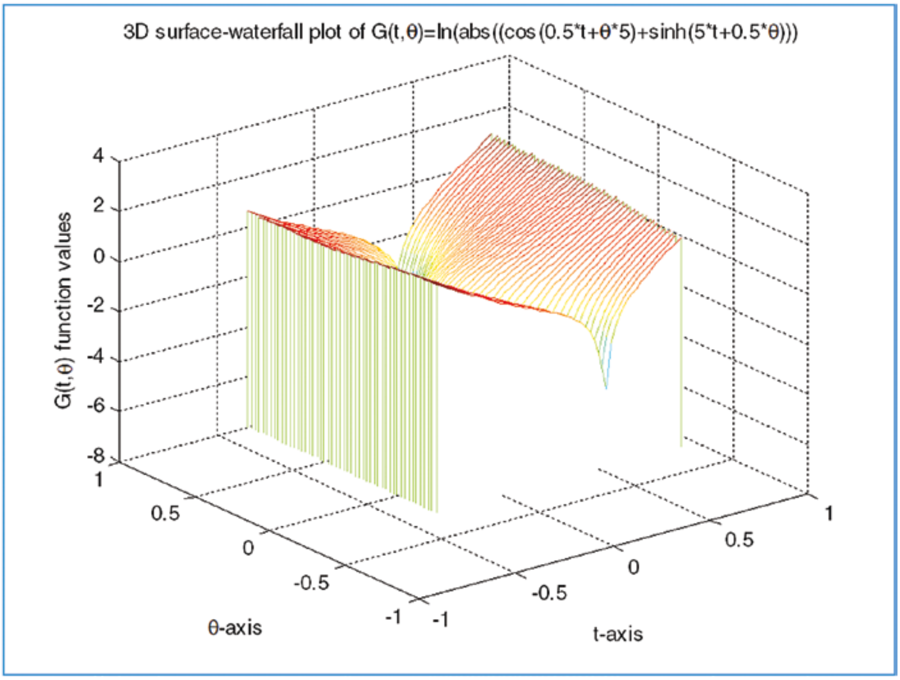


Figure 6-28. *3D surface-waterfall plot of $G(t, \theta) = ln3(|cos(0.5t + 5\theta) + cosh(5t + 0.5\theta)|)$*

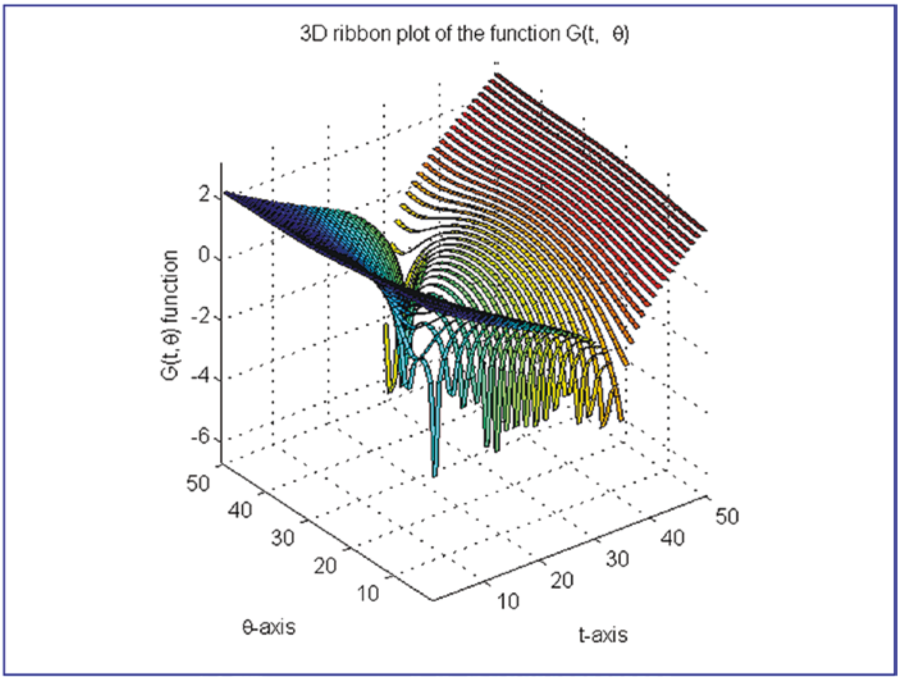


Figure 6-29. *3D ribbon plot of $G(t, \theta) = ln3(|cos(0.5t + 5\theta) + cosh(5t + 0.5\theta)|)$*

With 3D plot tools of surfaces, we have demonstrated the `mesh`, `surf (surface)`, `waterfall`, and `ezsurf, ribbon` plot functions. In addition to these, they `contour()`, `image()`, and `imagec()` functions can be employed in a similar manner.

3D Line Plots and Animations

There are several 3D space line plot tools (`plot3`, `comet3`, and `scatter3`), which are used to plot vector data points and space curves. The given vector data can be embedded in higher dimensions. 3D plotting methods are similar to plot tools employed in plot equations in 2D plots, which were described previously. Plot functions for 3D curves are `plot3`, `comet3`, `ezplot3`, and `scatter3`. They are implemented very much like the 2D plot functions, such as `plot` and `comet`. The `comet` and `comet3` functions draw plots by evolving them in action/animation. Let's consider the next example.

Example 21: Building 3D Line Plots and Animated 3D Line Plots with plot3(), comet3(), and ezplot3()

A popular amusement park ride known as the corkscrew has a helical shape. The parametric equations for a circular helix are defined by $x = a \cos t$; $y = a \sin t$; $x = bt$, where a is the radius of the helical path and b is a constant that determines the "tightness" of the path. In addition, if $b > 0$, the helix has the shape of a right-handed screw. If $b < 0$, then the helix is left-handed.

This example creates the three-dimensional plot of the helix in the following three cases and compares their appearance. Use $0 \leq t \leq 10\pi$ and $a = 1$.

a) $b = 0.1$; b) $b = 0.2$; c) $b = -0.1$;

```
% plot_ex21.m - Amusement ride - corkscrew plot in 3D
close all
t=0:pi/15:10*pi; a=1; x=a.*cos(t); y=a.*sin(t);
% case (a)
b1=0.1; z1=b1.*t;
% case (b)
b2=0.2; z2=b2.*t;
% case (c)
b3=-0.1; z3=b3.*t;
```

```
subplot(311); plot3(x, y, z1, 'r*-'); legend('b_1=0.1')
title('Corkscrew: b_1=0.1; b_2=0.2, b_3= -0.1');
subplot(312); plot3(x, y, z2, 'bs-'); legend('b_2=0.2')
subplot(313); plot3(x, y, z3, 'ko--');legend('b_3=-0.1')
xlabel('X'); ylabel('Y'); zlabel('Z');
figure(2); X=[x,x,x]; Y=[y,y,y]; Z=[z1,z2,z3];
comet3(X,Y,Z); % Animated plot
%% Alternative animated plot with ezplot
figure(3);
ezplot3('cos(t)','sin(t)', '0.1*t', [0,10*pi], 'animate'); hold on
ezplot3('cos(t)', 'sin(t)', '0.2*t', [0, 10*pi], 'animate'),
ezplot3('cos(t)', 'sin(t)', '-0.1*t', [0, 10*pi], 'animate')
```

After executing the script called `Amusement3D.m`, we obtain the plots displayed in Figure 6-30.

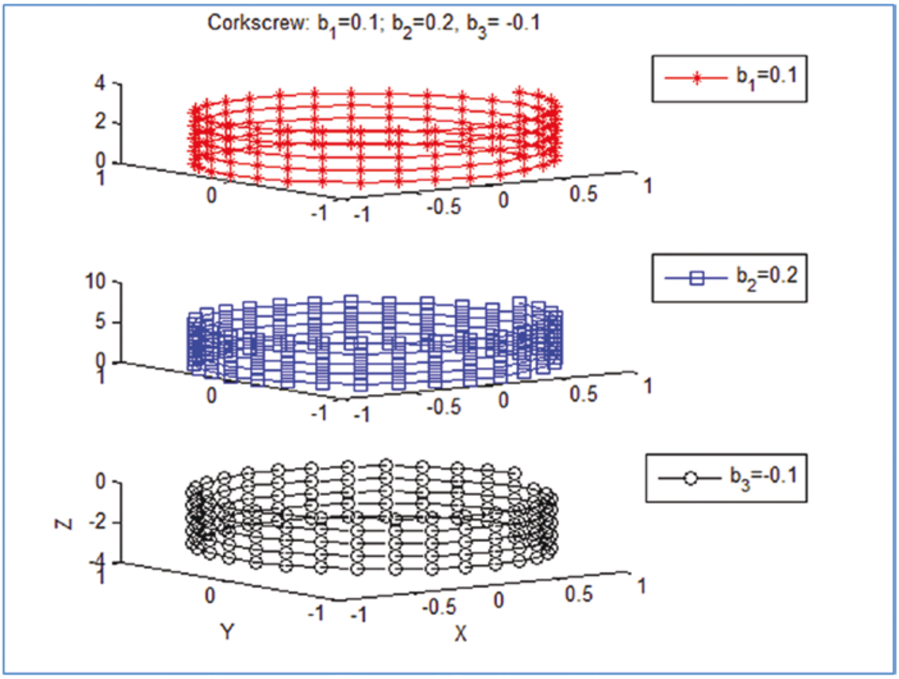


Figure 6-30. *Simulation plot of corkscrew amusement park rides $b_1 = 0.1$, $b_2 = 0.2$, $b_3 = -0.1$.*

With the 3D parametric curve plotter tools comet3 and ezplot3 shown in the previous script, we can obtain a simulation of the corkscrew amusement park ride in action. The comet3 and ezplot3 'animate' tools display simulated animation plots. There are also other 3D line and surface plot tools, including quiver, compass, feather, scatter3, stem3, and contour3.

Animated Plots

Plots can be animated with two functions, getframe and movie. There is also another tool called drawnow that can used to demonstrate this simulation process. It is a straightforward way to employ these tools and can be incorporated with other plot tools. Let's look at several exercises to see how to use these functions.

Example 22: Building an Animated Plot with getframe()

Given the function $f(x,y)=\log_{10}(|x|)tan\left(\frac{0.5y}{180}\right)$, $x=-13 \ldots 13; y=-13\ldots 13.$

We employ the getframe and movie tools to obtain an animated plot with the following script (plot_ex22.m).

```
% plot_ex22.m
for m=1:20
     [x, y]=meshgrid(linspace(-13, 13, 200));
     z=log10(abs(x)).*tan(m*y*pi/180);
     mesh(x, y, z)
     M(m)=getframe(gcf);
end
title({['3D surface-contour plot of the function' ],...
['f(x, y)=log10(abs(x))tan(0.5*y*\pi/180)']});
xlabel('X-axis'); ylabel('Y-axis');
zlabel('f(x, y) function values');
movie(M,2) % The plot movie is played twice
```

Example 23: Building an Animated Plot with drawnow

Given a function $f(t) = sin(t)\sqrt{\left(\left|\cos(t)\right|\right)},\ g(t) = cos(t)\sqrt{\left(\left|\sin(t)\right|\right)}$, $x = 0 \ldots 3\pi$;

For this example, we employ the `drawnow` function and write the following script (`plot_ex23.m`) to obtain a 2D animated plot.

```
% plot_ex23.m
for t=linspace(-2*pi, 2*pi, 200)
    f=sin(t)*sqrt(abs(cos(t)));
    g=cos(t)*sqrt(abs(sin(t))); drawnow
    plot(t,f, 'o-',t, g, 'mp--')
    hold all % used for colorful markers
end
title('Animated plot of the function')
xlabel('X-axis'); ylabel('Y-axis'); legend('f(t)', 'g(t)')
```

The `drawnow` command works very well with the plot (property) handle and the `refreshdata` commands.

Example 24: Building an Animated Plot with drawnow

Given a function $y(t) = \ sinc\,(e^{\sin(kt)}),\ \ t = 0 \ldots 5\pi,\ k = [1, 2]$.

```
% plot_ex24.m
t= 0:pi/100:5*pi; y = sinc(exp(sin(t)));
for k = 1:.01:2
    y = sinc(exp(sin(t.*k))); H = plot(t,y);H.LineStyle = ':';
    H.Color = [1 0 1];
    set(H, 'marker','o');
    set(H, 'markerfacecolor', [0 1 1]);
    set(H, 'markersize', 13);
    refreshdata(H,'caller')
    drawnow; pause(.005), shg
end
```

Example 25: Building an Animated Plot of a Projectile with getframe()

The height and speed of a projectile (such as a thrown ball) launched at a speed of v_0 at an angle θ to the horizontal are given by

$$h(t) = v_0\, t\sin(\theta) - 0.5gt^2$$

$$v(t) = \sqrt{v_0^2 - 2\,v_0 gt\sin(\theta) + g^2t^2}$$

Where g is the acceleration due to gravity. The projectile will strike the ground when $h(t) = 0$, which gives the time to hit $t_{hit} = \frac{2v_0\sin(\theta)}{g}$. Suppose that $\theta = 30^0$, $v_0 = 40\ m/s$ and $g = 9.81\ m/s^2$. Let's animate the ball's trajectory with getframe (movie), drawnow by choosing a sufficiently small step size for time.

Here is the solution script (plot_ex25.m).

```
% plot_ex25.m
% Study the projectile trajectory and velocity
clearvars; clc; close all
% Given data:
g=9.8;      % acceleration due to Earth gravity in m/s^2
theta=30;   % angle to the horizontal axis in degrees
v0=40;      % speed of the thrown ball in m/s
% t_hit is time needed for the thrown ball to hit the ground.
% h is height of the thrown ball relative to the ground in m.
% v is speed of the thrown ball over time.
% t is a time series for computation in [sec].
% dt is increment of total time.
t_hit=2*v0*sin(theta*pi/180)/g;
dt=t_hit/80; t=(0:dt:t_hit);
h=v0*t.*sin(theta*pi/180) - 0.5*g*t.^2;
v=sqrt(v0*v0-2*v0*g*t.*sin(theta*pi/180)+g*g*t.^2);
%% Animated plot of the projectile
dt=t_hit/100; t=(0:dt:t_hit);
for ii=1:length(t)
```

```
h(ii)=v0*t(ii).*sin(theta*pi/180) - 0.5*g*t(ii).^2;
plot(t(ii), h(ii), 'o:', 'markerfacecolor', 'y')
hold all
M(ii)=getframe(gcf);
end
% movie(M, 1)
%% Animated plot of the projectile velocity
for ii=1:length(t)
v(ii)=sqrt(v0^2-2*v0*g*t(ii).*sin(theta*pi/180)+g^2*t(ii).^2);
drawnow
plot(t(ii), v(ii), 'o:', 'markerfacecolor', 'c'), hold all
end
```

Summary

In this chapter, a few essential plot tools and functions were discussed. In particular, we demonstrated, with examples, how to employ the following plot commands and functions:

- 2D plot commands:
  - plot, plotyy, semilogx, semilogy, loglog, hist, title, axis, legend, grid, xlabel, ylabel, fplot, ezplot, xlim, and ylim
- 3D plot commands:
  - ezsurf, surface, meshc, meshgrid, ribbon, waterfall, plot3, comet3, colormap, view, and zlabel
- Additional plot-related commands:
  - subplot, close all, axis, axis vis3d, set, gcf, grid, hold, legend, clf, figure, title, text, gtext, and ginput
- Animated plot commands:
  - getframe, movie, refreshdata, and drawnow

Here are a few of the most common errors that occur while plotting:

- One of the most common errors made while plotting measured or computed data is that we try to plot two data sets with different sizes (e.g., as input vs. output). A plot cannot be obtained because of the mismatched data points. Therefore, we need to be careful about the size of our data sets when plotting them in an x vs. y plot.
- We sometimes confuse or do not assign legends correctly, and as a result, we get incorrect information from our plotted data sets.
- When we are dealing with large data sets composed of many rows and columns of data, we sometimes tend to confuse rows with columns and vice versa.
- Sometimes we try to plot complex numbers; however, MATLAB plots only the real part of complex numbers unless we plot them separately.

Exercises for Self-Testing

Exercise 1

Plot y *vs.* $f(y)$ of the following polynomial for $y = -3 \ldots 2$.

$$y^5 + 5y^4 + 3y^3 - 10y^2 - \left(10^{0.5}\right) = f(y)$$

1. Plot y vs. $f(y)$ with a 2.0 width solid line (style) in magenta, and with markers in a rectangle in yellow.
2. Add a plot title: '$y^5 + 5y^4 + 3y^3 - 10y^2 - (10^{0.5}) = f(y)$', axis labels, and grids.
3. Locate f_{min} by using `ginput`.
4. Add text (f_{min} =) and the value from Step 3 to the plot area.

Exercise 2

Plot the polynomial given in Exercise 1 using the `fplot` function:

1. Find the minimum values of the polynomial by using `ginput`.
2. Plot the found values (minimums) with a diamond marker and circle in blue and red, respectively.

Exercise 3

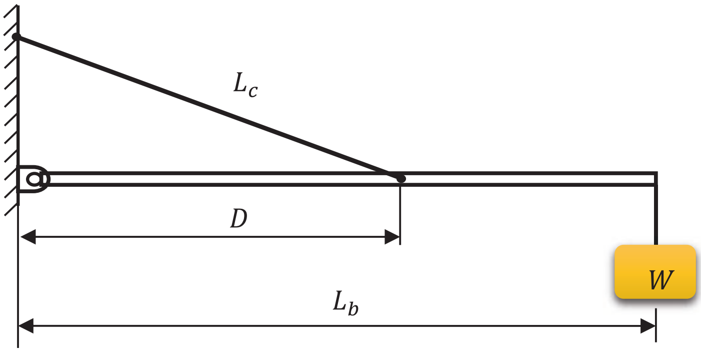
Given $f_1(y) = y^5 + 5y^4 + 3y^3 - 10y^2 - 10^{0.5}$

$$f_2(y) = 0.001y^5e^5 + 0.01y^3e^4 + 0.0125y^2e^3 - 0.0125y^1e^2 - 0.1\,e^{0.5}$$

1. Plot the two functions $f_1(y)$ and $f_2(y)$ in one plot area for $-3 \le y \le 2$ by using the `fplot` and `plot` functions. Compare the results of the two approaches. Insert all the necessary information (such as plot title, axis label, grid, line width, marker type, color, and so forth) to make the plot legible and informative for analysis.
2. Plot these two functions: y vs. $f_1(y)$ and y vs. $f_2(y)$ in one plot area for $-1 \le y \le 1$ by using `plotyy` or `yaxis` right/left so that $f_1(y)$ and $f_2(y)$ are in two separate vertical axes.
3. Locate the local minima of both plotted $f_1(y)$ and $f_2(y)$ functions in Step 2 using `ginput`.
4. Plot local minimum values found in Step 3 in the plot of Step 2.

Exercise 4

A cable of length *C* supports a beam of length *B*, so that it is horizontal when the weight *W* is attached to the beam end. The tension force *T* in the cable is given by $T = \frac{CBW}{D\sqrt{B^2 - D^2}}$, where *D* is the distance of the cable attachment point to the beam pivot.



1. Use the $W = 777\ N$, $B = 2.33\ m$, and $C = 3.77\ m$ element-wise operations and the function `min()` (Hint: `min(X, Y)`) to compute the value of D that minimizes the tension value.
2. Check the sensitivity of the solution by plotting T vs. D. How much can D vary from its optimal value before the tension T increases 13% above its minimum value?

Exercise 5

1. Use MATLAB to solve the following equations for x, y, and z as functions of the parameter c.:

$$x - 5y - 2z = 11c$$

$$6x + 3y + z = 13c$$

$$7x + 3y - 5z = 10c$$

2. Plot the solutions for x, y,and z versus c on the same plot, for $-10 \le c \le 10$. Note that the incremental change of c is 1.

Exercise 6

Plot the following two polynomials over the interval of $-6 \leq x \leq 13$.

$$f(x) = 0.003x^4 - 1.5x^3 - 13x^2 - 0.08x + 13;$$

$$h(x) = -0.03x^3 - 1.5x^2 - 0.04x + 9.0;$$

1. Plot *f*(*x*) *vs*. *x* in a separate plot and put a grid on the plot.
2. Using the `ginput` function, determine the coordinates of the peaks of the curves and add this information using `text` and `gtext`.

Exercise 7

Compute the following formulation: $h(\alpha) = \sin(\alpha)/\alpha$, where $\alpha = 0 : \frac{\pi}{40} : \pi$. Compute the values of *π* from the following formulations—(a) developed by Srinivasa Ramanujan in 1910 and (b) developed by Borwein-Plouffe in 1995:

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{m} \frac{(4k)!\, 1103 + 26390k}{(k!)^4\, 396^{4k}} \qquad (a)$$

$$\pi = \sum_{k=0}^{n} \frac{1}{16^k} \left[\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right] \qquad (b)$$

1. Compute *π* from (a) for m = 100, plot *α vs*. *h*(*α*)
2. Compute *π* from (b) for n = 10, plot *α vs*. *h*(*α*)
3. Build animated plots from Steps 1 and 2 using `drawnow` and `getframe + movie`.

Exercise 8

The volume *V* and paper surface area *A* of a conical paper cup are given by

$$V = \frac{1}{3}\pi r^2 h \text{ and } A = \pi r \sqrt{r^2 + h^2}$$

Where *r* is the radius of the base of the cone and *h* is the height of the cone.

1. By eliminating *h*, obtain the expression for *A* as a function of *r* and *V*.
2. Create a user-defined function that accepts *r* as the only argument and computes *A* for a given value of *V*. Declare *V* to be global within the function.

Exercise 9

A torus is shaped like a doughnut. If its inner radius is *a* and its outer radius is *b*, its volume and surface area are given by

$$V = \frac{1}{4}\pi^2(a+b)(b-a)^2$$

$$A = \pi^2\left(b^2 - a^2\right)$$

1. Create a user-defined function that computes *V* and *A* from the arguments *a* and *b*.
2. Suppose that the outer radius is constrained to be 2 inches greater than the inner radius. Write a script file that uses your function to plot *V* and *A* vs. a for $0.25 \le a \le 4$ [*in*].

Exercise 10

Create four anonymous functions to represent the function $6e^{3\cos x^2}$, which is composed of the functions $h(z) = 6e^z$, $g(y) = 3\cos y$ and $f(x) = x^2$. Use `fplot` to plot $6e^{3\cos x^2}$ over the range of $0 \le x \le 4$.

Exercise 11

Create an anonymous function for $20\,x^2 - 200x + 3$ and use it to plot the function in order to determine the approximate location of its minimum using `ginput`.

Exercise 12

Find the approximate roots of the equation $x^3 - 3x^2 + 5x\sin\left(\frac{\pi x}{4} - \frac{5\pi}{4}\right) + 3 = 0$, by plotting the equation with `fplot` and then using `ginput`.

Exercise 13

To compute the forces in structures, sometimes we must solve equations similar to the following. Use the `fplot` function to find all positive roots of this equation: $x \tan x = 7$.

Exercise 14

Cables are used to suspend bridge decks and other structures. If a heavy uniform cable hangs suspended from its two endpoints, it takes the shape of a catenary curve whose equation is $y = a\cosh\left(\frac{x}{a}\right)$. Where a is the height of the lowest point on the chain above some horizontal reference line, $x$is the horizontal coordinate measured to the right from the lowest point, and $y$is the vertical coordinate measured up from the reference line.

1. Let $a = 10\ m$ and plot the catenary curve for $-20\ x \leq 30\ [m]$. How high is each endpoint?
2. Let $a = 10w\ him$ with increment of 1m and plot the catenary curve for $-20\ tx \leq 30\ [m]$ using the loop control statements and automatic labeling tools of plotted data points with strings.
3. Build an animated plot of the computation results from Step 2 using `drawnow`.

Exercise 15

When a belt is wrapped around a cylinder, the relation between the belt forces on each side of the cylinder is as follows:

$$F_1 = F_2 e^{\mu\beta}$$

Where β is the angle of the wrap of the belt and μ is the friction coefficient. Write a script file that first prompts a user to specify β, μ and F_2 and then computes the force, F_1. Test your program with the values $=130^0$, $\mu = 0.3$ and $F_2 = 100$ N. The output force (F_1) has to be output in N in the Command Window. (Hint: Be careful with β! .)

1. Plot F_1 *vs.* β for $\beta = 30^0 \ldots 3o0^0$ for 30 linearly spaced data points.
2. Add the plot title, axis labels, marker type (pentagram in cyan), and marker size (8.0), and then set a line width at 1.50.
3. Add text to the plot area "F_1 is dependent on wrap angle β!" with a font size of 13.
4. Build an animated plot of the output force (F_1) for $\beta = 30^0 \ldots 3f0^0$ for 100 linearly spaced data points, $\mu = 0.1..0.3$ and $F_2 = 100$ N.

Exercise 16

Using estimates of rainfall, evaporation, and water consumption, the town engineer developed the following model of the water volume in the reservoir as a function of time.

$$V(t) = 10^9 + 10^8\left(1 - e^{-\frac{t}{100}}\right) - 10^7 t$$

Where V is the water volume in liters and t is time in days. Plot $V(t)$ versus t. Use the plot to estimate how many days it will take before the water volume in the reservoir is 50 % of its initial volume, $10^9 L$.

Exercise 17

Plot columns 2 and 3 of the following matrix A versus column 1. The data in column 1 is time (seconds). The data in columns 2 and 3 is force in N. Use the matrix axis mode to display the y-axis values in reverse order.

$$A = \begin{bmatrix} 0 & -8 & 6 \\ 5 & -4 & 3 \\ 10 & -1 & 1 \\ 15 & 1 & 0 \\ 20 & 2 & -1 \end{bmatrix}$$

Exercise 18

In certain kinds of structural vibrations, periodic force acting on the structure will cause the vibration amplitude to repeatedly increase or decrease with time. This phenomenon, called *beating*, also occurs in musical sounds. A particular structure's displacement is described by:

$$y(t)=\frac{1}{f_1^2-f_2^2}\left[\cos(f_2 t)-\cos(f_1 t)\right]$$

Where y is the displacement in inches and t is the time in seconds. Plot y versus t over the range $0 \le t \le 11\pi$ for $f_1 = 13\frac{rad}{sec}$ and $f_2 = 1.5\frac{rad}{sec}$. Make sure to select enough points to obtain an accurate plot of the process.

Exercise 19

A robot rotates about its base at two revolutions per minute while lowering its arm and extending its hand. It lowers its arm at the rate of 120^0 per minute and extends its hand at the rate of 5 *m*/*min*. The arm is 0.5 [*m*] long. The *xyz* coordinates of the hand are given by:

$$x=(0.5+5t)\sin\left(\frac{2\pi t}{3}\right)\cos(4\pi t)$$

$$y=(0.5+5t)\sin\left(\frac{2\pi t}{3}\right)\sin(4\pi t)$$

$$z=(0.5+5t)\cos\left(\frac{2\pi t}{3}\right)$$

Where t is time in minutes.

1. Obtain a 3D plot of the path of the hand for $0bt \le 0.2$ [*min*].
2. Simulate the arm's trajectory using `comet3` and `drawnow` and analyze which tool results in better visualization.

Exercise 20

Obtain surface and contour plots for the function $z = Ox^2 + 2\,xy + 3y^2$. This surface has the shape of a saddle. At its saddle-point at x = y = 0, the surface has zero slope, but this point does not correspond to either a minimum or a maximum. What types of contour lines correspond to the saddle-point?

Exercise 21

The following function describes oscillations in some mechanical structures and electric circuits:

$$z(t,\tau)=e^{-\frac{t}{\tau}}\sin(\omega t+\varphi)$$

In this function, t is time and ω is the oscillation frequency in radians per unit time. The oscillations have a period of $\frac{2\pi}{\omega}$, and their amplitudes decay in time at a rate determined by τ, which is called the *time constant*. The smaller τ is, the faster the oscillations die out.

Suppose that $\varphi = 0$, $\omega = 2.5$ and τ can have values in the range of $0.5n\tau \leq 10$ *sec*. Then the proceeding equation becomes

$$z(t,\tau)=e^{-\frac{t}{\tau}}\sin(2t)$$

Obtain a surface plot by `waterfall` and a contour plot of this function by `contour` to help visualize the effect of τ for $0.5 \leq t \leq 10$ *sec*. Label the axes by function name $z(t, \tau)$, with variable names of t and τ.

Exercise 22

The following equation describes the temperature distribution in a flat rectangular metal plate. The temperature on three sides is held constant at T_1 and T_2 on the fourth side (see the following figure). The temperature $T(x, y)$ as a function of the xy coordinates shown is given by

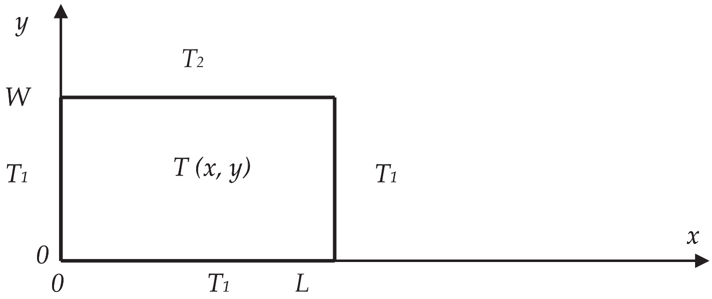
$$T(x,y)=(T_2-T_1)w(x,y)+T_1$$

Where

$$w(x,y)=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{2}{n}\sin\left(\frac{n\pi x}{L}\right)\frac{\sinh\left(\frac{n\pi y}{L}\right)}{\sinh\left(\frac{n\pi W}{L}\right)}$$

The given data for this problem is $T_1 = 22^0C$, $T_2 = 75^0C$, $W = L = 3\ [m]$.

Using a spacing of 0.05 for x and y, generate a surface mesh plot and a contour plot of the temperature distribution.



Exercise 23

Given a function $f(x,y)=\log_{10}(|x|)tan\left(\frac{0.5y}{180}\right)$, $x = -13...13$; $y = -13...13$;

Generate a 3D plot of $f(x, y)$ by using `meshc, ribbon`.

Exercise 24

Given a function $H(\theta, \varphi) = \log_{10}(|\theta|)\ tan(0.5\varphi)$, $\theta = -\pi...\pi$; $\varphi = -\pi...\pi$;

1. Plot the given function $H(\theta, \varphi)$ by using the 3D `surface` and `waterfall` plot functions on two subplots.

2. Build a 3D animated plot of $H(\theta, \varphi) = \log_{10}(|\theta|)\ tan\ (a * \varphi)$, with $\theta = -\pi...\pi$; $\varphi = -\pi...\pi$; and $a = -1.5..1.5$.

Exercise 25

Given these functions $h1 = cos\left(2t\right); h2 = sin\left(3t\right); h3 = cosh\left(\frac{t}{10}\right); t = 0\ldots 8\pi; \Delta t = \frac{\pi}{10}$.

1. Plot the given functions with `plot3` and `comet3`.
2. Build an animated plot of the given function by using `getframe` and `movie`.
3. Build an animated plot of the given function by using `drawnow`.

CHAPTER 7

Linear Algebra

This chapter includes a short introduction to linear algebra. It discusses some of the essential approaches to solving systems of linear equations, as well as various matrix operations (matrix inverse, determinant, sum, subtraction, division, multiplication, power, exponential, elementwise and array-wise operations, and so forth). It covers eigen-value problems and matrix factorizations/decompositions, such as Cholesky, Schur, LU, QR, and Singular Value Decomposition. It also includes built-in functions and scripts in MATLAB and Simulink models. Moreover, the chapter explains standard matrix generator functions of MATLAB, creating vector spaces, solving polynomials, and logical indexing of matrices, via examples in MATLAB and Simulink.

Introduction to Linear Algebra

Linear algebra is one of the more important branches of mathematic. It deals with vectors, vector spaces, linear spaces, matrices, and systems of linear equations. There is a wide range of linear algebra applications in engineering and scientific computing, including many fields of natural and social studies. Linear algebra starts with a system of linear equations for underdetermined, over-determined, and well-defined systems. If a given system is composed of m-linear equations with n-unknowns and $m \geq n$, that is solvable for unknowns. Consider the following linear system, formulated by the system of equations (Equation 7.1).

$$\begin{cases} a_{11}x_{11} & +\dots & +a_{1n}x_{1n} & =b_1 \\ \vdots & \vdots & \vdots & =\vdots \\ a_{m1}x_{m1} & +\dots & +a_{mn}x_{mn} & =b_m \end{cases} \qquad \text{(Equation 7.1)}$$

S. Eshkabilov, *Beginning MATLAB and Simulink*, https://doi.org/10.1007/978-1-4842-5061-7_7

The system of linear equations (Equation 7.1) is solvable directly for all cases when $m \geq n$. If $m < n$, there are more unknowns than the number of linearly independent equations, and such a system is called underdetermined and not solvable directly. If If $m > n$, there are more linearly independent equations, and such a system is called overdetermined and is solvable directly.

For the sake of simplicity, let's take $m = n$ and rewrite it (Equation 7.1):

$$\begin{cases} a_{11}x_1 & +\dots & +a_{1n}x_n & = b_1 \\ \vdots & \vdots & \vdots & = \vdots \\ a_{n1}x_1 & +\dots & +a_{nn}x_n & = b_n \end{cases} \qquad \text{(Equation 7.2)}$$

The given system of linear equations in Equation 7.2 can also be written in a matrix notation form.

$$[A]*\{X\}=[B] \qquad \text{(Equation 7.3)}$$

Where:

$$[A]=\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ : & \cdots & : \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}, \ [B]=\begin{bmatrix} b_1 \\ : \\ b_n \end{bmatrix}, \{X\} = \{x_1 \ \dots \ x_n\}.$$

Equation 7.3 can also be rewritten in the form of column matrices:

$$x_1\begin{bmatrix} A_1 \\ \vdots \\ A_{n1} \end{bmatrix}+\dots+x_n\begin{bmatrix} A_{1n} \\ \vdots \\ A_{nn} \end{bmatrix}=\begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} \qquad \text{(Equation 7.4)}$$

Where A and B are matrices and X is a vector of unknowns.

The system in Equation 7.3 for X (unknowns) can be solved with the next formulation.

$$\{X\}=[A]^{-1}*[B] \qquad \text{(Equation 7.5)}$$

Where $[A]^{-1}$ is the inverse of the matrix [A].

Matrix Properties and Operators

Matrices have several important properties and operators, such as determinant, diagonal, transpose, inverse, singularity, rank, and so forth.

The *determinant* of a matrix can be computed only if the given matrix is a square. For example:

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

The determinant of M will be computed with the following expression:

$$det(M) = aei + bfg + dhc - ceg - dbi - hfa$$

The MATLAB command for the determinant computation is `det()`. For example:

```
>> A =[      8      1      6;      3      5      7;      4      9      2]
A =
     8     1     6
     3     5     7
     4     9     2
>> det(A)
ans =
  -360
```

The *diagonal* of a matrix is composed of its element along its diagonals. For example, in the previous example, the diagonals are *aei* and *ceg*.

The MATLAB command for diagonal separation is `diag()`. For example:

```
>> A =[      8      1      6;      3      5      7;      4      9      2];
>> diag(A)
ans =
     8
     5
     2
```

The *transpose* of a matrix can be determined by the counter-clockwise rotation of a matrix by 90^0 (degrees). The transpose properties are as follows:

$$\left(M^T\right)^T = M$$

$$\left(M+B\right)^T = M^T + B^T$$

$$\left(kM\right)^T = kM^T$$

$$\left(MB\right)^T = B^T M^T$$

$$\left(M^{-1}\right)^T = \left(M^T\right)^{-1}$$

Where M and B are matrices of the same size, k is a scalar, and ^{T} and $^{-1}$ are the transpose and inverse operators.

The MATLAB command for the transpose operation is `transpose(), '`.
For example:

```
>> A =[      8     1     6;       3     5     7;       4     9     2];
>> transpose(A)
ans =
     8     3     4
     1     5     9
     6     7     2
>> A'
ans =
     8     3     4
     1     5     9
     6     7     2
```

Simulink Blocks for Matrix Determinant, Diagonal Extraction, and Transpose

Simulink has blocks that you can use to compute the matrix determinant, extract the matrix diagonal elements, and obtain the matrix transpose. The Determinant block (`[det(A) (3x3)]`) is present in Simulink‘s Aerospace Blockset/Utilities/Math Operations and it has a constraint and can only compute the determinant of 3-by-3 matrices.

Note Note that the block [`det(A) (3x3)`] from the Aerospace Blockset is limited; it can only compute the determinant of 3-by-3 matrices.

The block to extract the diagonal elements of a matrix is available in the DSP System Toolbox/MATH Functions/Matrices and Linear Algebra/Matrix Operations. The block to compute the matrix transpose is present in Simulink/Math Operations and the block name is Math Function. It has a few math functions embedded in it, including `exp` (by default), `log`, `10^u`, `magnitude^2, square`, `pow`, and `transpose`. Any of these math functions in the Math Function block can be chosen. You simply click the Apply and OK buttons of the block, and the chosen math function becomes available. These three blocks are shown in Figure 7-1.

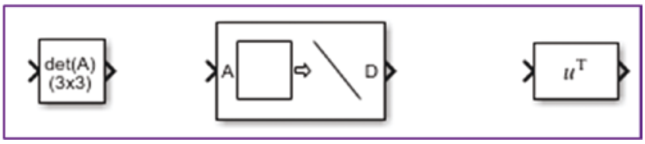


***Figure 7-1.** Simulink blocks used for determinant calculation, diagonal extraction, and transpose operation, from left*

These blocks have one input and one output port. Therefore, we need to add two additional blocks, one Constant block for input entry and one Display block to obtain/see the computation results. The Constant block can be taken from the Simulink Library Simulink/Sources or DSP System Toolbox/Sources. Similarly, the Display block can be taken from Simulink/Sinks or DSP System Toolbox/Sinks. Or alternatively, with the latest versions of MATLAB starting from 2018a, you can obtain all the necessary blocks by double-clicking (with the left mouse button) and typing the block name in the search box. As discussed in the previous chapters, in any Simulink model one signal source can be used as many times as necessary. There is no need to generate that signal within one model in order to use it with other blocks as an input signal. Moreover, to optimize the Simulink model, it is strongly advised you build a Simulink model with a fewer blocks in order to make your models more readable, comprehensive, and easy to edit. Therefore, this example uses one Constant block for input source [A]. Figure 7-2 shows the primary version of the Simulink model.

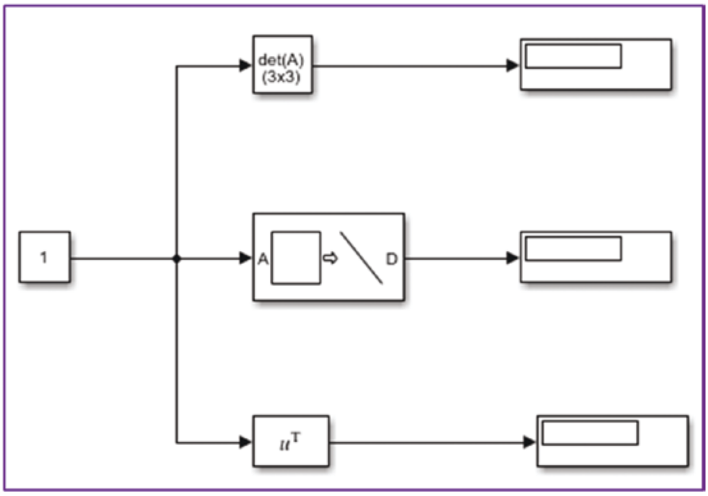
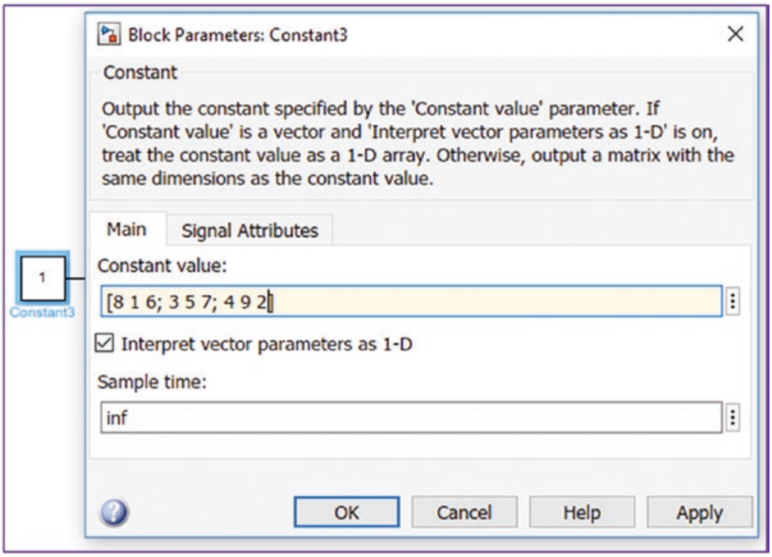


***Figure 7-2.** Simulink model to compute the determinant of a matrix, extract diagonal elements of a matrix, and perform a transpose on a matrix*

Let's use example matrix [A] to demonstrate these three Simulink blocks. The elements of the matrices [A] can be entered in two different ways:

- By typing all elements in the Constant block's Constant Value box, as shown in Figure 7-3. Click the Apply and OK buttons.



Block Parameters: Constant3

Constant

Output the constant specified by the 'Constant value' parameter. If 'Constant value' is a vector and 'Interpret vector parameters as 1-D' is on, treat the constant value as a 1-D array. Otherwise, output a matrix with the same dimensions as the constant value.

Main | Signal Attributes

Constant value: [8 1 6; 3 5 7; 4 9 2]

☑ Interpret vector parameters as 1-D

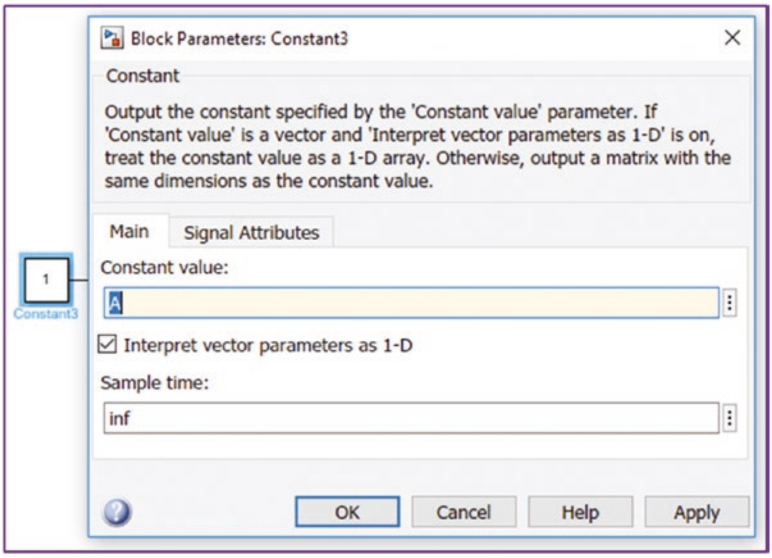
Sample time: inf

***Figure 7-3.** Entering matrix elements in a Constant block*

- By defining [A] via MATLAB's Command Window and workspace:

  ```
  >> A =[ 8  1  6;  3  5  7; 4  9  2];
  ```

  Then provide the variable name A in the Constant block's Constant Value box for [A], as shown in Figure 7-4.



Block Parameters: Constant3

Constant

Output the constant specified by the 'Constant value' parameter. If 'Constant value' is a vector and 'Interpret vector parameters as 1-D' is on, treat the constant value as a 1-D array. Otherwise, output a matrix with the same dimensions as the constant value.

Constant value: A

☑ Interpret vector parameters as 1-D

Sample time: inf

Figure 7-4. *Matrix [A], defined in the MATLAB workspace, called via the Constant block*

Then click the Apply and OK buttons. Note that we are not going to use the second method (see Figure 7-4) of defining matrix [A] elements in this example; it's just shown here for explanation purposes.

Finally, you'll get the complete model in which the matrix [A] elements are entered in the Constant block directly, as shown in Figure 7-3. After you complete the model, by pressing Ctrl+T from the keyboard or clicking the Run ▶ button in the Simulink model window, the complete model with its computed results will be created.

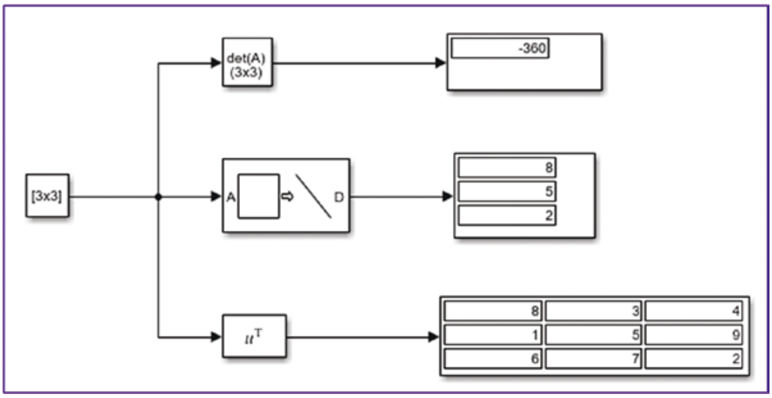


Figure 7-5. *Completed Simulink model that computes the determinant, extracts diagonal elements, and performs the transpose operation on the 3-by-3 matrix*

Note To see the simulation results in the Display block, it has to be resized/stretched. You left-click it and then drag with the mouse while holding the button.

The simulation results of the Simulink models match the ones from the MATLAB commands, such as. `det()`, `diag()`, and `transpose(),'`.

Matrix Inverse or Inverse Matrix

The *inverse matrix* has the following important property that

$$[A]*[A]^{-1}=[I]$$

where [*I*] is the identity matrix.

For example, $A=\begin{bmatrix}1 & 1\\ 3 & 4\end{bmatrix}$ has its inverse $A^{-1}=\begin{bmatrix}4 & -1\\ -3 & 1\end{bmatrix}$ and it's computed from the following:

$$A^{-1}=\frac{1}{\det(A)}\,adjugate(A)=\frac{1}{4*1-(-1*(-3))}*\begin{bmatrix}4 & -1\\ -3 & 1\end{bmatrix}=\begin{bmatrix}4 & -1\\ -3 & 1\end{bmatrix}.$$

The MATLAB command to compute the inverse of a matrix is `inv()`. For example,

```
>> A =[      8     1     6;      3     5     7;      4     9     2];
>> inv(A)
ans =
    0.1472   -0.1444    0.0639
   -0.0611    0.0222    0.1056
   -0.0194    0.1889   -0.1028
```

A given matrix is *singular* if it is square, if it does not have an inverse, and if it has a determinant of `0`.

Simulink Blocks for Inverse Matrix

The *matrix inverse* can also be calculated via several Simulink blocks with respect to a given matrix size, i.e., square matrix or rectangular. The inverse matrix or matrix inverse computing blocks are present in the DSP System and Aerospace Blockset Toolboxes of Simulink and can be accessed via the Simulink Library: the DSP System Toolbox/Math Functions/Matrices and Linear Algebra/Matrix Inverses, and the Aerospace Blockset/Utilities/Math Operations. Let's test the available blocks of this toolbox to compute the inverse of the matrix [A] shown in the previous example. Open a blank Simulink model and drag and drop the block from the libraries of the DSP System and Aerospace Blockset Toolboxes shown in Figure 7-6.

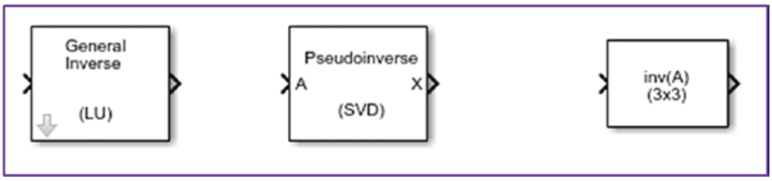


Figure 7-6. *Simulink blocks for computing the inverse matrix*

They are as indicated on the top of each block—General Inverse (LU), Pseudoinverse (SVD), and `inv(A)`—used to compute the matrix inverses based on LU factorization for square matrices, and pseudoinverse for rectangular matrices (i.e. `m>n`, or the number of rows is larger than the number of columns or vice versa). Theoretical aspects of the LU, SVD, and other matrix decomposition and transformation operations are highlighted in the subsection titled "Matrix Decomposition".

Note The block called [`inv(A) (3x3)`] from the Aerospace Blockset is limited in size and can only compute the inverse matrix of 3-by-3 matrices.

The three blocks have one input port for the entry matrix and one output port for the computed inverse. Add two additional blocks—one Constant block and one Display—by following the procedures. Figure 7-7 shows the primary version of the Simulink model.

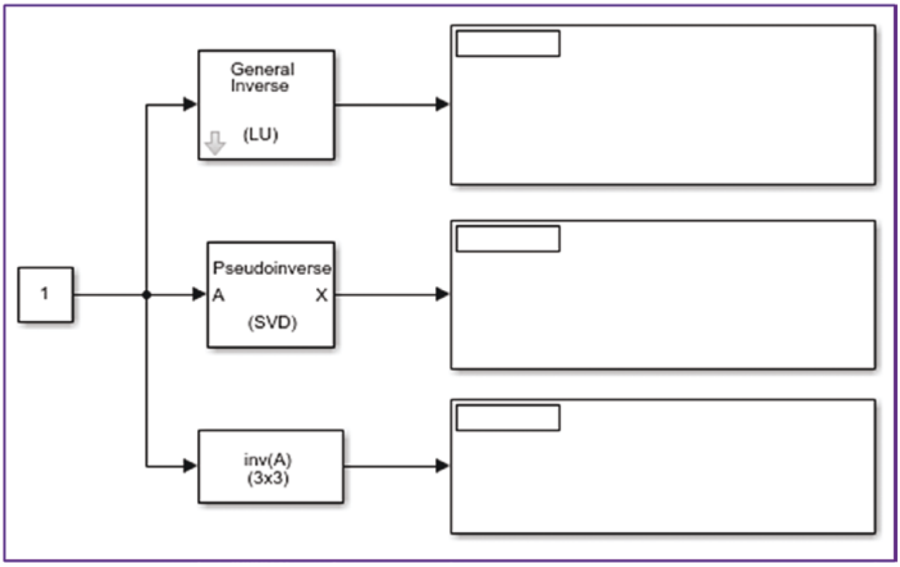
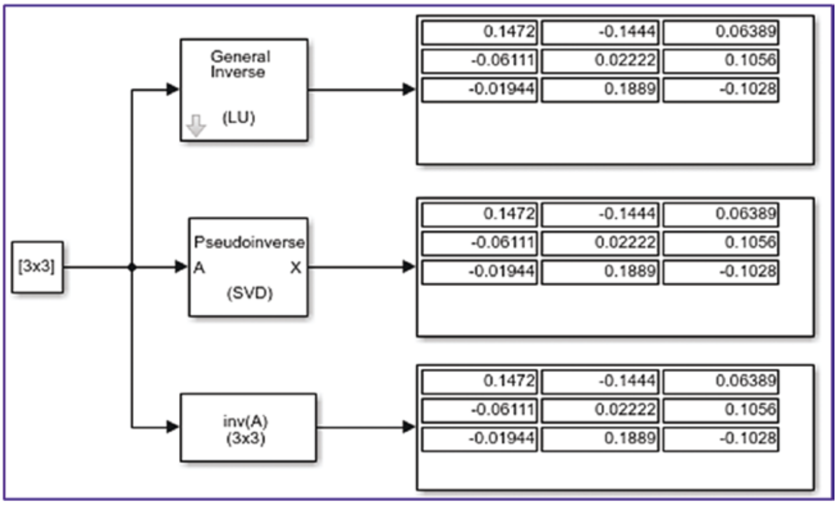


Figure 7-7. *Simulink model to compute the inverse matrix via three different blocks*

The elements of the matrices [A] can be entered in two ways: (1) by typing all the elements in the Constant block's Constant Value box and then clicking the Apply and OK buttons; or (2) by defining [A] via the MATLAB's Command Window and workspace.

Finally, you'll get the following complete model in which the matrix [A] elements are entered in the Constant block directly. After you complete the model, by pressing Ctrl+T from the keyboard or clicking the Run ▶ button in the Simulink model window, the finalized model with its computed results is created, as shown in Figure 7-8.



```
 0.1472   -0.1444   0.06389
-0.06111  0.02222   0.1056
-0.01944  0.1889   -0.1028
```

```
 0.1472   -0.1444   0.06389
-0.06111  0.02222   0.1056
-0.01944  0.1889   -0.1028
```

```
 0.1472   -0.1444   0.06389
-0.06111  0.02222   0.1056
-0.01944  0.1889   -0.1028
```

Figure 7-8. *The inverse matrix computed via three different blocks*

The computed inverse matrix (A^{-1}) values match the ones computed using MATLAB's `inv()` command, within four correct decimal places.

Another important operator of matrices is its *rank*. The *rank* of a matrix (e.g., [A]) is the maximum number of linearly independent row vectors of the matrix, which is the same as the maximum number of linearly independent column vectors. The [A] matrix is considered to have a full rank if its rank equals the largest possible for a matrix of the same dimensions. The [M] matrix is considered to be rank deficient if it does not have full rank. A matrix's rank determines how many linearly independent rows the system contains. The MATLAB command to compute the rank of a matrix is `rank()`. For example:

```
>> A =[8, 1, 6; 3, 5, 7; 4, 9, 2]; % Full rank matrix
>> rank(A)
ans =
     3
>> M =[8 0 6; -3, 0,  7; 0  0 2]   % Rank deficient matrix
M =
     8     0     6
    -3     0     7
     0     0     2
>> rank(M)
ans =
     2
```

Based on the rank, the systems (system matrices) can be full rank, overdetermined, and underdetermined.

Example 1: Solving a System of Linear Equations

The following example shows you how to solve a linear equation by using these formulations.

$$\begin{cases} 2x & +3y & +5z & =1 \\ -3x & -2y & +5z & = 2 \\ 4x & -7y & +6z & =3 \end{cases}$$

To solve this problem for unknowns, such as *x*, *y*, *z*, you apply Equations 7.3, 7.4, and 7.5 directly and then use the following operations.

$$\begin{bmatrix} 2 & 3 & 5 \\ -3 & -2 & 5 \\ 4 & -7 & 6 \end{bmatrix} * \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{, which can be rewritten as:}$$

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} 2 & 3 & 5 \\ -3 & -2 & 5 \\ 4 & -7 & 6 \end{bmatrix}^{-1} * \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{bmatrix} 0.0754 & -0.1738 & 0.0820 \\ 0.1246 & -0.0262 & -0.0820 \\ 0.0951 & 0.0852 & 0.0164 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -0.0262 \\ -0.1738 \\ 0.3148 \end{bmatrix}$$

$$Solution: \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \cong \begin{bmatrix} -0.0262 \\ -0.1738 \\ 0.3148 \end{bmatrix}$$

Alternative ways of solving this example include the reduced row echelon, Gauss elimination, and graphical methods. In the previous exercise, to compute the roots of the equations, matrix multiplication, summation, and inverse operations are used. There are a number of operators and built-in functions in MATLAB that can be used to solve a linear system of equations. They are as follows:

- `inv()`, which computes the inverse of a given matrix or the pseudo-inverse of the given system (used for over-determined systems).
- `\`, the backslash operator, which solves the system of linear equations directly. It's based on the Gaussian elimination method. This is one of the most powerful MATLAB operators (tools) for handling matrices.

- `mldivide()`, which is a built-in function similar to the \ backslash operator.
- `linsolve()`, which is is a built-in function similar to the \ backslash operator.
- `lsqr()`, which is a built-in function based on the Least Squares Method.
- `lu()`, which is a built-in function based on the Gauss Elimination Method.
- `rref()`, which is a built-in function based on the Reduced Row Echelon Method.
- `svd()`, which is a built-in function based on the Singular Value Decomposition.
- `chol()`, which is a built-in function based on the Cholesky Decomposition.
- `qr()`, which is a built-in function based on the Orthogonal Triangular Decomposition.
- `decomposition()`, which is a built-in function that automatically choses the decomposition method.
- `bicg()`, `cgs()`, `gmres()`, `pcg()`, `symmlq()`, and `gmr()`, which are built-in functions that are based on gradient methods.
- `solve()`, which is a built-in function from the Symbolic MATH toolbox.

Note Among these listed functions/commands and operators, some of them use the same computing algorithm and are alternatives to each other. For example, the \ backslash operator is an alternative to `mldivide()`.

First, denote the given system with the following notations.

$$\underbrace{\begin{bmatrix} 2 & 3 & 5 \\ -3 & -2 & 5 \\ 4 & -7 & 6 \end{bmatrix}}_{A} * \underbrace{\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}_{B}$$

The entries of [*A*] matrix (coefficients of the unknowns *x*, *y*, *z*) are defined and elements of [*B*] matrix are defined in the Command Window.

```
>> A=[2, 3, 5; -3, -2, 5; 4, -7, 6]
A =
     2     3     5
    -3    -2     5
     4    -7     6
>> B=[1; 2; 3]
B =
     1
     2
     3
```

Using inv() and (*).

```
>>Ai=inv(A)    %  [B] matrix is an inverse matrix of [A] matrix.
Ai =
    0.0754   -0.1738    0.0820
    0.1246   -0.0262   -0.0820
    0.0951    0.0852    0.0164
>> XYZ1=Ai*B    %   Solutions of the problem
XYZ1 =
     -0.0262
     -0.1738
      0.3148
```

This next example uses the backslash \ operator based on the Gaussian Elimination method. This approach is quite simple and efficient in terms of computation time.

```
>> XYZ2=A\B
XYZ2 =
   -0.0262
   -0.1738
    0.3148
```

Using `mldivide()`:

```
>>XYZ3=mldivide(A,B)
XYZ3 =
   -0.0262
   -0.1738
    0.3148
```

Using `linsolve()`:

```
>>XYZ4=linsolve(A,B)
XYZ4 =
   -0.0262
   -0.1738
    0.3148
```

Using `lsqr()`:

```
>>XYZ5=lsqr(A,B)
lsqr converged at iteration 3 to a solution with relative residual 6.6e-17.
XYZ5 =
   -0.0262
   -0.1738
    0.3148
```

Using lu():

```
>>[L, U, P] = lu(A); %L-lower; U-upper triangular; P-Permutation matrix
>> y = L\(P*B);
>> XYZ6 = U\y
XYZ6 =
   -0.0262
   -0.1738
    0.3148
```

Using rref():

```
>> MA = [A, B];       % Augmented matrix
>> xyz = rref(MA);
>> XYZ7= xyz(:,end)
XYZ7 =
   -0.0262
   -0.1738
    0.3148
```

Using chol():

```
>> [U, S, V]= svd(A);
>> XYZ8 = V*inv(S)*U'*B
XYZ8 =
   -0.0262
   -0.1738
    0.3148
```

Using chol():

```
>> [U, L] = chol(A);    % A has to be Hermitian positive definite
>> XYZ9 = U\(U'\B)      % U'*U = A
XYZ9 =
   -0.0262
   -0.1738
    0.3148
```

Using `qr()`:

```
>> [Q, R] = qr(A);
>> XYZ10  = R\Q.'*B
XYZ10 =
   -0.0262
   -0.1738
    0.3148
```

Using `decomposition()`:

```
>> XYZ11 = decomposition(A)\B
XYZ11 =
   -0.0262
   -0.1738
    0.3148
```

Using `bicg()` gradient methods:

```
>> XYZ12 = bicg(A, B)
bicg converged at iteration 3 to a solution with relative residual 3.1e-14.
XYZ12=
   -0.0262
   -0.1738
    0.3148
```

Using `solve()`, which is a Symbolic Math Toolbox function:

```
>> syms x y z
>> sol=solve(2*x+3*y+5*z-1, -3*x-2*y+5*z-2, 4*x-7*y+6*z-3);
>> XYZ13=[sol.x; sol.y; sol.z]
XYZ13 =
  -8/305
 -53/305
  96/305
>> XYZ13=double([sol.x; sol.y; sol.z])
XYZ13 =
   -0.0262
   -0.1738
    0.3148
```

All of the computed solutions are accurate within four decimal places of the employed operators and functions. In fact, the accuracy of the solutions and the computation time of each operator or function will differ. For instance, the inverse matrix calculation is not only costly in terms of computation time but is also less accurate. Moreover, among the studied methods, the last function of the Symbolic Math Toolbox, `solve()` is the slowest and least efficient method.

Note Note that the `decomposition()` function is available in the recent versions of MATLAB starting from MATLAB 2018b.

Simulink Modeling

In addition to the MATLAB commands shown and demonstrated previously, Simulink has several blocks by which the linear system of equations, such as [A]{x} = [B], can be solved. All of the solver blocks are present in the DSP System Toolbox and can be accessed via the Simulink Library: the DSP System Toolbox/Math Functions/Matrices and Linear Algebra/Linear System Solvers. Let's test some of the blocks here to solve the previous example, called Example 1. Open a blank Simulink model and drag and drop the block from the DSP System Toolbox library shown in Figure 7-9.

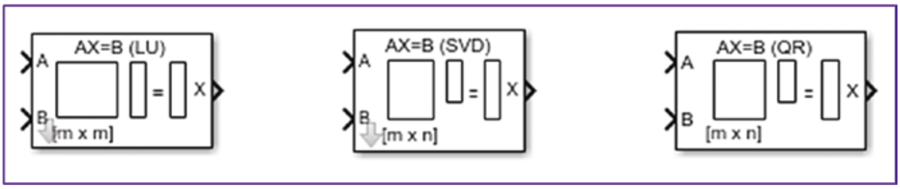


***Figure 7-9.** Simulink blocks used to solve a system of linear equations*

They are as indicated on the top of each block—LU, SVD, QR factorization and decomposition operation based solvers. All of them have two input ports for [A] and [B] and one output port for a solution {x}. Therefore, you need to add three additional blocks—two Constant and one Display block— which you add as explained previously in building Simulink models to compute determinant, transpose, and inverse of matrices. Figure 7-10 shows the primary version of the Simulink model.

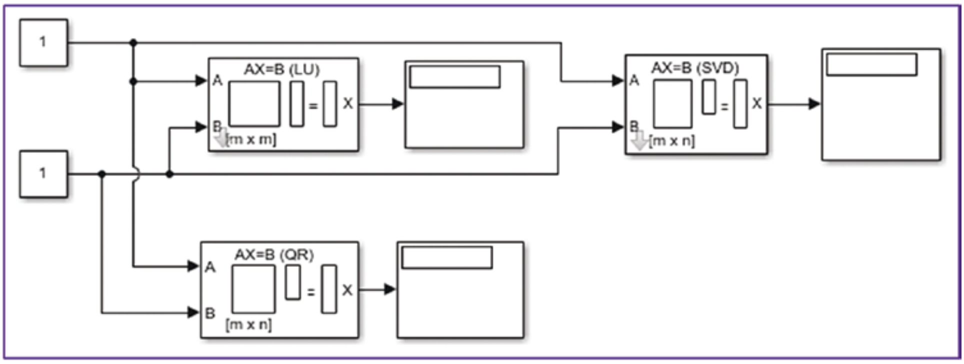
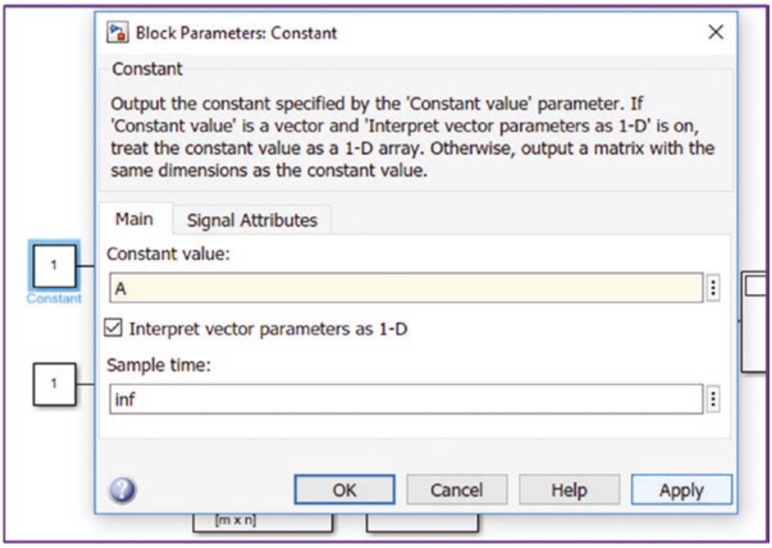


Figure 7-10. *Simulink model to solve a system of linear equations*

The elements of the matrices [A] and [B] can be inserted, as shown in Figure 7-3, directly in the Constant block's Constant Value window. Or you can define the elements of [A] and [B] via MATLAB's Command Window and workspace:

```
>> A=[2, 3, 5; -3, -2, 5; 4, -7, 6]
>> B=[1; 2; 3];
```

The variable names A and B are then entered in the fist and second Constant block's Constant Value box for [A] and [B], respectively, as shown in Figure 7-11. Click Apply and OK to complete the model.


Block Parameters: Constant

Constant

Output the constant specified by the 'Constant value' parameter. If 'Constant value' is a vector and 'Interpret vector parameters as 1-D' is on, treat the constant value as a 1-D array. Otherwise, output a matrix with the same dimensions as the constant value.

Constant value: A

☑ Interpret vector parameters as 1-D

Sample time: inf

Figure 7-11. *The variable names defined in the Constant block*

By pressing Ctrl+T from the keyboard or clicking the Run ▶ button in the Simulink model window, you'll obtain the complete model with its simulation results (see Figure 7-12). The computed results/solutions match the MATLAB solutions to four decimal places.

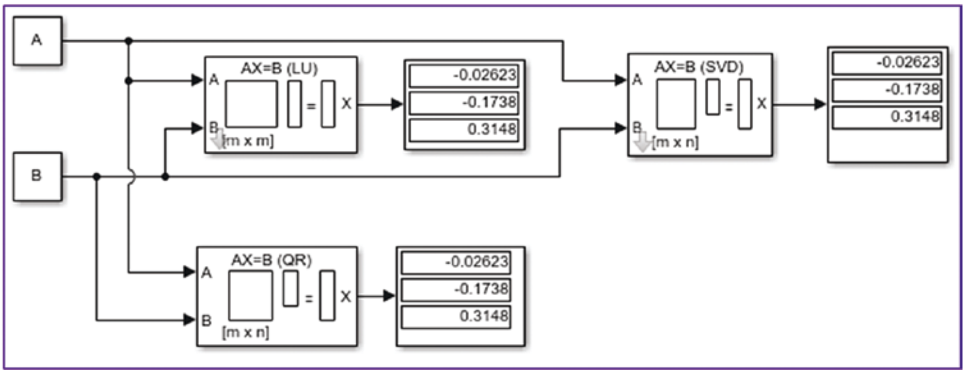
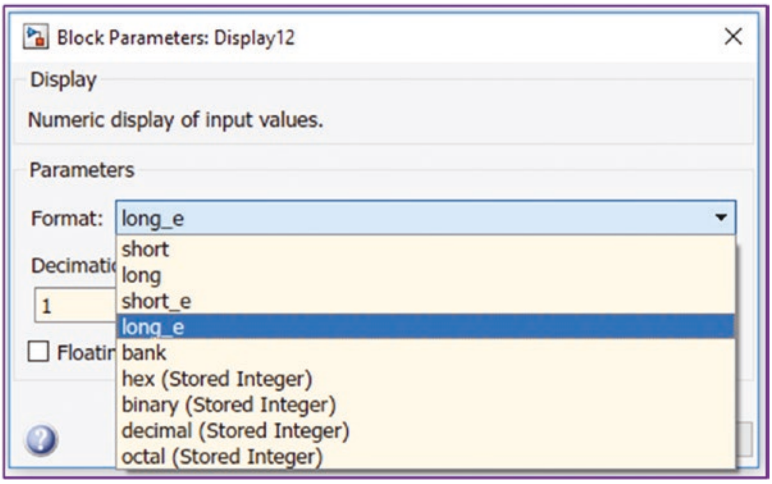


Figure 7-12. *Complete model with computed results*

Note that the variables (matrices) A and B are defined via MATLAB's Command Window.

In order to obtain more decimal places of the computed results with the Display block, the block parameters (Format Type) need to be tuned by selecting `long_e`, as shown in Figure 7-13.



Block Parameters: Display12

Display

Numeric display of input values.

Parameters

Format: long_e

Figure 7-13. *Adjusting the Display block's Format parameter*

Example 2: Embedding a MATLAB Function Block to Compute the Determinant and Solve Linear Equations

All of the afore-demonstrated MATLAB functions/commands used for computing matrix determinants, matrix inverses, or solutions of linear systems can be embedded in Simulink via the MATLAB Function block

. Let's take two MATLAB functions/commands used for computing a determinant of a matrix of any size with `det()` and solving with `linsolve()` and embed them into a Simulink model. For example:

$$A1 = \begin{bmatrix} 16 & 2 & -3 & 13 \\ -5 & 11 & 10 & -8 \\ 9 & 7 & -6 & 12 \\ -4 & 14 & 15 & 1 \end{bmatrix}, B1 = \begin{bmatrix} 3 \\ 2 \\ 4 \\ 5 \end{bmatrix}$$

Here are completed Simulink models. Figure 7-14 is built with three Content, two MATLAB Function, and two Display blocks.

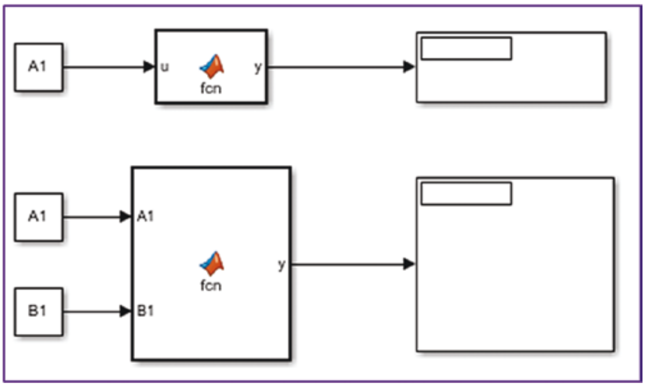


Figure 7-14. *Simulink models with MATLAB Function blocks to compute the determinant and solve a linear system of equations*

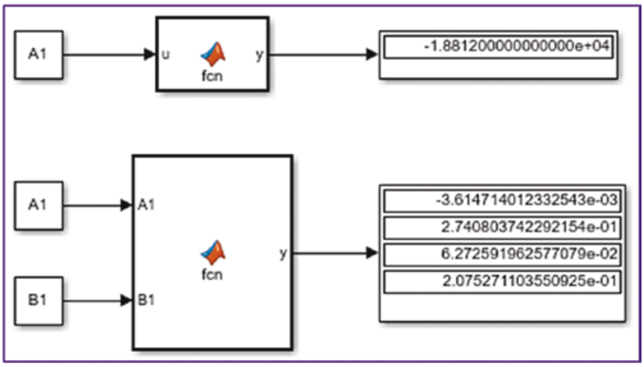
The input variables/entries for A1 and B1 are defined via the Command Window and MATLAB workspace in this model. To edit and type in the necessary script, you have to open the MATLAB Function block. It can be opened by double-clicking it, which opens the MATLAB editor window. The following function file scripts for the MATLAB Function blocks are typed in the MATLAB editor for the upper MATLAB Function block (with one input) and the lower one (with two inputs A1 and B1) models, respectively. After editing the codes of the blocks, save them. They will be saved under the created Simulink model and not as a separate MATLAB function file.

```
function y = fcn(u)
y = det(u);
```

And

```
function y = fcn(A1, B1)
y = linsolve(A1, B1);
```

The model is then completed and the finalized model is executed. Figure 7-15 shows the completed model with its computed results in the Display blocks. The upper Display block shows the determinant and lower one shows the solution of the given system.



```
-1.881200000000000e+04
```

```
-3.614714012332543e-03
2.740803742292154e-01
6.272591962577079e-02
2.075271103550925e-01
```

Figure 7-15. *Completed models with computed results*

The computed results of the Simulink model can be compared with the MATLAB.

```
>> A1=[16 2 -3 13 ; -5 11 10 -8; 9 7 -6 12; -4 14 15 1 ];
>> B1 = [3; 2; 4; 5];
>> det(A1)
ans =
      -18812
```

```
>> linsolve(A1, B1)
ans =
    -3.614714012332536e-03
     2.740803742292154e-01
     6.272591962577077e-02
     2.075271103550925e-01
```

The computed results from the determinant calculation and linear MATLAB solver match the Simulink model's results to 13 decimal places.

Example 3: Accuracy of Solver Functions of Linear Equations

Let's find out which one of the functions/tools (methods) highlighted in Example 1 is more accurate in computing the solutions. For this demonstration exercise, you'll take the following 13-by-13 [A] and 13-by-1 [B] matrices generated by the `magic()` and `randi()` (random integer) matrix generator functions of MATLAB. Moreover, the `norm()` function is used to compute the norm of the given linear system with its computed solutions. `LA_ex2.m` is the complete solution script:

```
%% Given 13-by-13 system of linear equations
clearvars; clc
A = magic(13);
B = randi([-169,169], 13,1); % Elements of B vary within [-169, 169]
%% 1-Way: inv()  or pinv()     %% INVERSE matrix method
x1a = inv(A)*B;
Err_INV = norm(A*x1a-B)/norm(B)     %#ok: ERROR checking
x1b = pinv(A)*B;
Err_PINV = norm(A*x1b-B)/norm(B)     %#ok: ERROR checking
%% 2-Way: \     %% backslash
x2  = A\B;
Err_BACKSLASH = norm(A*x2-B)/norm(B)    %#ok: ERROR checking
%% 3-Way: mldivide()          %% Left divide function
x3 = mldivide(A, B);
Err_MLDIVIDE = norm(A*x3-B)/norm(B)   %#ok: ERROR checking
%% 4-Way: Using linsolve()
x4 = linsolve(A, B);
```

```
Err_LINSOLVE = norm(A*x4-B)/norm(B)   %#ok: ERROR checking
%% 5-Way: Using lsqr()
x5 = lsqr(A, B);
Err_LSQR = norm(A*x5-B)/norm(B)   %#ok: ERROR checking
%% 6-Way: Using lu()
[L, U, P] = lu(A);
y = L\(P*B);
x6 = U\y;
Err_LU = norm(A*x6-B)/norm(B)   %#ok: ERROR checking
%% 7 - Way: Using rref()
MA = [A, B];
xyz = rref(MA);
x7= xyz(:,end);
Err_RREF = norm(A*x7-B)/norm(B)   %#ok: ERROR checking
%%  8 - Way: Using svd()
[U, S, V]= svd(A);
x8 = V*inv(S)*U'*B;
Err_SVD = norm(A*x8-B)/norm(B)   %#ok: ERROR checking
%% 9 - Way: Using chol()
[U, L] = chol(A);
x9 = U\(U'\B);
Err_CHOL = norm(A*x9-B)/norm(B)   %#ok: ERROR checking
%% 10 - Way: Using qr()
[Q, R] = qr(A);
x10  = R\Q.'*B;
Err_QR = norm(A*x10-B)/norm(B)   %#ok: ERROR checking
%% 11 - Way: Using decomposition()
x11 = decomposition(A)\B;
Err_DECOMPOSITION = norm(A*x11-B)/norm(B) %#ok: ERROR checking
%% 12 - Way: Using bicg()
x12 = bicg(A,B);
Err_BICG = norm(A*x12-B)/norm(B)   %#ok: ERROR checking
%% 13-Way: solve()            %% SOLVE() symbolic math method
x = sym('x', [1, 13]); x=x';
Eqn = A*(x); Eqn = Eqn - B;
```

```
Solution = solve(Eqn); SOLs = struct2array(Solution);
SOLs = double(SOLs); x13 = SOLs';
Err_SOLVE = norm(A*x13-B)/norm(B)   %#ok: ERROR checking
```

Here are the errors that were made while computing the solutions of the system with the employed methods.

```
Err_INV =
   5.8087e-16
Err_PINV =
   3.7982e-15
Err_BACKSLASH =
   3.0569e-16
Err_MLDIVIDE =
   3.0569e-16
Err_LINSOLVE =
   3.0569e-16
lsqr converged at iteration 7 to a solution with relative residual 3.5e-07.
Err_LSQR =
   3.4959e-07
Err_LU =
   3.0569e-16
Err_RREF =
   1.1576e-05
Err_SVD =
   3.7982e-15
Err_CHOL =
    2.2400
Err_QR =
   7.0615e-16
Err_DECOMPOSITION =
   3.0569e-16
bicg stopped at iteration 13 without converging to the desired tolerance
1e-06
because the maximum number of iterations was reached.
The iterate returned (number 13) has relative residual 9.6e-06.
```

```
Err_BICG =
   9.5856e-06
Err_SOLVE =
   1.5109e-16
```

From the computed errors, it is clear that the RREF(), BICG(), and LSQR() functions make errors within the margin of $10^{-5}...10^{-7}$ and all other methods make errors within the margin of $10^{-15}...10^{-16}$ while computing the solutions of this given system.

Example 4: Efficiency of Solver Functions of Linear Equations

This example demonstrates which one of the shown ways is more efficient in terms of computation time. For this demonstration, you'll consider two large matrices of 1000-by-1000 and 1000-by-1, generated by the random integer number generator function randi() to generate the elements of matrices [A] and [B]. In addition, to record the elapsed time of each computation method, the [tic, toc] functions are used. Here is the complete solution script, called LA_ex3.m:

```
clearvars
A=randi([-100,100],1000); B=randi([-100, 100], 1000, 1);
%% 1) inv() or pinv()
tic; Ai = inv(A); xyz1=Ai*B; T_inv=toc
%% 2) bacslash oerator: \
clearvars
A=randi([-100,100],1000); B=randi([-100, 100], 1000, 1);
tic; xyz2 = A\B; T_backslash = toc
%% 3) mldivide()
clearvars
A=randi([-100,100],1000); B=randi([-100, 100], 1000, 1);
tic; xyz3= mldivide(A, B); T_mld = toc
%% 4) linsolve()
clearvars
A=randi([-100,100],1000); B=randi([-100, 100], 1000, 1);
tic; xyz4 = linsolve(A, B); T_linsolve = toc
%% 5) lsqr()
clearvars
A=randi([-100,100],1000); B=randi([-100, 100], 1000, 1);
```

```
tic; xyz5 = lsqr(A, B); T_lsqr = toc
%% 6) lu()
clearvars
A=randi([-100,100],1000); B=randi([-100, 100], 1000, 1);
tic; [L, U, P]=lu(A); y=L\(P*B); xys6=U\y; T_lu=toc
%% 7) rref()
clearvars
A=randi([-100,100],1000); B=randi([-100, 100], 1000, 1);
tic; MA = [A, B];xyz7 = rref(MA); XYZ7=xyz7(:, end); T_rref=toc
%% 8) svd()
clearvars
A=randi([-100,100],1000); B=randi([-100, 100], 1000, 1);
tic; [U S V] = svd(A); xyz8 = V*inv(S)*U'*B; T_svd=toc
%% 9) chol()
clearvars
A=randi([-100,100],1000); B=randi([-100, 100], 1000, 1);
tic; [U L]= chol(A); xyz9 = U\(U'\B); T_chol=toc
%% 10) qr()
clearvars
A=randi([-100,100],1000); B=randi([-100, 100], 1000, 1);
tic; [Q R] = qr(A); xyz10 = R\Q.'*B ; T_qr=toc
%% 11) decomposition()
clearvars
A=randi([-100,100],1000); B=randi([-100, 100], 1000, 1);
tic; xyz11 = decomposition(A)\B; T_decom = toc

%% 12) bicg()  Gradient methods
clearvars
A=randi([-100,100], 1000); B=randi([-100, 100], 1000, 1);
tic; xyz12 = bicg(A, B); T_bicg=toc
%% 13) solve()
A=randi([-100,100],100); B=randi([-100, 100], 100, 1);
tic;
x = sym('x', [1, 100]); x=x';
Eqn = A*(x); Eqn = Eqn - B;
```

```
Solution = solve(Eqn); SOLs = struct2array(Solution);
SOLs = double(SOLs); x13 = SOLs';
T_solve=toc
```

Here are the elapsed computation time values from the simulations.

```
T_inv =
    0.0390
T_backslash =
    0.0173
T_mld =
    0.0171
T_linsolve =
    0.0171
lsqr stopped at iteration 20 without converging to the desired tolerance
1e-06
because the maximum number of iterations was reached.
The iterate returned (number 20) has relative residual 0.24.
T_lsqr =
    0.0236
T_lu =
    0.0235
T_rref =
   10.1406
T_svd =
    0.4263
T_chol =
    0.0330
T_qr =
    0.1045
T_decom =
    0.0459
bicg stopped at iteration 20 without converging to the desired tolerance 1e-06
because the maximum number of iterations was reached.
The iterate returned (number 0) has relative residual 1.
```

```
T_bicg =
    0.0195
T_solve =
   14.6306
```

From these computations, it is clear that `linsolve()`, `mldivide`, and `\` (the backslash operator) (Gaussian elimination method) are the fastest among all the tested methods. The slowest and computationally costliest one is the `solve()` operator of the Symbolic MATH even when the size of the system was 10 times smaller. It is worth noting that the reduced row echelon method called `rref()` is the next slowest, after the `solve()` operator.

Let's consider another example in order to solve these four different methods, which are `\`, `linsolve(), inv()`, and `solve()`, discussed previously.

Example 5: Solving Linear Equations ([A]{x} = [b]) by Changing Values of [b]

This exercise is composed of two parts:

1. Solve the given linear system for unknowns *a*, *b*, and *c*.

$$\begin{cases} 0.072a - c = -12 \\ 0.12b - c = -9 \\ a + b = 50 \end{cases}$$

2. Solve the given system for unknowns *a*, *b*, and *c*. The third equation's value changes in the range of 50…250.

$$\begin{cases} 0.072a - c = -12 \\ 0.12b - c = -9 \\ a + b = 50\ldots250 \end{cases}$$

The system is rewritten in a matrix form as [*A*]{*x*} = [*B*] and then solved directly for unknowns *a*, *b*, and *c*. Here is the solution script (`LA_Ex4.m`).

```
% PART I.
% The given system is written from the Ax=B as [A]*[abc]=[B]
A=[.072, 0, -1; 0, .12, -1; 1 1 0];
B=[-12, -9, 50];
```

```
abc1=A\B'                %#ok  % BACKSLASH \
abc2 = linsolve(A,B')    %#ok  % LINSOLVE()
abc3 = inv(A)*B'         %#ok  % INV
% SOLVE() in symbolic MATH
syms a b c
abc4=solve(0.072*a-c+12, 0.12*b-c+9, a+b-50);
abc4=double([abc4.a; abc4.b; abc4.c]) %#ok  % SOLVE()
%% Part II. %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% BACKSLASH \ ; LINSOLVE(); INV
tic
Bk=50:250;
a=zeros(numel(Bk),1); b=zeros(numel(Bk),1); c=zeros(numel(Bk),1);
A=[.072, 0, -1; 0, .12, -1; 1 1 0];
for ii=1:numel(Bk)
    B=[-12; -9; Bk(ii)];
    abc=A\B;
    a(ii)=abc(1,:);
    b(ii)=abc(2,:);
    c(ii)=abc(3,:);
end
Time1=toc;
fprintf('Computation time with BACKSLASH: %3.3f  \n', Time1)
clearvars
tic
Bk=50:250;
a=zeros(numel(Bk),1); b=zeros(numel(Bk),1); c=zeros(numel(Bk),1);
A=[.072, 0, -1; 0, .12, -1; 1 1 0];
for ii=1:numel(Bk)
    B=[-12; -9; Bk(ii)];
    abc=linsolve(A,B);
    a(ii)=abc(1,:);
    b(ii)=abc(2,:);
    c(ii)=abc(3,:);
end
Time2=toc;
```

```
fprintf('Computation time with LINSOLVE:  %3.3f  \n', Time2)
clearvars
tic
Bk=50:250;
a=zeros(numel(Bk),1); b=zeros(numel(Bk),1); c=zeros(numel(Bk),1);
A=[.072, 0, -1; 0, .12, -1; 1 1 0];
for ii=1:numel(Bk)
    B=[-12; -9; Bk(ii)];
    abc=inv(A)*B;
    a(ii)=abc(1,:); b(ii)=abc(2,:); c(ii)=abc(3,:);
end
Time3=toc;
fprintf('Computation time with INV:       %3.3f  \n', Time3)
%% SOLVE() from symbolic math
clearvars
tic;
Bk=50:250;
a1=zeros(numel(Bk),1);b1=zeros(numel(Bk),1); c1=zeros(numel(Bk),1);
syms a b c
for ii=1:numel(Bk)
    abc=solve(0.072*a-c+12,0.12*b-c+9,a+b-Bk(ii));
    a1(ii)=double(abc.a);
    b1(ii)=double(abc.b);
    c1(ii)=double(abc.c);
end
Time4=toc;
fprintf('Computation time with SOLVE:     %3.3f  \n', Time4)
```

Here are the results of the calculations from Part 1:

```
abc1 =

   15.6250
   34.3750
   13.1250
```

```
abc2 =

   15.6250
   34.3750
   13.1250

abc3 =

   15.6250
   34.3750
   13.1250

abc4 =

   15.6250
   34.3750
   13.1250
```

Here are the results of the script from Part 2:

```
Computation time with BACKSLASH: 0.002
Computation time with LINSOLVE:  0.002
Computation time with INV:       0.002
Computation time with SOLVE:    22.066
```

From the computation time spent to compute solutions of the given linear system with three variables and 201 possible cases using four ways, it is clear that the least efficient way of solving linear equations is using the Symbolic Math toolbox's `solve()` function. The backslash operator (`\`), `linsolve()`, and `inv()` methods all performed similarly. The solver `linsolve()`, `\`, and `inv()` methods are more than 11033 times more efficient and faster than the `solve()` function.

Matrix Operations

This section covers general mathematical operations and computations of matrices, vectors, and eigenvectors. Many numerical examples are used to explain the matrix operations. Table 7-1 lists the matrix operations their command syntax.

***Table 7-1.** Matrix Operators in Two Equivalent Formulations*

Operation Name	MATLAB First Way	MATLAB Second Way
Matrix multiplication	`A*B`	`mtimes(A,B)`
Array-wise multiplication	`A.*B`	`times(A,B)`
Matrix right division	`A/B`	`mrdivide(A,B)`
Array-wise right division	`A./B`	`rdivide(A,B)`
Matrix left division	`A\B`	`mldivide(A,B)`
Array-wise left division	`A.\B`	`ldivide(A,B)`
Matrix power	`A^B`	`mpower(A,B)`
Array-wise power	`A.^B`	`power(A,B)`
Complex transpose	`A'`	`ctranspose(A)`
Matrix transpose	`A.'`	`transpose(A)`
Binary addition	`A+B`	`plus(A,B)`
Unary plus	`+A`	`uplus(A)`
Binary subtraction	`A-B`	`minus(A,B)`
Unary minus	`-A`	`uminus(A)`
Determinant	`det(A)`	`det(A)`
Rotate by 90^0	`rot90(A)`	`rot90(A)`
Replicate and tile an array n times	`repmat(A, n)`	`repmat(A, n)`
Flip matrix left/right	`fliplr(A)`	`fliplr(A)`
Flip matrix in up/down	`flipud(A)`	`flipud(A)`

Basic MATLAB unit data is in the array type format. Matrices and vectors can be employed in many cases to define input and output, local data, and function inputs and outputs. Moreover, they can be used to combine separate scalars into one signal and process multidimensional input and output signals. An array is defined by a single name and a collection of data arranged by rows and columns, as shown here.

Row #1 ⇨	A_{11}	A_{12}	A_{13}
Row #2 ⇨	A_{21}	A_{22}	A_{23}
Row #3 ⇨	A_{31}	A_{32}	A_{33}
Row #4 ⇨	A_{41}	A_{42}	A_{43}
	⇧ Column #1	⇧ Column #2	⇧ Column #3

Let's look at some numerical examples. They perform matrix operations with scalars, such as addition, subtraction, power, multiplication, and division, including array-wise (elementwise) operations in the Command Window.

```
>> A=[8,1,6; 3,5,7; 4,9,2]  % Matrix 3-by-3
A =
         8     1     6
         3     5     7
         4     9     2
>> a = 2; b = 2+3i; c = 5j;
>> B = A^a % NB:  Difference between .^ and ^
B =
    91    67    67
    67    91    67
    67    67    91

>> C = A.^a % Elementwise op. NB:  Difference between .^ and ^
C =
    64     1    36
     9    25    49
    16    81     4
>> D = A*a+B/b
D =
  30.0000 -21.0000i  12.3077 -15.4615i  22.3077 -15.4615i
  16.3077 -15.4615i  24.0000 -21.0000i  24.3077 -15.4615i
  18.3077 -15.4615i  28.3077 -15.4615i  18.0000 -21.0000i
```

```
>> E = C./c
E =
   0.0000 -12.8000i   0.0000 - 0.2000i   0.0000 - 7.2000i
   0.0000 - 1.8000i   0.0000 - 5.0000i   0.0000 - 9.8000i
   0.0000 - 3.2000i   0.0000 -16.2000i   0.0000 - 0.8000i
>> F = C/c
E =
   0.0000 -12.8000i   0.0000 - 0.2000i   0.0000 - 7.2000i
   0.0000 - 1.8000i   0.0000 - 5.0000i   0.0000 - 9.8000i
   0.0000 - 3.2000i   0.0000 -16.2000i   0.0000 - 0.8000i>>
```

Example: Performing Matrix Operations

Given six arrays: $A\ (4-\text{by}-3),\ \ B(3-\text{by}-4),\ \ C(4-\text{by}-4)$, D(4 – by – 3), $E(3-by-3)$, *and* $F(3-by-3)$.

$$A=\begin{bmatrix}2 & -3 & 1\\ 3 & 2 & 5\\ 1 & 3 & 4\\ -3 & -2 & 3\end{bmatrix}, B=\begin{bmatrix}3 & 4 & -2 & 1\\ 2 & 5 & 4 & -6\\ 4 & -3 & 1 & 2\end{bmatrix},\ C=\begin{bmatrix}16 & 2 & 3 & 13\\ 5 & 11 & 10 & 8\\ 9 & 7 & 6 & 12\\ 4 & 14 & 15 & 1\end{bmatrix},\ D=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 4 & 3 & 1\\ -2 & -3 & 1\end{bmatrix},$$

$$E=\begin{bmatrix}8 & 1 & 6\\ 3 & 5 & 7\\ 4 & 9 & 2\end{bmatrix}, F=\begin{bmatrix}3 & 7 & 3\\ 3 & 2 & 8\\ 9 & 2 & 1\end{bmatrix}$$

Let’s perform some matrix operations—such as summation, subtraction, multiplication, power, scalar multiplication, square root, mean, round, standard deviations, and replicate/rotate/flip matrix—from the Command Window.

```
>> A=[2 -3 1; 3 2 5; 1 3 4; -3 -2 3] ;
>> B=[3,4,-2 1;2,5,4,-6;4,-3, 1,2] ;
>> C=[16,2,3,13;5,11,10,8;9 4 7 14;6 15 12 1] ;
>> D=[1 2 3; 2 3 4; 4 3 1; -2 -3 1] ;
>> E=[8, 1, 6; 3, 5, 7; 4, 9, 2];
>> F=[3 7 3; 3 2 8; 9 2 1];
>> M_AB = A*B
M_AB =
     4   -10   -15    22
    33     7     7     1
    25     7    14    -9
    -1   -31     1    15
>> M_BA = B*A
M_BA =
    13    -9    18
    41    28    25
    -6   -19    -1
>> M_S = M_AB-C
M_S =
   -12   -12   -18     9
    28    -4    -3    -7
    16     3     7   -23
    -7   -46   -11    14
>> M_S = M_BA-C
Matrix dimensions must agree.
```

```
>> CM=C*M_S     % Not equivalent to M_S*C
CM =
  -179  -789  -416   243
   352  -442  -141  -150
    18  -747  -279    88
   533  -142   -80  -313
>> CM1=M_S*C    % Not equivalent to C*M_S
CM1 =
  -360   -93  -174  -495
   359  -105   -61   283
   196  -252  -149   307
  -357  -354  -390  -599
```

```
>> CM2=M_S.*C  % Elementwise operation: NOT equivalent to M_S*C
CM2 =
  -192   -24   -54   117
   140   -44   -30   -56
   144    12    49  -322
   -42  -690  -132    14
>> MDE=M_S./C  % Elementwise operation: NOT equivalent to M_S/C
MDE =
   -0.7500   -6.0000   -6.0000    0.6923
    5.6000   -0.3636   -0.3000   -0.8750
    1.7778    0.7500    1.0000   -1.6429
   -1.1667   -3.0667   -0.9167   14.0000
>> MD=M_S/C   % Not equivalent to M_S./C
CM =
    1.9275    8.5704   -5.6271   -5.8414
    1.4496   -6.4076    1.5420    3.8277
   -1.0389  -10.8246    5.0116    6.9401
   -4.9118  -12.9118   12.6765    3.6765
>> M_AD = A.*D % Elementwise operation: matrix multiplication
M_AD =
     2    -6     3
     6     6    20
     4     9     4
     6     6     3
>> MM_AD = A*D % Error due to size mismatch of [A] and [D]

Error using  *
Incorrect dimensions for matrix multiplication. Check that the
number of columns in the first matrix matches the number of rows
in the second matrix. To perform elementwise multiplication, use
'.*'.
>> M_EF=E.*F     % Elementwise multiplication of square matrices
M_EF =
    24     7    18
     9    10    56
    36    18     2
```

```
>> MM_EF=E*F     % Square matrices can be multiplied matrix-wise
MM_EF =
    81    70    38
    87    45    56
    57    50    86
>> Csqrt=sqrt(C) % Not equivalent to sqrtm(C)
Csqrt =
    4.0000    1.4142    1.7321    3.6056
    2.2361    3.3166    3.1623    2.8284
    3.0000    2.0000    2.6458    3.7417
    2.4495    3.8730    3.4641    1.0000
>> Csqrt=sqrtm(C) % Not equivalent to sqrt(C)
Csqrt =
  Columns 1 through 3
   3.8335 - 0.0167i   0.0738 + 0.7839i   0.1262 + 0.3666i
   0.3251 + 0.0011i   2.6850 - 0.0526i   1.6850 - 0.0246i
   1.3123 - 0.0237i   0.7322 + 1.1107i   1.9687 + 0.5194i
   0.5925 + 0.0373i   2.0922 - 1.7477i   1.7997 - 0.8172i
  Column 4
   1.7975 - 1.1337i
   1.1359 + 0.0761i
   1.8178 - 1.6064i
   1.3466 + 2.5276i
>> C_E1 = expm(C)   % Matrix exponential not equal to exp(C)
C_E1 =
   1.0e+14 *
    1.5718    1.3711    1.3622    1.5295
    1.5718    1.3711    1.3622    1.5295
    1.5718    1.3711    1.3622    1.5295
    1.5718    1.3711    1.3622    1.5295
```

```
>> C_E2 = exp(C) % Exponential of a matrix: not equal to expm(C)
C_E2 =
   1.0e+06 *
    8.8861    0.0000    0.0000    0.4424
    0.0001    0.0599    0.0220    0.0030
    0.0081    0.0001    0.0011    1.2026
    0.0004    3.2690    0.1628    0.0000

>> S=[A(1,1:3); B(2,1:3);C(3,2:4)]; % Created from the existed
>> Y=[A(1), 1.3];
>> Arot90=rot90(A)      % Matrix rotate

Arot90 =

     1     5     4     3
    -3     2     3    -2
     2     3     1    -3
>>Crep=repmat(C, 2,1)   % Matrix replication/copy

Crep =
     5    11    10     8
     9     4     7    14
     6    15    12     1
    16     2     3    13
     5    11    10     8
     9     4     7    14
     6    15    12     1
>>Bflip=fliplr(B)       % Matrix flip

Bflip =

     1    -2     4     3
    -6     4     5     2
     2     1    -3     4
```

```
>> Cud=flipud(Crep)     % Matrix flip up or down

Cud =

     6    15    12     1
     9     4     7    14
     5    11    10     8
    16     2     3    13
     6    15    12     1
     9     4     7    14
     5    11    10     8
    16     2     3    13
```

Many of these matrix operations can also be performed in the Simulink environment. Let's use the previous examples to demonstrate how and what Simulink uses for matrix operations and manipulations.

The Simulink Library contains the blocks for sum, multiplication/division, power, exponent, and concatenation, as shown in Figure 7-16.

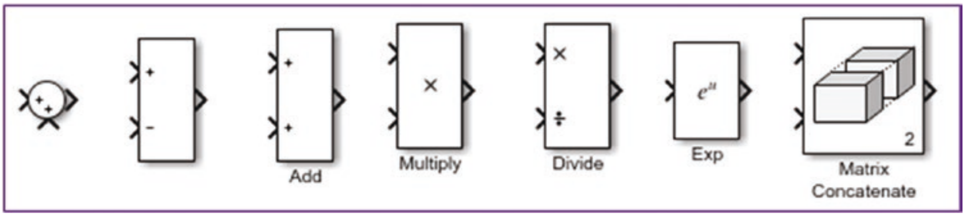
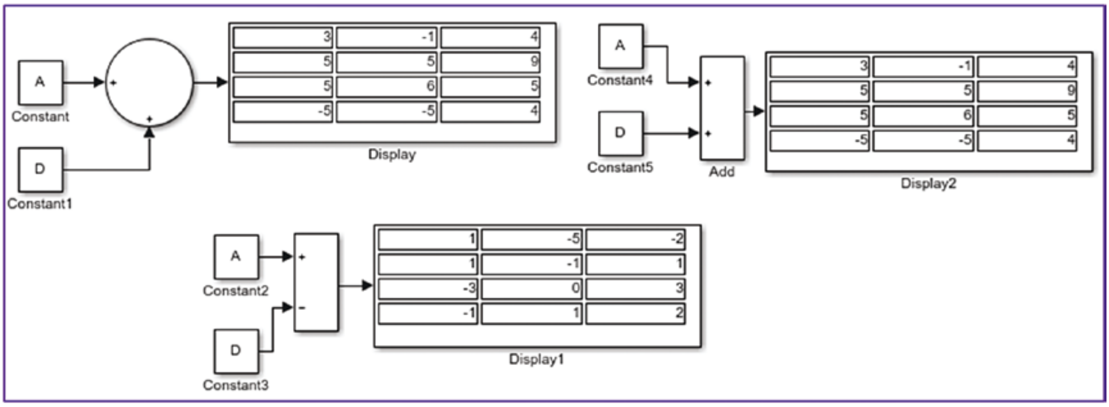


Figure 7-16. *Matrix operation blocks in the Simulink Library*

First define the [A] and [D] matrices in the Command Window:

```
>> A=[2 -3 1; 3 2 5; 1 3 4; -3 -2 3] ;
>> D=[1 2 3; 2 3 4; 4 3 1; -2 -3 1] ;
```

Now compute the sum and subtraction of matrices [A] and [D], as shown in Figure 7-17.



```
3  -1  4
5   5  9
5   6  5
-5 -5  4
```

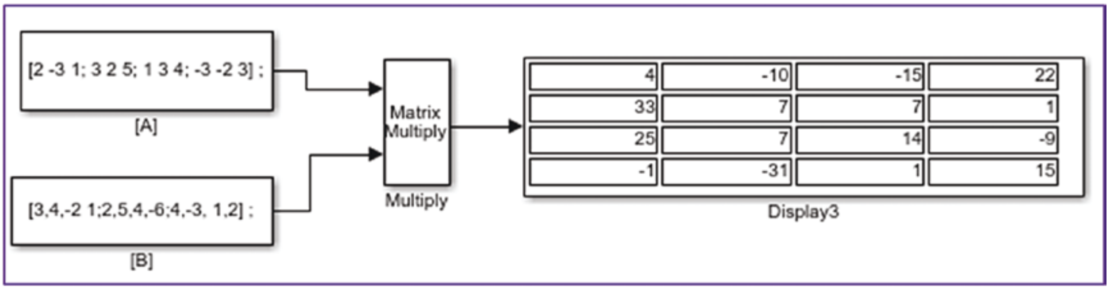
```
3  -1  4
5   5  9
5   6  5
-5 -5  4
```

```
1  -5  -2
1  -1   1
-3  0   3
-1  1   2
```

Figure 7-17. *Matrix sum and subtraction operations in Simulink*

Note that matrices [A] and [D] are defined via the Command Window and workspace. The computed sums match the ones calculated using MATLAB's Command Window.

Here are the results of multiplication (see Figure 7-18), exponent, and square (see Figure 7-20) of the matrices.

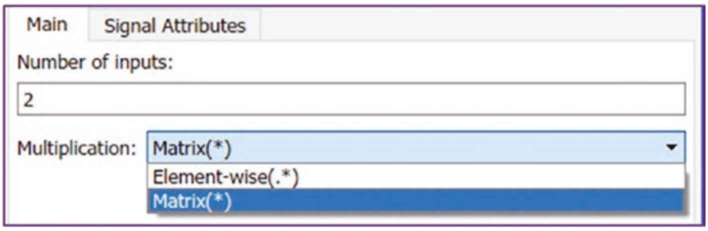


```
[2 -3 1; 3 2 5; 1 3 4; -3 -2 3] ;
[3,4,-2 1;2,5,4,-6;4,-3, 1,2] ;
```

```
 4  -10  -15  22
33    7    7   1
25    7   14  -9
-1  -31    1  15
```

Figure 7-18. *Matrix multiplication in Simulink*

Note that for the matrix multiplication operation shown in Figure 7-18, the Multiply block change from the Element-wise (.*) multiplication to the matrix (*) multiplication, as shown in Figure 7-19.

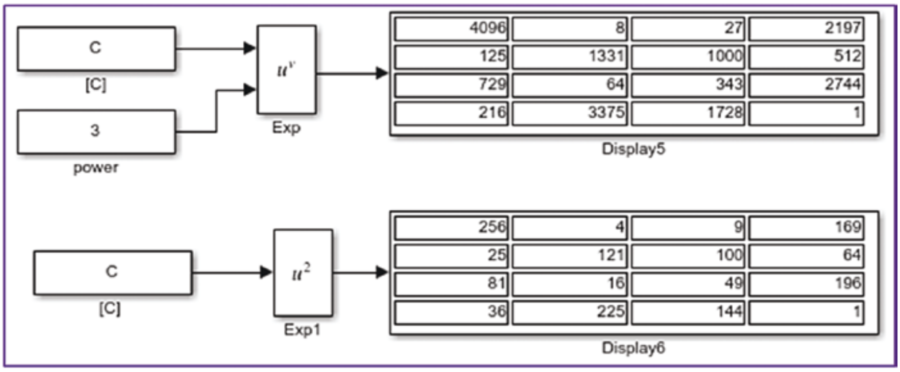


Number of inputs: 2

Multiplication: Matrix(*)

Figure 7-19. *Setting up the Matrix Multiply block for matrix multiplication (*) or element-wise multiplication (.∗)*

Otherwise, the multiplication operation will not be performed due to the mismatched sizes of [A] and [B]. Again, the computed results match the ones from MATLAB.

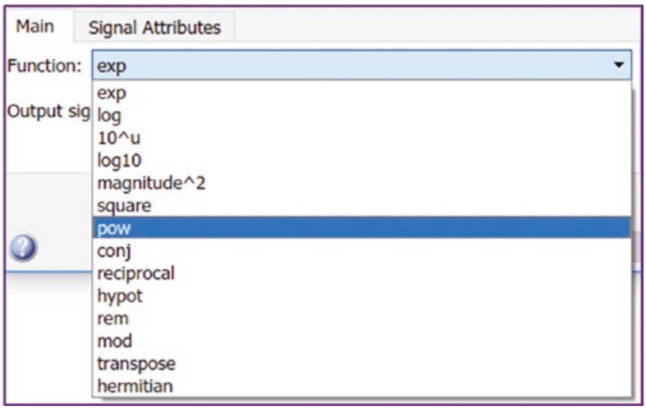


```
4096     8    27  2197
 125  1331  1000   512
 729    64   343  2744
 216  3375  1728     1
```

```
 256     4     9   169
  25   121   100    64
  81    16    49   196
  36   225   144     1
```

Figure 7-20. *Matrix exponential and square operation blocks*

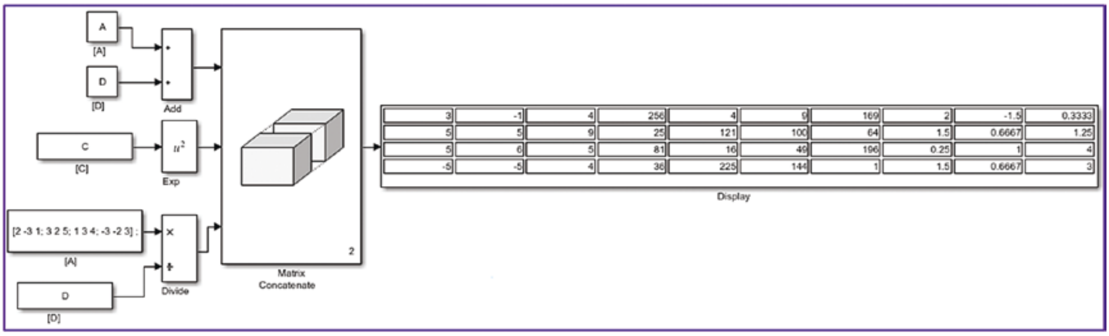
Note that in the operations in Figure 7-20, the exponential and power operations are performed with one block (one Math Function block), by choosing its Function type [pow] in u^v and [square] in u^2 (see Figure 7-21).



Function: exp

Figure 7-21. *How to set the Math Function block for matrix operations*

Now by using the Matrix Concatenate block, we create a new matrix (4-by-10) from the computed the matrix sum (4-by-3), square (4-by-4), and matrix division (4-by-3). See Figure 7-22.



3	-1	4	256	4	9	169	2	-1.5	0.3333
5	5	9	25	121	100	64	1.5	0.6667	1.25
5	6	5	81	16	49	196	0.25	1	4
-5	-5	4	36	225	144	1	1.5	0.6667	3

Figure 7-22. *The Matrix Concatenate block performs matrix concatenation*

As demonstrated, Simulink blocks perform various matrix operations, much like MATLAB functions. However, there are computationally costly simulations with matrix and array operations in which Simulink models might be slower than MATLAB scripts. For example when computing discrete Fourier transforms, Simulink models are much slower than MATLAB. For some matrix and array operations, the MATLAB Fcn block or the Interpreted MATLAB Fcn block can be used in Simulink modeling.

In addition to these matrix operations, there are a few other operations by which you can create new matrices. For instance, you can take out diagonals of existing matrices with diag(A) or take out selected elements of matrices and create a new matrix.

```
>> E=[8, 1, 6; 3, 5, 7; 4, 9, 2];
>> F=[3 7 3; 3 2 8; 9 2 1];
>> EF = [diag(E), diag(F)]
EF =
     8     3
     5     2
     2     1
```

Standard Matrix Generators

MATLAB has numerous standard array and matrix generators, which can be used to generate a wide range of matrices. For instance, eye(n), eye(k, m), ones(m), ones(m, k), zeros(l), zeros(l,k), magic(k), pascal(k), pascal(k, m), rand(m), rand(k, m), randi(n,m,k), repmat(A, r, c), blkdiag(A, B, C), sparse(m,n), and many more. For example:

```
>> eye(3)
ans =
     1     0     0
     0     1     0
     0     0     1
>> magic(5)         % Magic matrix in a size of 5 by 5
ans =
    17    24     1     8    15
    23     5     7    14    16
     4     6    13    20    22
    10    12    19    21     3
    11    18    25     2     9
```

```
>> A=pascal(4)       % Pascal matrix in a size of 4 by 4
A =
     1     1     1     1
     1     2     3     4
     1     3     6    10
     1     4    10    20

>> A=pascal(4,2)       % Pascal matrix in a size of 4 by 4
A =
    -1    -1    -1    -1
     3     2     1     0
    -3    -1     0     0
     1     0     0     0
>> zeros(3)            % Zero matrix in a size of 3 by 3
ans =
     0     0     0
     0     0     0
     0     0     0
>> zeros(2,3)          % Zero matrix in a size of 2 by 3
ans =
     0     0     0
     0     0     0
>> ones(3)             % Ones matrix in a size of 3 by 3
ans =
     1     1     1
     1     1     1
     1     1     1
>> ones(3,2)           % Ones matrix in a size of 3 by 2
ans =
     1     1
     1     1
     1     1
```

```
>> eye(4,3)            % Unit diagonal matrix of size 4 by 3
ans =
     1     0     0
     0     1     0
     0     0     1
     0     0     0
>> eye(4,5)
ans =
     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
>> rand(2)
ans =
   0.814723686393179   0.126986816293506
   0.905791937075619   0.913375856139019
>> rand(2,3)
ans =
   0.957166948242946   0.800280468888800   0.421761282626275
   0.485375648722841   0.141886338627215   0.915735525189067
>>A=round(rand(2))
A =
     0     1
     0     0
>>A_rep=repmat(A, 2, 3) % replicating the matrix A by making its
           % replication 2 times of rows and 3 times of columns
A_rep =
     0     1     0     1     0     1
     0     0     0     0     0     0
     0     1     0     1     0     1
     0     0     0     0     0     0
```

```
>> randi([-13, 13], 5) % Random integers within [-13, 13]
ans =
    -7    10    12     0    10
     6    -8     5     1    -2
   -11     5    -9    -6     4
    -9    11    -6     8     3
   -10     0     3    -1    -2
>> C=eye(2); B=magic(3); A=ones(4);
>> D=blkdiag(A,B,C)    % combine matrices in diagonal directions to
     % create a block diagonal matrix.
D =
     1     1      1     1     0     0     0     0     0
     1     1      1     1     0     0     0     0     0
     1     1      1     1     0     0     0     0     0
     1     1      1     1     0     0     0     0     0
     0     0      0     0     8     1     6     0     0
     0     0      0     0     3     5     7     0     0
     0     0      0     0     4     9     2     0     0
     0     0      0     0     0     0     0     1     0
     0     0      0     0     0     0     0     0     1

>> K=reshape(randperm(9), 3,3) % Change the size (reshape) of array % to
make 3 by 3 matrix by random permutation
K =
     4     9     5
     3     8     6
     7     1     2
```

In addition, there are a few dozen matrix generation functions. They are the gallery of test matrices, such as binomial, cauchy, clement, invol, house, krylov, leslie, lesp, neumann, poisson, ris, rando, smoke, wilk, and many more. In general, the command syntax of these matrices is as follows:

```
[A, B, C,...] = gallery(matname,P1,P2,...);
[A, B, C,...] = gallery(matname,P1,P2,..., classname);
A=gallery(3);
B=gallery(5);
```

To get more information about the gallery of matrices, type this in the Command Window:

```
>> help gallery
>> doc gallery
```

Here are several examples of how to employ gallery matrices:

```
>> L=gallery('leslie', s, x)  % This is the 3-by-3 Leslie pop.
% matrix taken from the model with average birth numbers s(1:n)
and          % survival rates x(1:n-1)
L =
            0            2            0
          0.5            0            0
            0         0.78            0
>> C = gallery('chebspec',3,1) % Chebyshev spectral differentiation
% matrix of order 3
C =
     -0.33333           -1      0.33333
            1      0.33333           -1
      -1.3333            4      -3.1667

>> L=gallery('cauchy', s, y) % Returns 3-by-3 matrix, C(i,j) =
%1/(s(i)+y(j)). Arguments s and y are vectors of length 3. If you
         % pass in scalars for s and y, they are interpreted as vectors 1:s
         % and 1:y.
L =
            2       1.2821      0.76923
          0.4      0.35971      0.30303
            2       1.2821      0.76923
>> B = gallery('krylov', rand(3)) % Returns 3 by 3 Krylov matrix
B =
            1       1.3462       1.6804
            1       1.2107       1.578
            1       1.3387       1.7209
```

```
>> B = gallery('house',rand(3,1))  % Householder matrix
B =
       2.1404
      0.28584
       0.7572
>> B = gallery('ris',4)  % Returns a symmetric 4-by-4Hankel
matrix      %with elements A(i,j) = 0.5/(n-i-j+1.5)
B =
    0.1429    0.2000    0.3333    1.0000
    0.2000    0.3333    1.0000   -1.0000
    0.3333    1.0000   -1.0000   -0.3333
    1.0000   -1.0000   -0.3333   -0.2000
>>gallery('smoke', 2) %Smoke matrix - complex, with "smoke ring"
% pseudo-spectrum.

ans =

  -1.0000 + 0.0000i   1.0000
   1.0000             1.0000
```

These standard and gallery matrices have special properties that can be of great use in various numerical simulations and analysis problems. For instance, these standard matrices—`ones()`, `eye()`, `zeros()`, `rand()`, `randn()`—are used very often for signal processing, data analysis, and memory allocation in large computations.

Vector Spaces

In signal processing, numerical analyses, and building computer simulation models, vector spaces are very important. For instance, the logarithmic space is used for digital signal processing when frequencies go over a unit circle. There are several straightforward ways by which vectors, vector spaces, and arrays with equal spaces between their elements can be created. Let's suppose that we need to create a vector W that begins with a value $w1$ and ends with $w2$, as shown in Figure 7-23.

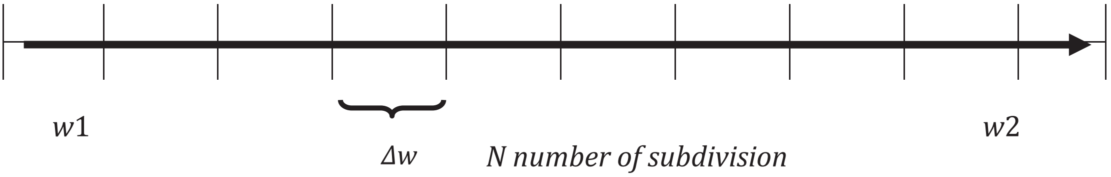


Figure 7-23. *Vector space*

If the size Δw is known, then the space can be expressed by $W = w1$: Δw: $w2$. For instance, the whole space can be defined in terms of $w1 = 1$, $w2 = 13$, $\Delta w = 0.1$ by:

```
>> w=1:0.1:13;
```

Moreover, if N number of points between the start and end boundaries of a space are known, the linear space function `linspace()` can be used.

```
>>w=linspace(1, 13, N); % This creates a linear space of w array
            % with equally spaced k number of elements
```

Note If *N* is not specified in the `linspace()` command, its default value is 100.

This example generates sound waves with a sine function.

```
>> fs=3e4;          % Sampling frequency
>> f1=100; f2=200;  f3=300;  f4=400;  f5=500;  f6=600;
>> t=0:1/fs:5;
>> x=sin(2*pi*t*f1)+sin(2*pi*t*f2)+sin(2*pi*t*f3)+
   sin(2*pi*t*f4)+...          +sin(2*pi*t*f5);
>> [m, n]=size(x);  % Check the size of the created vector space
>> sound(x, fs)     % Play a created sound & hear from sound cards
```

The `linspace` command creates linearly spaced vector spaces/arrays. In MATLAB, there is another similar function, called `logspace`, that creates logarithmic scaled vector spaces. For example, you use the following command to create logarithmic space of the x array containing 130 logarithmically spaced elements (here, $N = 130$) between boundary points 0 and 13:

```
>> x=logspace(1,13,130);
```

Likewise, use this command to create 50 logarithmic spaced points between 0 and π:

```
>> s=logspace(0, pi);
```

Note If *N* is not specified in `logspace()`, then its default value is 50.

Polynomials Represented by Vectors

For numerical simulations in MATLAB, polynomials are represented via vectors using coefficients of polynomials in descending order. For instance, a fifth order polynomial is given by:

$$12x^5 + 13x^4 - 15x^2 + 17x - 13$$

That is defined as a vector space in the following manner:

```
>> f = [12  13 0 -15 17 -13];
```

Note, that MATLAB reads vector entries as a vector of length *n+1* as an *n*-th order polynomial. Thus, if any of the given polynomial misses any coefficients, zero has to be entered for its coefficient. For instance, in the previous example, `0` is entered for the coefficient of x^3.

There are several functions that can be used to compute the roots of polynomials. They are as follows:

- Using the `roots()` MATLAB function
- Using the `zero()` Control System Toolbox function
- Using the `solve()` Symbolic MATH Toolbox function

You find roots of the given polynomial using the base MATLAB function, `roots()`.

```
>>x_sols=roots(f)

x_sols =

  -1.2403 + 0.9412i
  -1.2403 - 0.9412i
   0.7941
   0.3015 + 0.6869i
   0.3015 - 0.6869i
```

Note that the given polynomial has only one real value root and four complex valued roots.

The roots are computed by using the solve() function of MATLAB to find symbolic solutions of the polynomial, and then solutions are converted (note that conversion may be not necessary) to obtain a shorter number of decimal point numeric data using the double() function with the following entries in the Command Window:

```
>> syms x
>> Sol=solve(12*x^5+13*x^4-15*x^2+17*x-13)

Sol =

 root(z^5 + (13*z^4)/12 - (5*z^2)/4 + (17*z)/12 - 13/12, z, 1)
 root(z^5 + (13*z^4)/12 - (5*z^2)/4 + (17*z)/12 - 13/12, z, 2)
 root(z^5 + (13*z^4)/12 - (5*z^2)/4 + (17*z)/12 - 13/12, z, 3)
 root(z^5 + (13*z^4)/12 - (5*z^2)/4 + (17*z)/12 - 13/12, z, 4)
 root(z^5 + (13*z^4)/12 - (5*z^2)/4 + (17*z)/12 - 13/12, z, 5)

>> double(Sol)

ans =

   7.9411e-01 + 0.0000e+00i
  -1.2403e+00 + 9.4121e-01i
  -1.2403e+00 - 9.4121e-01i
   3.0155e-01 + 6.8690e-01i
   3.0155e-01 - 6.8690e-01i
```

Roots can be computed by using zero(), which is a function of Control Toolbox of MATLAB:

```
>>f_tf=tf(f,1, 's')

Transfer function:
12 s^5 + 13 s^4 - 15 s^2 + 17 s - 13
```

```
>>x_sols=zero(f_tf)

x_sols =

  -1.2403 + 0.9412i
  -1.2403 - 0.9412i
   0.7941
   0.3015 + 0.6869i
   0.3015 - 0.6869i
```

Note in this case, a transfer function (ratio of two polynomials) with a denominator of 1 in the s domain is created first. Then the values of the variable s are computed, which would make the polynomial equal to zero.

Simulink Model Based Solution of Polynomials

To solve polynomials via Simulink modeling, use the MATLAB Fcn block, the Constant block to input the polynomial coefficients, and the Display block to see the computed roots. Figure 7-24 shows the complete model saved as `Polynomial_Solver.slx`.

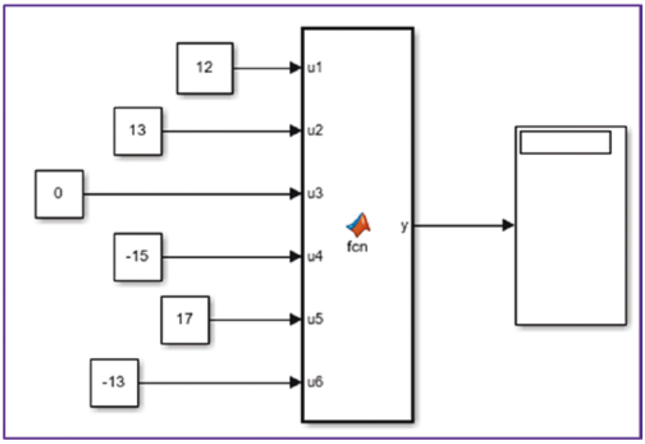


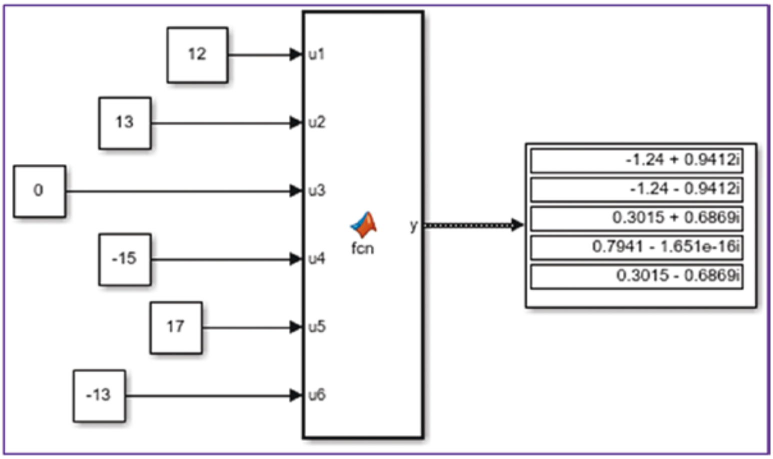
***Figure 7-24.** Simulink model to solve the polynomial* $12x^5 + 13x^4 - 15x^2 + 17x - 13 = 0$

The MATLAB Function block has the following command syntax embedded in it.

```
function y = fcn(u1, u2, u3, u4, u5, u6)
y = roots([u1, u2, u3, u4, u5, u6]);
```

The MATLAB Fcn block calls the MATLAB function `roots()` and computes the roots of the polynomial with respect to its coefficients given by the input variables u1, u2, … u6 since we are solving a fifth order polynomial. As it is, this model does not run and there are two more issues related to the size of the variables and solver type. First of all, the *solver type* has to be a fixed step size type. That can be adjusted via Simulation ➤ Model Configuration Parameters ➤ Solver Selection ➤ Fixed Step Solver. By default, the solver is a variable type.

Second, you need to change the size of the output variable y. You can do that by clicking on the ▤ Model Explorer ➤ [Model Hierarchy] ➤ Polynomial_Solver.slx ➤ MATLAB Function ➤ y Output ➤ Size. Set the size to 5 and click Apply. (The fifth order polynomial has five roots.) After clicking the Run button from the menu of the Simulink model window or pressing Ctrl+T from the keyboard, you'll see the results as displayed in Figure 7-25.



```
-1.24 + 0.9412i
-1.24 - 0.9412i
0.3015 + 0.6869i
0.7941 - 1.651e-16i
0.3015 - 0.6869i
```

Figure 7-25. *Complete model with computed roots of the polynomial* $12x^5 + 13x^4 - 15x^2 + 17x - 13 = 0$

The computed roots of the given polynomial match the ones computed by the MATLAB commands to four decimal places.

Eigen-Values and Eigen-Vectors

Eigen-values and eigen-vectors have broad applications, not only in linear algebra, but also in many engineering problems. For instance, they are used with vibrations, modal analysis, control applications, robotics, and so forth.

Definition 1. An *eigen-value* and *eigen-vector* of a square matrix A are, respectively, a scalar λ and a nonzero vector ν that satisfy

$$Av = \lambda v \tag{Equation 7.6}$$

Definition 2. Given a linear transformation A (a square matrix), a non-zero vector ν is defined to be an *eigen-vector* of the transformation if it satisfies the following *eigen-value* equation for some scalar λ:

$$Av = \lambda v \tag{Equation 7.7}$$

In this case, the scalar λ is called an *eigen-value* of A corresponding to the *eigen-vector* ν**.**

$$\underbrace{\begin{bmatrix} a & b & c \\ d & f & g \\ h & i & j \end{bmatrix}}_{[A]} \underbrace{\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}}_{[X]} = \underbrace{\lambda}_{[\lambda]} * \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{[X]}$$

$$[\ A\ ]*[\ \boldsymbol{X}\ ]-[\ \lambda]*[\ I\ ]*[\ \boldsymbol{X}\ ]=0 \tag{Equation 7.8}$$

Where $[I]$ is the identity matrix. Now by rearranging, the next formulation can be written:

$$\left([A]-[\lambda]*[I]\right)*[X]=0 \tag{Equation 7.9}$$

Assuming that there is an inverse matrix of the coefficient of $[X]$, *i.e.* $([A] - [\lambda] * [I])$.

$$\left([A]-[\lambda]*[I]\right)^{-1} \tag{Equation 7.10}$$

There can be other solutions apart from a trivial solution $[X] = 0$. So, this means $([A] - [\lambda] * [I]) = 0$ is obtained via determinant of this matrix equal to 0.

$$det\left\{[A]-[\lambda]*[I]\right\}=0 \tag{Equation 7.11}$$

The left side of Equation 7.11 is called a characteristic polynomial. So, when this equation is expanded, it will lead to a polynomial equation of λ. Use the following example to compute eigen-values and eigen-vectors:

$$\begin{cases} 2.3x_1 + 3.4\,x_2 - \quad 5x_3 = 0 \\ \;\;3x_1 + 2.4\,x_2 - \;1.5x_3 = 0 \\ \;2\,x_1 - 0.4\,x_2 - 7.2\,x_3 = 0 \end{cases}$$

Now, the given system's equations are written in matrix form.

$$\begin{bmatrix} 2.3 & 3.4 & 5 \\ 3 & 2.4 & -1.5 \\ 2 & -0.4 & -7.2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Eigen-values of this transformation matrix are defined to be:

$$det \begin{bmatrix} 2.3-\lambda & 3.4 & 5 \\ 3 & 2.4-\lambda & -1.5 \\ 2 & -0.4 & -7.2-\lambda \end{bmatrix} = 0$$

$$-7.884 + 49.12\,\lambda - 2.5\,\lambda^2 - \lambda^3 = 0$$

Solutions of this characteristic polynomial equation are $\lambda_1 = -8.434$; $\lambda_2 = 0.162$; $\lambda_3 = 5.772$.

Further, three eigen-vectors are computed by plugging in each eigen-value one by one into the equation. Hand calculations of eigen-values and eigen-vectors for larger systems are tedious and time consuming. For very large systems of linear equations, it is infeasible to compute eigen-values and eigen-vectors with hand calculations. All of these computations can be performed with a single built-in function of MATLAB, called `eig(A)`:

```
>> A=([2.3, 3.4, 5; 3, 2.4, -1.5; 2, -0.4, -7.2])
A =
    2.3000    3.4000    5.0000
    3.0000    2.4000   -1.5000
    2.0000   -0.4000   -7.2000
```

```
>> [v, lambda]=eig(A)
v =
    0.4725   -0.7649    0.6510
   -0.2479   -0.6366   -0.7276
   -0.8458   -0.0983    0.2164

lambda =
   -8.4345         0         0
         0    5.7726         0
         0         0    0.1619
>> A*v - v*lambda % Verify: eigen-vectors and eigen-values;
ans =
  1.0e-014 *

    0.9326    0.4441    0.1346
   -0.2220   -0.0888    0.1402
   -0.3553   -0.0222    0.0520
% Results are very close
```

Note that there are several different syntax forms of the `eig()` function to compute eigen-values and eigen-vectors of square arrays, and there is another command, called `eigs(A)`, to compute eigen-values and eigen-vectors.

```
d = eig(A)
d = eig(A,B)
[V,D] = eig(A)
[V,D] = eig(A,'nobalance')
[V,D] = eig(A,B)
[V,D] = eig(A,B,flag)
```

To evaluate the largest eigen-values and eigen-vectors:

```
d     = eigs(A)
[V,D] = eigs(A)
[V,D,flag] = eigs(A)
eigs(A,B)
eigs(A,k)
eigs(A,B,k)
eigs(A,k,sigma)
eigs(A,B,k,sigma)
eigs(A,K,sigma,opts)
eigs(A,B,k,sigma,opts)
```

Matrix Decomposition

The matrix decompositions have broad and valuable applications in many areas of linear algebra and engineering problem solving, for instance, solving linear equations, linear least squares, non-linear optimization, Monte-Carlo simulation, experimental data analysis, modal analysis, circuit design, filter design, and many more. There are a few types of matrix transformations and decompositions, including QR, LU, LQ, Cholesky, Schur, Singular Value decomposition, and so forth. We very briefly already discussed the command syntaxes of QR, LU, LQ, `chol()` Cholesky, and `svd()` Singular Value decompositions while solving the systems of linear equations. This section explains how to compute matrix decompostions by using MATLAB's built-in functions.

QR Decomposition

QR decomposition is also called orthogonal-triangular decomposition. It's the process of factoring out a given matrix as a product of two matrices. They are traditionally called the Q and R matrices, and they are the orthogonal matrix Q and the upper triangular matrix R.

$$A = QR \qquad \text{(Equation 7.12)}$$

$$Q^T Q = I \qquad \text{(Equation 7.13)}$$

Where *Q* is an orthogonal matrix; Q^T is a transpose of *Q*; *R* is an upper triangular matrix; and *I* is an identity matrix. The QR decomposition is based on the Gram-Schmidt method. More details of the Gram-Schmidt method can be found from the source of Wikipedia [1]. In MATLAB for the QR decomposition computation, there is a function called `qr()`. It has a few different syntax methods that evaluate Q, R, and other relevant matrices.

```
[Q,R] = qr(A)%Produces upper triangular matrix R & unit matrix Q
[Q,R] = qr(A,0) %Produces the economy-size decomposition
[Q,R,E] = qr(A) %Produces Q, R and permutation matrix E =>A*E = Q*R
[Q,R,E] = qr(A,0)%Produces economy-size decomposition: A(:,E) = Q*R
X = qr(A)  %Produces matrix X. triu(X) is upper triangular factor R
X = qr(A,0) %alike X = qr(A);
R = qr(A)  % Used when A is a sparse matrix and computes a Q-less
% QR decomposition and returns R.
```

Example: Computing QR Decomposition of a 5-by-5 Matrix

Let's take matrix [A] of size 5x5 generated from a normally distributed random number generator, called randn(). Compute the QR decompositions of the [A] matrix.

```
>> format short
>> A=randn(5,5)
A =
    1.2280   -0.5261    0.6770    0.9856    0.8005
    1.8766    1.3087    0.1504    2.0377   -0.0944
   -0.8958    0.6341    0.5682   -0.9155   -1.2224
    0.2791    1.0490   -0.1173    0.7488    1.2225
   -0.7456    0.3522    0.1206   -0.4902    0.1131
>> [Q, R]=qr(A)
Q =
   -0.4829    0.4184   -0.6628   -0.3755   -0.1068
   -0.7380   -0.5121   -0.0038    0.2917    0.3286
    0.3523   -0.4417   -0.7048    0.3635   -0.2279
   -0.1098   -0.5418    0.1633   -0.5900   -0.5654
    0.2932   -0.2720   -0.1932   -0.5419    0.7135
R =
   -2.5428   -0.5003   -0.1895   -2.5283   -0.8486
         0   -1.8346   -0.0140   -0.4991    0.2301
         0         0   -0.8922    0.2012    0.5091
         0         0         0   -0.2847   -1.5550
         0         0         0         0   -0.4484
```

```
>> [Q, R]=qr(A, 0)
Q =
   -0.4829    0.4184   -0.6628   -0.3755   -0.1068
   -0.7380   -0.5121   -0.0038    0.2917    0.3286
    0.3523   -0.4417   -0.7048    0.3635   -0.2279
   -0.1098   -0.5418    0.1633   -0.5900   -0.5654
    0.2932   -0.2720   -0.1932   -0.5419    0.7135
R =
   -2.5428   -0.5003   -0.1895   -2.5283   -0.8486
         0   -1.8346   -0.0140   -0.4991    0.2301
         0         0   -0.8922    0.2012    0.5091
         0         0         0   -0.2847   -1.5550
         0         0         0         0   -0.4484

>> [Q,R,E]=qr(A)
Q =
   -0.3790    0.4944    0.1187   -0.7732   -0.0060
   -0.7836   -0.3819   -0.4391    0.0709    0.2058
    0.3521   -0.5445   -0.3906   -0.5784   -0.3041
   -0.2879   -0.4732    0.7478   -0.0438   -0.3634
    0.1885   -0.2990    0.2851   -0.2465    0.8562
R =
   -2.6005   -0.8385   -0.9904   -0.1179   -2.4722
         0   -1.7067    0.4852   -0.0126    0.4692
         0         0    1.5604   -0.2610   -0.3322
         0         0         0   -0.8660   -0.1267
         0         0         0         0   -0.0886
E =
     0     0     0     0     1
     0     1     0     0     0
     0     0     0     1     0
     1     0     0     0     0
     0     0     1     0     0
```

```
>> A*E
ans =

    0.9856   -0.5261    0.8005    0.6770    1.2280
    2.0377    1.3087   -0.0944    0.1504    1.8766
   -0.9155    0.6341   -1.2224    0.5682   -0.8958
    0.7488    1.0490    1.2225   -0.1173    0.2791
   -0.4902    0.3522    0.1131    0.1206   -0.7456

>> Q*R
ans =
    0.9856   -0.5261    0.8005    0.6770    1.2280
    2.0377    1.3087   -0.0944    0.1504    1.8766
   -0.9155    0.6341   -1.2224    0.5682   -0.8958
    0.7488    1.0490    1.2225   -0.1173    0.2791
   -0.4902    0.3522    0.1131    0.1206   -0.7456
```

LU Decomposition

The LU decomposition or factorization is also called a modified form of the Gauss Elimination method and was introduced by Alan Turing [2]. It is defined as follows:

$$A = LU \qquad \text{(Equation 7.14)}$$

Where *A* is a rectangular matrix and *L* and *U* are the lower and upper triangular matrices, respectively. In MATLAB, the LU decomposition is evaluated using the following syntax of the built-in function `lu()`:

```
Y = lu(A)       %Produces matrix Y, for sparse A. Y contains only L
[L,U] = lu(A)   %Produces U and L
[L,U,P] = lu(A) %Produces U & L with a unit diagonal & permutation
                % matrix P
[L,U,P,Q] = lu(A)     % Produces U, L, and row permutation matrix P
                      % and column reordering matrix Q, so that P*A*Q = L*U
[L,U,P,Q,R] = lu(A)   % Produces U,L, & permutation matrices P and %Q,
                        diagonal scaling matrix R so that P*(R\A)*Q = L*U
                        for sparse %non-empty A.
```

```
[...] = lu(A,'vector') %Produces the permutation information in two %row
vectors p and q. A user can specify from 1 to 5 outputs.
[...] = lu(A,thresh)
[...] = lu(A,thresh,'vector')
```

Example: Computing LU Composition of a 3-by-3 Pascal Matrix

Let's compute L, U, and other (P, Q, R) matrices from any given rectangular matrix. For this task, you write a small script called `LU_decomposition.m` with the MATLAB's built-in function, `lu()`. The script takes one user entry (input), which has to be a rectangular matrix. You'll employ in this script another built-in function of MATLAB, called `issparse()`. It identifies whether the user-entered matrix is a sparse matrix or not.

```
% LU_decomposition.m
A=input('Enter rectangular matrix: ');
if  issparse(A)
    Y = lu(A)            %#ok
    [L,U,P,Q] = lu(A)    %#ok
    disp('  oops more  ')
    [L,U,P,Q,R] = lu(A)  %#ok
    [L, U, P, Q, R] = lu(A,'vector') %#ok
else
    [L,U] = lu(A)        %#ok
    [L,U,P] = lu(A)      %#ok
    % Check evaluation results:
    ERROR=P*A-L*U        %#ok
    [L,U,P] = lu(A, 'vector')        %#ok
end
```

Run the script `LU_decomposition.m` and enter a standard matrix, called `pascal(3)`, as an input matrix.

```
Enter rectangular matrix: pascal(3)

L =
    1.0000         0         0
    1.0000    0.5000    1.0000
    1.0000    1.0000         0
```

```
U =
    1.0000    1.0000    1.0000
         0    2.0000    5.0000
         0         0   -0.5000
L =
    1.0000         0         0
    1.0000    1.0000         0
    1.0000    0.5000    1.0000
U =
    1.0000    1.0000    1.0000
         0    2.0000    5.0000
         0         0   -0.5000
P =
     1     0     0
     0     0     1
     0     1     0

ERROR =
     0     0     0
     0     0     0
     0     0     0
L =
    1.0000         0         0
    1.0000    1.0000         0
    1.0000    0.5000    1.0000

U =
    1.0000    1.0000    1.0000
         0    2.0000    5.0000
         0         0   -0.5000
P =
     1     3     2
```

Rerun the script and use a sparse matrix of size 3-by-3 as input.

```
Enter rectangular matrix: sparse(3)

Y =

   (1,1)        3
L =

   (1,1)        1
U =
   (1,1)        3
P =
   (1,1)        1
Q =
   (1,1)        1
  oops more

L =
   (1,1)        1
U =
   (1,1)        1
P =
   (1,1)        1
Q =
   (1,1)        1
R =
   (1,1)        3
```

Cholesky Decomposition

The Cholesky decomposition is particularly important for Monte Carlo simulations and Kalman filter designs. This type of matrix factorization is only applicable to square matrices and to Cholesky triangles, which are decompositions of positive and definite matrixes that is decomposed into a product of a lower triangular matrix and its transpose. The Cholesky decomposition [3, 4] can be expressed via the following formulation.

$$A = U^T U \qquad \text{(Equation 7.15)}$$

Where A is a square matrix and U and $U^T$are an upper triangular matrix and its transpose, respectively. This formulation can be written with lower triangular matrix (L) and its transpose (L^T) as well.

$$A = LL^T \qquad \text{(Equation 7.16)}$$

In MATLAB, the Cholesky decompositions are evaluated using the following syntax options of the MATLAB's built-in function, `chol()`.

```
R = chol(A) % Produces an upper triangular matrixRsatisfying:R'*R=A
L = chol(A,'lower') % Produces a lower triangular matrix R satisfying:
                    % L*L'=A
[R,p] = chol(A) %Produces an upper triangular matrix R and p is 0
[L,p] = chol(A,'lower') %Produces lower triangular matrix R&p is 0
[R,p,S] = chol(A) % When A is a sparse matrix, produces a permutation
                  % matrices S and R, and p that can be zero or non-zero
[R,p,s] = chol(A,'vector') % Produces the permutation information %as a
vector 's'
[L,p,s] = chol(A,'lower','vector') % Produces a lower triangular matrix
                                   % L and a permutation vector 's'
```

Note Using `chol` (the Cholesky decomposition operator) is preferable over the `eig` (Eigen-value and eigen-vector) operator for determining positive definiteness.

To evaluate the Cholesky decompositions of any given matrix (a user entered matrix), you write the next script, called `Chol_decoposition.m`, by considering the requirements and properties of the Cholesky decompositions to compute decompositions of any matrix with respect to the formulations in Equations 7.15 and 7.16. It takes one input, which is a user entry matrix. Note that in this script, we used `disp()`, `size()`, `det()`, `run()`, and a pop-up dialog box command, `warndlg()`.

```
% Chol_decomposition.m
clearvars; clc
disp('Note your matrix must be square & positive definite!!!')
disp('NB: Positive means all determinants must be positive.')
disp('You can enter as matrix elements ')
disp('or define your matrix 1st, ')
disp('and then just enter your matrix name')
disp('      ')
A=input('Enter a given Matrix: ');
[rows, cols]=size(A);
for k=1:rows
    % Determinants are computed
    Det_A(k)=det(A(1:k, 1:k));
end
if rows==cols
  if true(Det_A>0)==1
     if issparse(A)
            [R,p,S] = chol(A) %#ok
            [R,p,s] = chol(A,'vector');
            [L,p,s] = chol(A,'lower','vector');
else
            R = chol(A) %#ok % Upper triangular matrix R: R'*R=A
            L = chol(A,'lower') %#ok % Lower triangular matrix R.
            [R,p] = chol(A);
% Verify:
Error_up  = A-R'*R;
Error_low = A-L*L';
disp('Error is with upper triangular matrix: ')
disp(Error_up)
disp('Error is with lower triangular matrix:')
disp(Error_low)
       end
else
warndlg('Sorry your matrix is not positive and definite!')
warndlg('Try again!!!')
run('Chol_decomposition')
    end
end
```

You can test the script with different input entries (matrices). Let's use a 4-by-4 standard matrix generated with pascal().

```
Note your matrix must be square & positive definite!!!
You can enter as matrix elements
or define your matrix 1st,
and then just enter your matrix name

Enter a given Matrix: pascal(4)

R =
     1     1     1     1
     0     1     2     3
     0     0     1     3
     0     0     0     1

L =
     1     0     0     0
     1     1     0     0
     1     2     1     0
     1     3     3     1

Error is with upper triangular matrix:
     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0

Error is with lower triangular matrix:
     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0
```

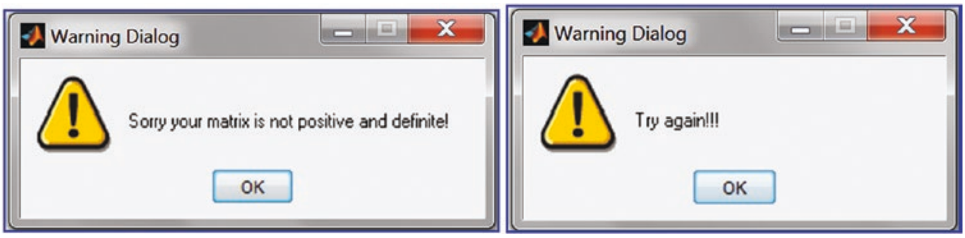
Now, consider a magic matrix of size 3-by-3.

```
>> run('Chol_decomposition')
Note your matrix must be square & positive definite!!!
You can enter as matrix elements
```

```
or define your matrix 1st,
and then just enter your matrix name
```

```
Enter a given Matrix: magic(3)
```

After running the script with an input entry of a magic square matrix of size 3-by-3, the warning dialog boxes shown in Figure 7-26 appear.



Sorry your matrix is not positive and definite!

Try again!!!

Figure 7-26. Warnings showing that the input matrix is not positive and definite and so cannot compute the Cholesky decompositions

The `Chol_decoposition.m` script identifies the Cholesky decomposition properties and computes the Cholesky decomposition of a user-entered matrix. It detects a matrix type and works for given square and positive definite matrices with the MATLAB built-in function `chol()`.

Schur Decomposition

The Schur decomposition has many applications in numerical analyses, including image-processing areas in combination with other matrix decompositions or factorization tools. The Schur decomposition of a complex square matrix [A] is defined as a matrix decomposition [5]:

$$Q^H AQ = T = D + N \qquad \text{(Equation 7.17)}$$

Where Q is a unitary matrix, Q^H is a conjugate transpose of Q, and T is an upper triangular matrix that's equal to sum of a matrix $D = \ diag\,(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n)$ a diagonal matrix consisting of eigen-values λ_i of A,and strictly upper triangular matrix N. The Schur decomposition can be computed via the MATLAB's built-in function, `schur()`.

```
T = schur(A)       % Produces the Schur matrix of A
T = schur(A, flag) % Produces the Schur matrix for two cases.
%{
for real matrix A, returns a Schur matrix T in one of two forms depending
on the value of flag:

'complex'  T is triangular and is complex if A has complex eigenvalues.
'real'     T has the real eigen-values on the diagonal and the complex eigen-
           values in 2-by-2 blocks on the diagonal. 'real' is the default.

%}
[U,T] = schur(A,...)
```

Let's look at several examples of standard matrices and compute their Schur decompositions:

```
>> A=magic(5); B=pascal(3); C=round(randn(5,5)*10);
>> T=schur(A)
T =
   65.0000    0.0000   -0.0000   -0.0000   -0.0000
         0  -21.2768   -2.5888    2.1871   -3.4893
         0         0  -13.1263   -3.3845   -2.8239
         0         0         0   21.2768    2.6287
         0         0         0         0   13.1263
>> T=schur(B)
T =
    0.1270         0         0
         0    1.0000         0
         0         0    7.8730
>> T=schur(C)
T =
  -30.8100    7.2703    8.3016   -6.8105   -2.3533
         0   12.1987  -20.5925    3.4241    6.1840
         0    4.7364   12.1987   21.5183   -7.7496
         0         0         0    2.7859    6.9060
         0         0         0         0   12.6268
```

```
>> [T, U]=schur(A, 'complex')
T =
   -0.4472    0.0976   -0.6331    0.6145   -0.1095
   -0.4472    0.3525    0.7305    0.3760    0.0273
   -0.4472    0.5501   -0.2361   -0.6085    0.2673
   -0.4472   -0.3223    0.0793   -0.3285   -0.7628
   -0.4472   -0.6780    0.0594   -0.0535    0.5778

U =
   65.0000    0.0000   -0.0000   -0.0000   -0.0000
         0  -21.2768   -2.5888    2.1871   -3.4893
         0         0  -13.1263   -3.3845   -2.8239
         0         0         0   21.2768    2.6287
         0         0         0         0   13.1263

>> [T, U]=schur(B, 'real')
T =
   -0.5438   -0.8165    0.1938
    0.7812   -0.4082    0.4722
   -0.3065    0.4082    0.8599

U =
    0.1270         0         0
         0    1.0000         0
         0         0    7.8730

>> [T, U]=rsf2csf(U,T) % Convert real Schur form to complex Schur form

T =
    0.0914   -0.0882         0
   -0.6667   -0.6912   -0.2790
    1.5247    1.5808   -7.5604
U =
    0.2645    0.0569    0.0744
         0   -1.3538    0.0172
   -0.3065         0    0.9971
```

Singular Value Decomposition (SVD)

The SVD has many applications in signal processing, statistics, and image processing areas. It is formulated as a product of three matrices, which are an orthogonal matrix (U_{ii}), a diagonal matrix (D_{ij}), and the transpose of an orthogonal matrix (V_{jj}), if a given matrix A_{ij} is an i by j sized real matrix with $i > j$.

$$A_{ij} = U_{ii} D_{ij} V_{jj}^{\ T} \qquad \text{(Equation 7.18)}$$

Where, $U_{ii}^T U_{ii} = I, V_{jj}^{\ T} V_{jj} = I$. Diagonal entries of D_{ij} are known as singular values of A_{ij}. Moreover, there are a few other important properties of the SVD.

- Left-singular vectors of A_{ij} are eigen-vectors of $A_{ij}A_{ij}*$
- Right-singular vectors of A_{ij} are eigen-vectors of $A_{ij}*A_{ij}$
- Non-zero singular values (on the diagonal entries of D_{ij}) of A_{ij} are square roots of the non-zero eigen-values of both $A_{ij}*A_{ij}$ and $A_{ij}A_{ij}*$

There are a few ways to evaluate the SVD, singular values, and vectors of any given matrix. You use `svd()` and `svds()`, which are MATLAB built-in functions.

```
s = svd(A)        %Produces a vector of singular values
[U,D,V] = svd(A) %Produces a diagonal matrix D of the same dimension
%as A, with nonnegative diagonal elements in decreasing order, and
% unitary matrices U and V so that X = U*D*V'.
[U,D,V] = svd(A,0) % Produces the "economy size" decomposition. If A
% is  m-by-n with m > n, then SVD computes only the first n columns of
%U and D is n-by-n.
s = svds(A)
s = svds(A,k)
s = svds(A,k,sigma)
s = svds(A,k,'L')
s = svds(A,k,sigma,options)
[U,D,V] = svds(A,...)
[U,D,V,flag] = svds(A,...)
```

Now, take two matrices (of size 2-by-3 and 3-by-3) and evaluate their SVDs.

```
>> A=ceil(randn(2,3)*10); B=pascal(3);
>> A
A =
    26     2   -19
    -6     0    -4
>> s = svd(A)
s =
   32.3640
    6.7506
>>[U,D,V] = svd(A)
U =
   -0.9968    0.0801
    0.0801    0.9968
D =
   32.3640         0         0
         0    6.7506         0
V =
   -0.8156   -0.5774   -0.0366
   -0.0616    0.0237    0.9978
    0.5753   -0.8161    0.0549
>> [U,D,V] = svd(A,0)
U =
   -0.9968    0.0801
    0.0801    0.9968
D =
   32.3640         0         0
         0    6.7506         0
V =
   -0.8156   -0.5774   -0.0366
   -0.0616    0.0237    0.9978
    0.5753   -0.8161    0.0549
```

```
>> s = svds(A)
s =
   32.3640
    6.7506

>> s = svds(B)
s =
    7.8730
    1.0000
    0.1270

>> s = svds(B, k)
s =
    7.8730
    1.0000
>> s = svds(A,k,'L')
s =
   32.3640
    6.7506
```

Logic Operators, Indexes, and Conversions

MATLAB uses logic 1 and logic 0 for system variables to denote logic values for true and false, respectively. Variables of logical values are distinguished by a logical data type. Table 7-2 is a list of logic operators and their operational functions used in MATLAB.

Table 7-2. *Logical Expressions and Operators in MATLAB*

Operator	Operation
`true, false`	Setting logical value
`& (and), | (or), ~ (not), xor, any, all`	Logical operations
`&&, ||`	Short-circuits operations
`bitand, bitcmp, bitor, bitmax, bitxor, bitset, bitget, bitshift`	Bitwise operations
`==(eq), ~=(ne), <(lt), >(gt), <=(le), >=(ge)`	Relational operations
`strcmp, strncmp, strcmpi, strncmpi`	String comparisons

Note To get a complete list of relational operators, their functions, and how to use them, type `>> help relop` in the Command Window.

Logical Indexing

Logic operators are one of the most central and essential keys to any programming language. Logic operators introduce another method for accessing data in MATLAB variables. For instance, given a magic matrix [A] of size 5-by-5, say you need to separate out the elements of [A] that are equal to or less than 13.

```
>> A=magic(5)
A =
    17    24     1     8    15
    23     5     7    14    16
     4     6    13    20    22
    10    12    19    21     3
    11    18    25     2     9
>> A_index=(A<13 | A==13) % Logical indexing
A_index =
  5×5 logical array
   0   0   1   1   0
   0   1   1   0   0
   1   1   1   0   0
   1   1   0   0   1
   1   0   0   1   1
>> A(A_index)            % Elements are taken out
ans =
     4
    10
    11
     5
     6
    12
     1
     7
    13
     8
     2
     3
     9
>> % Or directly:
>> A(A<13 | A==13);
```

Let's explore the logical indexing properties further via examples to select matrix elements.

```
>> E = eye(5)
E =
     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1
>> A = magic(5); A(E)
Array indices must be positive integers or logical values.
```

```
>> E_L = logical(eye(5))
E_L =
  5×5 logical array
   1   0   0   0   0
   0   1   0   0   0
   0   0   1   0   0
   0   0   0   1   0
   0   0   0   0   1
>> A(E_L)            % With the logical matrix E, diagonal elements of A
ans =
    17
     5
    13
    21
     9
```

Note The previous example demonstrates the identity matrix [E] (whose elements are 1s and 0s), which is not equivalent to the logic matrix [E_L] (whose elements are also 1s and 0s).

Moreover, there are a number of functions/commands (e.g., the `is*()` command) that can be used to find out whether the input is of a specified type of variable contains any elements of a particular type, or whether such a variable or file exists, and so forth. All of these functions can be used for logical indexing. Let's look at a few simple examples:

```
>> x=13; isnumeric(x)     % whether x is a numeric data or not?
ans =
         1
>> x=13; islogical(x)     % whether x is a logical data or not?
ans =
         0
>> x=13; islogical(x>110) % whether the operation (x>110) is logic or not
ans =
          1
>> x = 13; isempty(x)  % whether x is an empty or not
ans =
         1
>> x = [ ]; isempty(x) % whether x is an empty or not
ans =
 logical
   1
>> x = [1 , 2; 3, 4]; iscell(x)  % whether x is a cell array or not
 ans =
  logical
   0
```

```
>> x = [1, 0; 0, 4];
>> X_x = x/0; isnan(X_x)  % whether any elements of X_x are not-a-number
ans =
  2×2 logical array
   0   1
   1   0
>> exist('X_x', 'var')    % whether the variable called X_x exists or not
ans =
     1
```

In the previous example, zero divide by zero (0/0) is defined to be NaN (i.e. not-a-number) in MATLAB.

Note The logical indexing operations have particular importance in matrix/array operations, programming, data analysis, and processing since they can be used to sort out, locate, or change particular elements of matrices/arrays/data sets.

Example: Logical Indexing to locate and Substitute Elements of A Matrix

Let's look at another example. Given a matrix [A] of size 3-by-3 with some elements equal to infinity.

$$A = \begin{bmatrix} 17 & \infty & -6 \\ 5 & -3 & 11 \\ \infty & 13 & \infty \end{bmatrix}$$

How do you substitute the elements equal to inf with 1000 and all negative valued elements with 0? This task can be solved easily using logical indexing operations.

```
>> A=[17   Inf     -6 ; 5 -3   11; Inf  13  Inf]  % [A] is entered
A =
    17   Inf    -6
     5    -3    11
   Inf    13   Inf
```

```
>> A_inf = (A==1/0)    % Find out which elements of A are equal to inf
A_inf =
  3×3 logical array
   0   1   0
   0   0   0
   1   0   1
>> A(A_inf)=1000      % Set the found elements equal to 1000
A =
          17        1000          -6
           5          -3          11
        1000          13        1000
>> A_0 = A<0        % Find out which elements of A are negative
A_0 =
  3×3 logical array
   0   0   1
   0   1   0
   0   0   0
>> A(A_0)=0          % Set the found elements equal to 0
A =
          17        1000           0
           5           0          11
        1000          13        1000
```

Note The division of any value by 0 gives the value of `Inf` in MATLAB.

Let's look at another example. Given a matrix [A] of size 4-by-5 with `NaN` (not a number) and `inf` (infinity) elements. How do you substitute `NaN` elements with 0 and `inf` with 100? This task can be solved easily with logical indexing similar to the afore-demonstrated example.

```
>> A = [2 -3 -2 -3 1; Inf -2  3 -1  NaN; -3 0 Inf 3 2; 3 NaN 0 2 Inf]
A =
     2    -3    -2    -3     1
   Inf    -2     3    -1   NaN
    -3     0   Inf     3     2
     3   NaN     0     2   Inf
```

```
>> A_nan=isnan(A)
A_nan =
  4×5 logical array
   0   0   0   0   0
   0   0   0   0   1
   0   0   0   0   0
   0   1   0   0   0
>> A(A_nan)=0
A =
     2    -3    -2    -3     1
   100    -2     3    -1     0
    -3     0   100     3     2
     3     0     0     2   100
```

Note that this section contains rather simple and small examples to demonstrate how easily you can substitute specific (valued) elements of a matrix using logical indexing operations. This technique (logical indexing or relational operators) can be applied to matrices, arrays, and data sets of any size. Therefore, the logical indexing is particularly useful in analysis and processing of large data sets. It is fast and efficient and does not require any additional effort to program with loop (`for ... end`, `while ... end`) and conditional (`if .. elseif ... else ... end`) operators.

Conversions

There are many examples in signal processing where you need to convert something. Analog to digital converters and vice versa, data processing and analysis, and programming when analog signal data format or type needs to be converted into digital or vice versa. For instance, to resolve memory issues in image processing, you might need to convert decimal (double) formatted data into binary numbers. That can be easily accomplished in MATLAB using `DEC2BIN().` Conversely, `BIN2DEC()` is used to convert binary strings into decimal (double) type of data. `DEC2BIN(D)` returns the binary representation of *D* as a string. *D* must be a non-negative integer smaller than 2^{52}. `DEC2BIN(D,N)` produces a binary representation with at least *N* bits.

Another conversion example is character conversion. You need to convert numbers into character strings and vice versa. MATLAB uses the `CHAR()` command to convert numbers into ASCII/ANSI formatted characters, `DOUBLE()` to convert characters and symbolic representations of numbers into double precision format, STR2NUM() to convert strings into binary numbers, and NUM2STR() to convert any number into a string. Let's consider several examples of employing these conversion commands:

```
>> dec2bin(11) % DEC2BIN - Convert decimal integer to a binary string
ans =
         1011
>> dec2bin(23)
10111
>> dec2bin(22)
ans =
         10110
>> x=13.125/5.5;
>> dec2bin(x)
10
>> dec2bin(11.11)
ans =
         1011
>> dec2bin(11)
ans =
         1011
>> bin2dec('1101') % Converts a binary number into decimal one
ans =
    13
>> bin2dec('10110')
ans =
    22
>>dec2bin(64)
ans =
10000000
>> char(bin2dec('10000000'))
ans =
    @
>>  G='MatLab'
G =
MatLab
```

```
>> G0=G+0
G0 =
    77    97   116    76    97    98
>> d2bG0=dec2bin(G0)
d2bG0 =
1001101
1100001
1110100
1001100
1100001
1100010
>> b2dG0=bin2dec(d2bG0)
b2dG0 =
    77
    97
   116
    76
    97
    98
>> char(b2dG0)'
ans =
MatLab
>> num2str(123)
ans =
    '123'
>> num2str('matlab')
ans =
    'matlab'
>> ans+0
ans =
   109    97   116   108    97    98
```

Example: Creating Character Strings with char()

Create the following letters in a progressive format by writing a script that has one input argument that has to be an integer. All the other letters need to be generated programmatically.

a

b c

d e f

g h i j

k l m n o

These characters can be generated in several ways. First, you need to determine the ASCII/ANSI numeric representation of a. Then you can generate all the other letters.

```
>> format short
>> double('a')
ans =
    97
>> char(97)
ans =
    'a'
>> double('b')
ans =
    98
```

The letter as numeric representation in ASCII/ANSI is 97, b is represented by 98, and so forth. Based on these, you can generate linear space of integers starting at 97 and convert them to character strings one row at a time. In other words, you display one character on the first row, two characters on the second, three in the third row, etc. Here is the complete script (print_character.m), which prints the letters in progressive order.

```
% print_character.m
% Part 1.
Start = 97;
```

```
for ii = 1:5
    for jj = 1:ii
        fprintf(char(Start));
        Start = Start+1;
    end
    fprintf('\n')
end
```

Here is the result of the script.

```
a
bc
def
ghij
klmno
```

Let's consider the following example, which prints a series of uppercase characters:

ABCDEF

GHIJK

LMNO

PQR

ST

U

This example is similar to the previous example with a few small differences—it requires uppercase characters, starts with six letters, and reduces in the following rows.

Again, you can determine the numerical representation of `A` in ANSI/ASCII with the following commands:

```
>> double('A')
ans =
    65
>> 'A' + 0    % An alternative way:
ans =
    65
```

So now you know that the numerical representation of A is 65. You can then edit the script (print_characters.m) by introducing two small changes and then write this script.

```
%% Part 2
Start = 65;
for ii = 6:-1:1
    for jj = 1:ii
        fprintf(char(Start));
        Start = Start+1;
    end
    fprintf('\n')
end
```

When you execute this script, you obtain the following output in the Command Window:

```
ABCDEF
GHIJK
LMNO
PQR
ST
U
```

Via a few examples, this section discussed logic operators, conversions, and indexing issues briefly. Applications of the issues of conversions are demonstrated via more extended examples in other chapters.

Summary

This chapter introduced linear algebra, matrix operations, vector spaces, polynomials, methods of solving linear systems of equations, and matrix decompositions and conversions. Via examples, you learned how to use MATLAB's built-in functions and commands, how to develop Simulink blocks in association with the MATLAB Command Window, and how to use functions and the MATLAB Fcn block. The following MATLAB functions were discussed and explained in examples.

- Matrix operations +,-, *, and /
- Elementwise operations .*, .^, and ./
- Backslash operator (\) and mldivide()
- Solving linear equations with linsolve()
- Matrix inverse operators inv() and pinv()
- Eigen-values and eigen-vectors eig()
- Polynomial solvers roots(), solve(), and zero()
- Symbolic math equation solver solve()
- Standard matrices and gallery matrices, magic(), gallery(), and sparse()
- Vector spaces linspace() and logspace()
- Matrix operations and factorization methods, such as QR, LU, Cholesky, SVD, Schur: qr(), lu(), chol(), svd(), schur(), and decomposition()
- Logical operators (<=, ~=, >=, |, &..., is*(),) and indexing options
- Conversion tools and operators (bin2dec, dec2bin, double, and char)

References

[1] Wikipedia, http://en.wikipedia.org/wiki/Gram-Schmidt_process, viewed on September 19, 2013.

[2] Bunch, James R.; Hopcroft, John (1974), "Triangular Factorization and Inversion by Fast Matrix Multiplication," *Mathematics of Computation* 28: 231–236, ISSN 0025-5718.

[3] Gentle, J. E. "Cholesky Factorization." §3.2.2 in *Numerical Linear Algebra for Applications in Statistics.* Berlin: Springer-Verlag, pp. 93-95, 1998.

[4] Nash, J. C. “The Choleski Decomposition.” Ch. 7 in *Compact Numerical Methods for Computers: Linear Algebra and Function Minimisation, 2nd ed.* Bristol, England: Adam Hilger, pp. 84-93, 1990.

[5] Mathworld , `http://mathworld.wolfram.com/SchurDecomposition.html`, viewed on September 20, 2013.

Exercises for Self-Testing

Exercise 1

Solve the following equations for variables *x*, *y*, and *z*

$$\begin{cases} 3x+5y+4z=-2 \\ -2x+3y-2z=2 \\ x+6\left(y-\dfrac{z}{2}\right)=0 \end{cases}$$

1. Use the backslash (\) operator or `mldivide()` to solve the given system of equations.
2. Use the inverse matrix method `inv()` to solve the given system of equations.
3. Use the `linsolve()` function to solve the given system of equations.
4. Use the `solve()` function to solve the given system of equations.
5. Use `chol()` to solve the given system of equations.
6. Use Simulink blocks to solve the given system of equations.
7. Compute errors by computing norms for each of the methods.

Exercise 2

Solve the following equations, using the matrix inverse.

$$\begin{cases} 2q_1 + 9q_2 + 3q_3 = 15 \\ 13q_1 + 2q_2 - 5q_3 = 11 \\ q_1 - 2q_2 + 2q_3 = 9 \end{cases}$$

1. Use the inverse matrix method `inv()` to solve the given system of equations.
2. Use the least squares method `lsqr()` to solve the given system of equations.
3. Use the Gauss Elimination method with the `lu()` function to solve the given system of equations.
4. Use the `solve()` function to solve the given system of equations.
5. Use Simulink blocks to solve the given system of equations.
6. Compare the accuracy (to eight decimal places) of each solution.

Exercise 3

Solve the following equations:

$$\begin{cases} 2.5x_1 - x_2 + 3.3x_3 = 5 \\ -2.2x_1 + 2x_2 - 5x_3 = -2 \\ x_1 - 2x_2 + 2.5x_3 = 3 \end{cases}$$

1. Use the inverse matrix method `qr()` to solve the given system of equations.
2. Use the reduced row echelon method `rref()` to solve the given system of equations.
3. Use the `decomposition()` function to solve the given system of equations.
4. Use the `solve()` function to solve the given system of equations.
5. Use Simulink blocks to solve the given system of equations.
6. Compare the accuracy (to 10 decimal places) of these four methods.

Exercise 4

Solve the following equations.

$$\begin{cases} 2.5x_1 - x_2 + 3.3x_3 + 1.3x_4 - 0.3x_5 = 11 \\ -1.2x_1 + 2x_2 - 5x_3 - 2.1x_4 + 5.2x_5 = -2 \\ x_1 - 2x_2 + 2.5x_3 = 3 \\ 2x_1 + x_2 - 5x_3 - 3x_4 - 2.4x_5 = -4 \\ 3x_1 - 2.2x_4 - 1.5x_5 = -5 \end{cases}$$

1. Use the inverse matrix method `inv()` to solve the given system of equations.
2. Use the singular decomposition `svd()` method to solve the given system of equations.
3. Use the `linsolve()` function to solve the given system of equations.
4. Use the `solve()` function to solve the given system of equations.
5. Use Simulink blocks to solve the given system of equations.
6. Compare the accuracy (to 13 decimal places) of these four methods.

Exercise 5

Given:

$$3x + 6y - cz = 0$$
$$2x + 4y - 6z = 0$$
$$x + 2y - 3z = 0$$

- Find for which values of *c* the set of equations has a trivial solution
- Find for which values of *c* the set of equations has an infinite number of solutions
- Find relations between *x*, *y*, and *z*.

Exercise 6

Find the inverse of the given matrix:

$$A = \begin{bmatrix} 3 & 6 & -12 \\ 2 & 4 & -6 \\ 1 & 2 & -3 \end{bmatrix}$$

- Explain why the given matrix does not have an inverse.
- Compute the determinant of the matrix.
- Find eigen-values and eigen-vectors of the given system by using `eig()`.
- Find eigen-values by using `roots()`.

Exercise 7

Find the inverse of the given matrix:

$$A = \begin{bmatrix} 3 & 6 & -2 \\ 1 & 2 & -4 \\ 0 & 1 & -3 \end{bmatrix}$$

- Compute determinant of the matrix;
- Find eigen-values and eigen-vectors of the given system using `eig()`.
- Find eigen-values using `roots()`.

Exercise 8

Find a solution to the following set of equations representing an underdetermined system, using the left division (\ backslash) method and the pseudo-inverse method (`pinv`). Compare your obtained results and discuss the differences.

$$\begin{gathered} 2.5x_1 - x_2 + 3.3x_3 + 1.3x_4 - 0.3x_5 = 11 \\ -1.2x_1 + 2x_2 - 5x_3 - 2.1x_4 + 5.2x_5 = -2 \\ x_1 - 2x_2 + 2.5x_3 = 3 \end{gathered}$$

Exercise 9

Solve the following set of equations using the backslash (\) operator, as well as the `linsolve()`, `inv()`, `lsqr()`, and `solve()` functions:

$$
\begin{gathered}
2x-3y=5\\
6x+10y=70\\
10x-4y=53
\end{gathered}
$$

Exercise 10

Show why there is no solution to the following set of equations:

$$
\begin{gathered}
-2x-3y=2\\
-3x-5y=7\\
5x-2y=-4
\end{gathered}
$$

Exercise 11

Create the matrix [*C*] from the given two [*A*] and [*B*] matrices by using logic operators. Explain why some of the elements of new array are zeros.

$$
C=\begin{bmatrix}0&0&-3\\1&0&0\\0&0&1\\1&0&-2\\2&1&0\\0&0&1\end{bmatrix},\ A=\begin{bmatrix}3&6&-3\\1&2&-2\\0&1&1\\1&3&-2\\2&1&-1\\1&0&1\end{bmatrix},\ B=\begin{bmatrix}1&2&-3\\1&1&-3\\-1&-2&1\\1&1&-1\\2&2&-2\\-2&0&1\end{bmatrix}
$$

Hints: Use logic operators (`<`,`=`) and element-wise matrix multiplication.

Exercise 12

Perform the following:

- Obtain an eye matrix of the size 5-by-5 from the magic matrix of the size of 5-by-5.
- Create a square eye matrix of the size 10-by-10 from the random square matrix of the size 10-by-10.
- Obtain a replicated square matrix of size 3-by-9 from the gallery matrix pascal() of size 3-by-3.

Exercise 13

Solve the following equations and discuss the solutions for two cases: $a = 13$ and $a = 29$.

$$\begin{cases} q_1 + q_2 = 1 \\ 13q_1 + 23q_2 = a \\ q_1 - 2q_2 = 9 \end{cases}$$

Write a script with logic and loop operators (`if`, `break`, `for`, and `end`) to find such value of a that gives real solutions to these equations. Consider that a has an integer value that lies within 1 to 50.

Hints: Use the `rank()` function and backslash (`\`) operators.

Exercise 14

Solve the following polynomials with `roots()`, `solve()`, `zero()`, and the Simulink model.

- $2u^9 - 3u^8 + 5u^4 - 13u^2 - 131 = 0$
- $y^7 + 5y^5 - \frac{13y^4}{4} - 11y^3 + 9y + 3 = 0$
- $5x^5 + \frac{4x^4}{11} + \frac{3x^3}{13} - x^2 - 269x = 13$

Exercise 15

Create a logarithmic spaced array (a row vector) *B* of numbers starting with 10 and ending with 100, and create *BB* column vector from a row vector *B*.

Exercise 16

Play a sound that is defined in the next expression:

$$S(t)=\cos(2\pi tf_1)+\cos(2\pi tf_2)+\cot(2\pi tf_3)+\tan(2\pi tf_4)+\tan(2\pi tf_5),$$

Where $f_s = 10000$ *Hz* (*sampling frequency*); $t = 13$ *sec.* (*time length*); $f_1 = 100$ *Hz* (1*st signal*); $f_2 = 200$ *Hz* (2*nd signal*); $f_3 = 300$ *Hz* (3*rd signal*); $f_4 = 600$ *Hz* (4*th signal*); $f_5 = 700$ *Hz* (5*th signal*).

Exercise 17

Answer the following questions using MATLAB:

- What are the binary representations of decimal numbers 123, 123.123, 321, 321.123, 223, 322, 333, and 333.3?
- Why are the binary representations of 123 vs. 123.123, 321 vs. 321.123, and 333 vs. 333.3 the same?

Exercise 18

Answer the following questions using MATLAB:

- What are the decimal representations of the binary numbers 1001, 01010, 111100, 0101011?
- What are character representations of the binary numbers 1001, 01010, 111100, 0101011?

Exercise 19

Write a script that takes one input number (an integer) and prints out the following characters in the order in the Command Window.

```
A
BCD
EFGHI
JKLMNOP
QRSTUVWXY
```

Exercise 20

Use numeric values of matrices [A] and [B] from Exercise 11 to evaluate the QR, LU, LQ, Cholesky, Schur, and Singular Value Decompositions. Explain why some of the decompositions (matrix factorizations) of [A] and [B] cannot be computed.

Exercise 21

Create the Hilbert matrix of size 5-by-5 using gallery matrix functions and compute Cholesky decomposition using the `Chol_decoposition.m` script. Edit the script (`Chol_decoposition.m`) in order to make it compute only the lower triangular matrix of Cholesky decomposition.

Exercise 22

Create the Riemann matrix of size 3-by-3 using gallery matrix functions and compute its QR, LU, LQ, Cholesky, Schur, and Singular Value Decompositions.

Exercise 23

Perform the following:

- Create the 4-by-4 random matrix with normalized columns and specified singular values using gallery matrix functions. Hint: Use `randcolu`.
- Compute the QR, LU, LQ, and Decompositions of the matrix you just created.

Exercise 24

Perform the following:

- Create one 5-by-5 random matrix with random integer elements varying in the range of 1 to 13 and name it `A_mat`.
- Create one 5-by-5 Krylov matrix using a matrix gallery of Krylov and name it `K_mat`.

- Create logic valued 5-by-5 matrix called `Logic_A` by using logic operation ($A\_mat \geq K\_mat$) and elementwise matrix multiplication from `A_mat` and `K_mat`.

Exercise 25

Create the following 10-by-10 matrix:

```
AaA =

    17    24     1     8    15    17    24     1     8    15
    23     5     7    14    16    23     5     7    14    16
     4     6    13    20    22     4     6    13    20    22
    10    12    19    21     3    10    12    19    21     3
    11    18    25     2     9    11    18    25     2     9
    17    24     1     8    15    17    24     1     8    15
    23     5     7    14    16    23     5     7    14    16
     4     6    13    20    22     4     6    13    20    22
    10    12    19    21     3    10    12    19    21     3
    11    18    25     2     9    11    18    25     2     9
```

Hint: Use `magic()` and `repmat()`.

CHAPTER 8

Ordinary Differential Equations

Many modeling problems with engineering applications can be formulated using ordinary differential equations (ODEs). There are a few different definitions of differential equations. One of the simplest is “A differential equation is any equation which contains derivatives, either ordinary derivatives or partial derivatives,” as given in source [1]. From this definition, we can derive two types of differential equations—ordinary differential equations (ODEs) and partial differential equations (PDEs). ODEs contain one type of derivative or one independent variable and PDEs, on the contrary, contain two or more derivatives or independent variables. For example, first order ODEs can be expressed by:

$$\frac{dy}{dx} = f\left(y,\ x\right) \qquad \text{(Equation 8.1)}$$

Where $y(x)$ is a dependent variable whose values depend on values of the independent variable of x. Another good example of ODEs is Newton’s Second Law of Motion, formulated by:

$$ma = \frac{dp}{dt} = \frac{mdv}{dt} = F\left(t,v\right) \qquad \text{(Equation 8.2)}$$

Where $F(t, v)$ is force, which is a function of time (t) and velocity (v); $\frac{dv}{dt}$ is a velocity change rate (acceleration) of a moving object; m is the mass of a moving object; a is an acceleration of a moving object; p is momentum; and $\frac{dp}{dt}$ is its derivative. This formulation of Newton’s Second Law can be also rewritten the following way.

$$\frac{md}{dt}\left(\frac{dx}{dt}\right) = \frac{md^2x}{dt^2} = F\left(t,\ x,\ \frac{dx}{dt}\right) \qquad \text{(Equation 8.3)}$$

S. Eshkabilov, *Beginning MATLAB and Simulink*, https://doi.org/10.1007/978-1-4842-5061-7_8

Where the derivative $\left(\frac{dx}{dt}\right)$ of the displacement (x) of a moving object is the velocity (v). In other words, the velocity is the rate of change of the displacement $x(t)$ of a moving object in time. This can be visualized with the flowchart displayed in Figure 8-1.

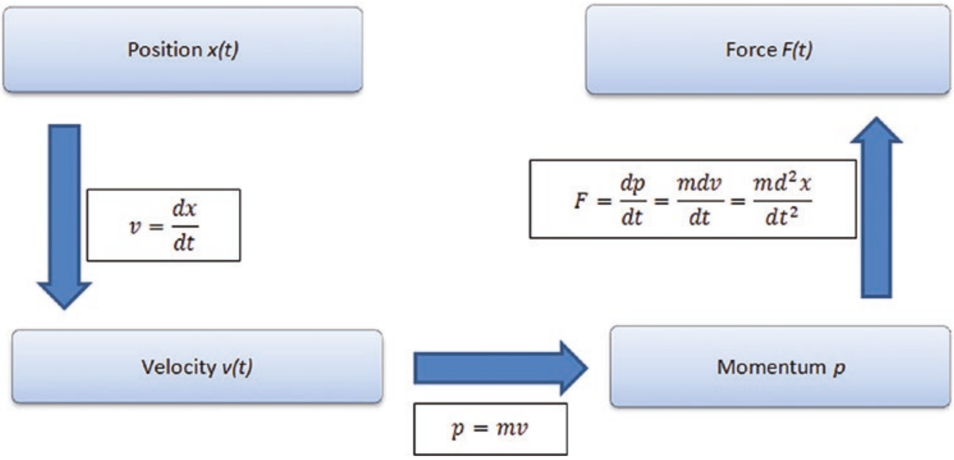


Figure 8-1. *Flowchart expressing motion and exerted force of a moving object*

Classifying ODEs

There are two classifications of ODE-related problems:

- Initial Value Problems (IVPs). $\ddot{x} = xt - 3\dot{x}$ with initial conditions $x(0) = 3,\ \dot{x}(0) = 1$
- Boundary Value Problems (BVPs). $\ddot{x} = xt - 3\dot{x}$ with boundary conditions $x(0) = 3,\ \ x(2) = 1.50$

Initial Value Problems (IVPs) are defined with ODEs together with a specified value, called the initial condition, of the unknown function at a given point in the solution domain. In the IVP of ODEs, there can be a unique solution, no solution, or many solutions. By definition, the IVP of ODEs can be explicitly or implicitly defined. Most of the IVP are explicitly defined. We start with explicitly defined IVPs and then move to implicitly defined ones. Besides by solution type—how solution values change over the solution search space—the IVP are divided into stiff and non-stiff problems. Moreover, ODEs are grouped into two categories, linear and non-linear, and divided into two groups, homogeneous and non-homogeneous.

Here are some specific examples of different ODE types, categories, and groups.

- Stiff ODEs: $\begin{cases} \dot{x} = -0.04x + 10^4 yz \\ \dot{y} = 0.04x - 10^4 yz - 3*10^7 y^2 \quad t \in [0, 40] \\ \dot{y} = 3*10^7 y^2 \end{cases}$
- Non-stiff ODEs: $\dot{y} + 2y = 2t$, $\ddot{w} + \dot{w} = 5$
- Linear ODEs: $\frac{dv}{dt} = 9.81 - 0.198v$, $\ddot{x} + 3\dot{x} + 5x = 0$
- Non-linear ODEs: $\frac{dv}{dt} = 9.81 - 0.198v^2$, $\ddot{x} + 3x\dot{x} + 5x^2 = 0$
- Homogeneous ODEs: $\dot{y} + 2y = 0$, $\ddot{x} + 3\dot{x} + 5x = 0$
- Non-homogeneous ODEs: $\dot{y} + 2y = \sin(x)$, $\ddot{x} + 3\dot{x} + 5x = e^{2t}$

Here are several examples of ODEs and their application areas.

Example 1: Unconstrained Growth of Biological Organisms

This is an exponential growth problem that describes unconstrained growth of biological organisms (such as bacteria). This behavior can also describe real estate or investment values, membership increase of a popular networking site, growth in retail businesses, positive feedback of electrical systems, and generated chemical reactions. The problem is formulated by the following first order ODE:

$$\frac{dy}{dt} = \mu y \text{ has a solution: } y(t) = y_0 e^{\mu t}$$

Example 2: Radioactive Decay

This refers to exponential decay that describes many phenomena in nature and in engineering, such as radioactive decay, washout of chemicals in a reactor, discharge of a capacitor, and decomposition of material in a river. It's expressed using this first order ODE:

$$\frac{dy}{dt} = -\mu y \text{ has a solution: } y(t) = y_0 e^{-\mu t}$$

Examples 1 and 2 are two simple examples of first order ODEs.

Example 3: Newton's Second Law

The motion of a falling object is expressed in the following equation using Newton's Second Law.

$$\frac{md^2y}{dt^2} = mg - \frac{\gamma dy}{dt}$$

This is a second order ODE that has a solution in the form of:

$$y(t) = C_1 e^{-\left(\frac{\gamma t}{m}\right)} - \frac{m(mg - g\gamma t)}{\gamma^2} + C_2$$

Where m is the mass of the falling object; g is gravitational acceleration; and γ is an air-drag coefficient of a falling object. Three parameters—m, g, and γ—are constant, and two parameters change over time—$\frac{d^2y}{dt^2}$ (acceleration) and $\frac{dy}{dt}$ (velocity). In the solution of a falling object, C_1 and C_2 are arbitrary numbers that are dependent on the initial conditions. In other words, they can be computed considering the initial condition of a falling object.

There are a few methods that evaluate analytical solutions of ODEs, including separation of variables, introduction of new variables, and others. We look at specific examples of these types of ODEs to see how to evaluate their analytical solutions and compute numerical solutions. We do this by employing different techniques in the MATLAB/Simulink environment and writing scripts and building models. In computing analytical solutions of ODEs, we explain using specific examples how to use built-in functions of the Symbolic Math Toolbox[1] and the MuPAD[2] Notebooks. We put more emphasis on the Symbolic Math Toolbox's command syntax rather than the MuPAD Notebooks. In future releases of MATLAB, MuPAD Notebooks will be removed.

For obvious reasons, considerable effort is placed on numerical solution methods rather than analytical solution search tools. It is not always possible or is too costly to evaluate analytical solutions of ODEs. Therefore, a numerical solution search is often best. There are a number of numerical methods. They are Euler (forward, backward, modified), Heun, mid-point rule, Runge-Kutta, Runge-Kutta-Gill, Adams-Bashforth, Milne, Adams-Moulton, Taylor series, and trapezoidal rule methods.

[1] Symbolic Toolbox is a registered trademark of The MathWorks Inc.

[2] MuPAD is a registered trademark of The MathWorks Inc.

Some of these methods are explicit and others are implicit. In order to demonstrate how to employ these methods, we first describe their formulations and then work on their implementation algorithm for writing scripts (programs) explicitly. We do not attempt to derive any of the formulations used in these numerical methods. There are many literature sources [see 2, 3, 4, 5] that explain the theoretical aspects of these methods.

In solving the IVP using numerical methods, we start at an initial point (initial conditions) and then take a step (equal step-size or varying step-size) forward in time to compute the following numerical solution. Some of the previously named numerical methods (e.g. Euler's methods) are single-step methods and others (Runge-Kutta, Adams-Bashforth, Milne, Adams-Moulton, and the Taylor series) are multi-step methods. Single-step methods refer to only one previous point and its derivative to determine the current value. Other methods, such as Runge-Kutta methods, take some intermediate steps to obtain a higher-order step and then drop off values before taking the next step. Unlike single-step methods, multi-step methods keep and use values from the previous steps instead of discarding them. This way, multi-step methods link a few previously obtained values (solutions) and derivative values. All of these methods, i.e., the single-step and multi-step methods, are assessed based on their accuracy and efficiency in terms of the computation time and resources (e.g., machine time) spent to compute numerical solutions for specific types of IVPs of ODEs. Nevertheless, it remains true that most solutions of the first, second, or higher order IVPs cannot be found by analytical means. Therefore, we need to employ various numerical methods.

Analytical Methods

The Symbolic Math Toolbox (or MuPAD Notebooks) has several functions that are capable of evaluating analytical solutions of many analytically solvable ODEs. There are two commands (built-in functions) by which analytical solutions of some ODEs can be evaluated—`dsolve` and `ilaplace/laplace`.

Note that in this subsection we demonstrate—via a few examples of first and second order ODEs and systems of coupled differential equations—how to compute analytical solutions of ODEs.

DSOLVE

A very first ODE solver tool for computing an analytical (or general) solution of any given ODE in MATLAB is dsolve. It can be used with the following general syntax:

```
Solution = dsolve(equation)
Solution = dsolve(equation, conditions)
Solution = dsolve(equation, conditions, Name, Value)
[y1,...,yN] = dsolve(equations)
[y1,...,yN] = dsolve(equations, conditions)
[y1,...,yN] = dsolve(equations, conditions, Name, Value)
```

Example 1: Using dsolve

Given an ODE $\dot{y}+2ty^2=0$, note that the initial and boundary conditions are not specified.

```
>> y_solution=dsolve('Dy=-2*y^2*t')


Y_solution=-1/(C3-t^2)
```

Note that C3 is defined from the initial or boundary conditions of the given ODE.

There is also an alternative command. In later versions of MATLAB (starting with MATLAB 2012), we can solve the given problem by using the following command syntax:

```
>>syms y(t); y_sol=dsolve(diff(y) == -2*y^2*t)
 y_sol =
              0
 -1/(- t^2 + C3)
```

Example 2: Plotting from dsolve

Given an ODE $\dot{y}+2ty^2=0$ with the initial condition $y(0) = 0.5$:

```
>> Solution=dsolve('Dy=-2*y^2*t', 'y(0)=0.5')
Solution =
 1/(t^2 + 2)
```

The alternative command syntax with later versions of MATLAB is as follows:

```
>> syms y(t); Solution=dsolve(diff(y) == -2*y^2*t, y(0)==0.5)
Solution =
 1/(t^2 + 2)
```

The analytical solution is in a symbolic formulation and thus can be plotted with `ezplot`. (See Figure 8-2.)

```
>> ezplot(Solution)
```

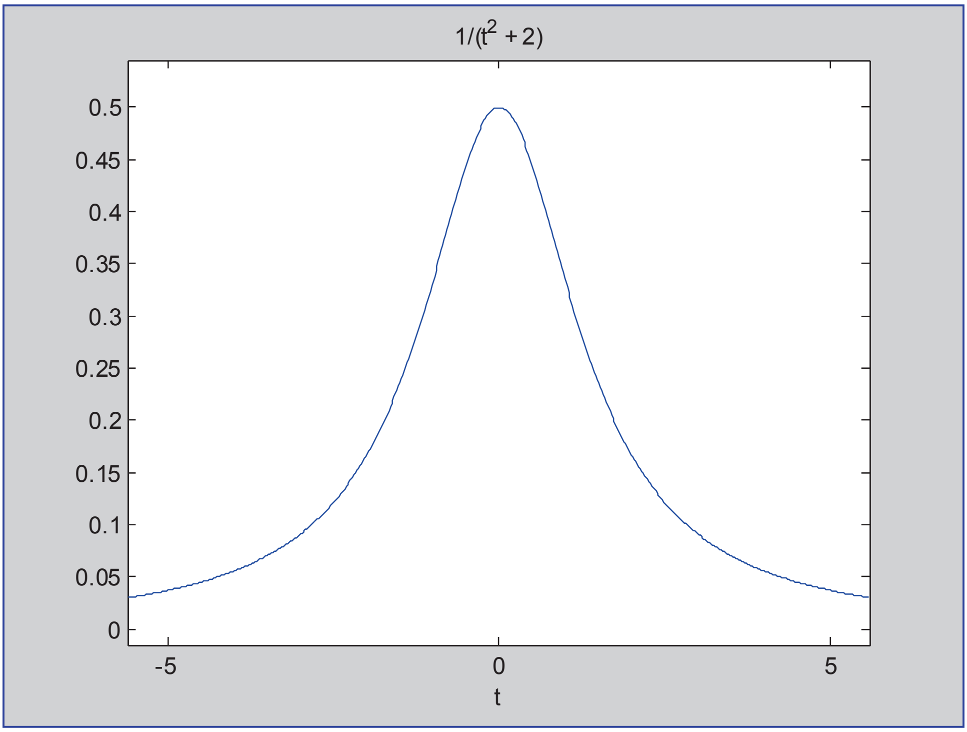


Figure 8-2. *Analytical solution of* $\dot{y}+2ty^2=0,\ y(0)=0.5$

Numerical values of the analytical solution (the equation) can be computed by vectorizing (parameterizing) the symbolic formulation (the solution):

```
>> ysol=vectorize(solution)
ysol =
1./(t.^2 + 2)
>> t=(-5:.1:5); ysol_values=eval(ysol);
```

Example 3: Adding an Unspecified Parameter

Given $\dot{y} + kty^2 = 0,\ y(0) = 0.5$, note that this exercise has one unspecified parameter, *k*.

```
>> syms k
>> solution=dsolve('Dy=-k*y^2*t', 'y(0)=0.5')
solution =
1/((k*t^2)/2 + 2)
```

Here is the alternative command syntax:

```
>> syms y(t) k; solution=dsolve(diff(y) == -k*y^2*t, y(0)==0.5)
solution =
1/((k*t^2)/2 + 2)
```

Note Options in `dsolve` need to be set appropriately depending on the problem type. In MATLAB 2008-2010 or earlier versions, we should set `IgnoreAnalyticConstraints` to none in order to obtain all possible solutions.

For example:

```
solution=dsolve('Dy=-k*y^2*t', 'y(0)=0.5', ... 'IgnoreAnalyticConstraints',
'none')
```

Note For MATLAB 2012 or later versions, we should set `IgnoreAnalyticConstraints` to `false` in order to get all possible correct answers for all the argument values. Otherwise, `dsolve` may output an incorrect answer due to its pre-algebraic simplifications.

For example:

```
solution=dsolve(diff(y)==-k*y^2*t, y(0)==0.5, ... 'IgnoreAnalyticConstraints',
false)
```

Second-Order ODEs and a System of ODEs

There are a myriad of processes and phenomena that are expressed via second order differential equations. For example, simple harmonic motions of a spring-mass system, motions of objects with some acceleration (Newton's Second Law), damped vibrations, current flows in resistor-capacitor-inductance circuits, and so forth. In general, second order ODEs are expressed in two different forms—homogeneous (Equation 8.4) and non-homogeneous (Equation 8.5).

$$\ddot{y}+p(x)\dot{y}+q(x)y=0 \qquad \text{(Equation 8.4)}$$

$$\ddot{y}+p(x)\dot{y}+q(x)y=g(x) \qquad \text{(Equation 8.5)}$$

Note that the homogeneous ODEs in Equation 8.4 always have one trivial solution, which is, $y(x) = 0$ and it satisfies the givens in Equation 8.4. With respect to the independent functions $p(x)$, $q(x)$, $g(x)$, the ODEs can be linear or non-linear. In some cases, the independent functions $p(x)$, $q(x)$, $g(x)$ can be constant values or non-constant values.

Let's consider several examples of second order ODEs to see how to compute general and particular solutions with MATLAB's Symbolic Math Toolbox.

Example 1: dsolve with a Second-Order ODE

Given an ODE $\frac{d^2u}{dt^2}+100u=2.5\sin(10t),\ u(0)=0,\ \frac{du(0)}{dt}=0:$

```
usol=dsolve('D2u+100*u=2.5*sin(10*t)', 'u(0)=0', 'Du(0)=0');
pretty(usol)

%% Alternative syntax
syms u(t)
Du = diff(u);
u(t) = dsolve(diff(u, 2)==2.5*sin(10*t)-100*u, u(0)==0, Du(0) == 0);
pretty(u(t))
```

The output from executing the two short scripts/commands is:

```
3 sin(10 t)   sin(30 t)
  ----------- - --------- -
      320          320

               / t   sin(20 t) \
     cos(10 t) | - - --------- |
               \ 8      160    /
```

Example 2: System ODEs

Given a system of ODEs: $\begin{cases} y_1' = y_2 \\ y_2' = -y_1 - 0.125y_2 \end{cases}$, $y_1(0)=1, y_2(0)=0$;

The given problem is a system of two first order ODEs. This problem can be solved directly with dsolve, similar to the previous examples:

```
%% System of two 1st order ODEs solved with dsolve
yt=dsolve('Dy1=y2', 'Dy2=-y1-0.125*y2','y1(0)=1', 'y2(0)=0');
pretty(yt.y1)
pretty(yt.y2)
```

```
%% Alternative syntax
syms y1(t) y2(t)
z=dsolve(diff(y1,1)==y2, diff(y2,1)==(-y1-0.125*y2), y1(0)==1, y2(0)==0);
pretty(z.y1), pretty(z.y2)
```

The computed analytical solutions of the problem displayed in the command window are:

```
     /    1/2   \             /    1/2   \
     | 255    t |      1/2    | 255    t |
  cos| -------- |   255    sin| -------- |
     \    16    /             \    16    /
  --------------- + ----------------------
          1/16                    1/16
```

```
   exp(t)              255 exp(t)

                 /    1/2   \
        1/2      | 255    t |
   16 255    sin| -------- |
                 \    16    /
 - -------------------------
                   1/16
        255 exp(t)
```

The computed analytical solutions are:

$$y_1(t) = \frac{cos\left(\frac{\sqrt{255}t}{16}\right)}{\sqrt[16]{e^t}} + \frac{\sqrt{255}sin\left(\frac{\sqrt{255}t}{16}\right)}{255\sqrt[16]{e^t}}$$

$$y_2(t) = -\frac{16\sqrt{255}sin\left(\frac{\sqrt{255}t}{16}\right)}{255\sqrt[16]{e^t}}$$

Example 3: Unsolvable Solutions Using dsolve

Given a second order ODE: $2\ddot{y} + 3\dot{y}^3 - |y|\cos(100t) = 2$, $y(0) = 0$ and $\dot{y}(0) = 0$.

```
>> Solution_dsolve=dsolve('2*D2y+3*(Dy^3)-cos(100*t)*abs(y)-2=0', ...
'y(0)=0','Dy(0) =0');
Warning: Explicit solution could not be found.
> In dsolve at 194
```

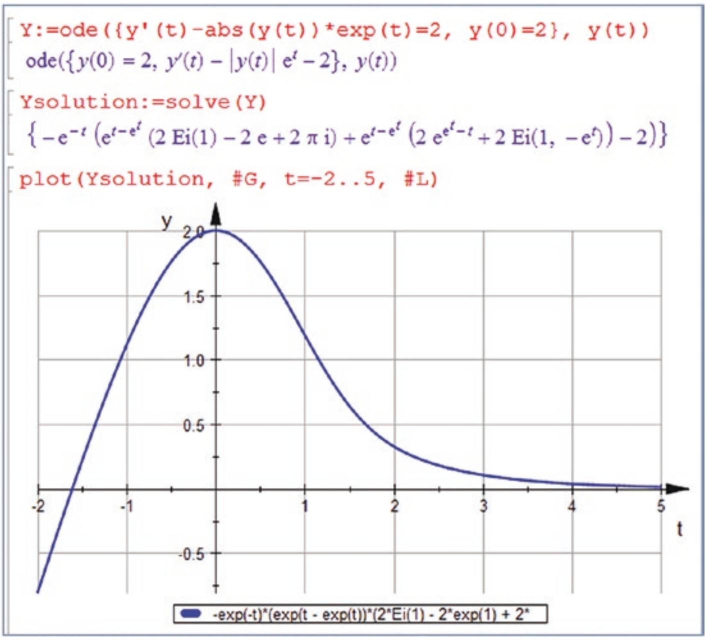
This is a good approach to ODEs that cannot be solved analytically by using dsolve.

Let's consider several examples of computing analytical solutions of ODEs in MuPAD Notebooks. We can open MuPAD by typing this command in the command window:

```
>> mupad
```

Example 4: Computing an Analytical Solution

$\dot{y} - |y|e^{t} = 2,\ y(0) = 2.$ Figure 8-3 shows the commands used to compute an analytical solution of the given exercise.



```
Y:=ode({y'(t)-abs(y(t))*exp(t)=2, y(0)=2}, y(t))
```

$\mathrm{ode}(\{y(0) = 2,\ y'(t) - |y(t)|\, e^{t} - 2\},\ y(t))$

```
Ysolution:=solve(Y)
```

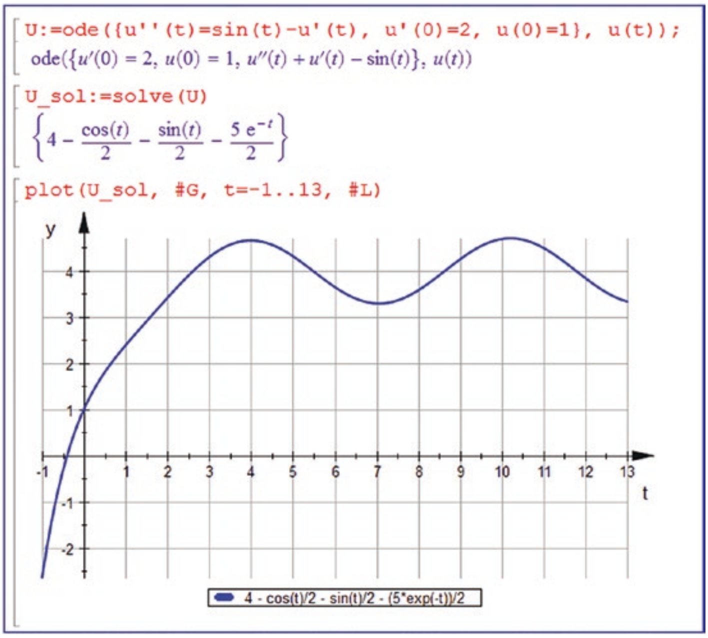
$\{-e^{-t}\,(e^{t-e^{t}}\,(2\,\mathrm{Ei}(1) - 2\,e + 2\,\pi\,i) + e^{t-e^{t}}\,(2\,e^{e^{t}-t} + 2\,\mathrm{Ei}(1,\ -e^{t})) - 2)\}$

```
plot(Ysolution, #G, t=-2..5, #L)
```

Figure 8-3. *MuPAD commands to solve* $\dot{y} - |y|e^{t} = 2,\ y(0) = 2$ *analytically*

Example 5: An Interesting ODE

Let's consider a second order ODE, as follows: $\ddot{y} + \dot{y} = \sin t$ with $y(0) = 1,\ \dot{y}(0) = 2.$



```
U:=ode({u''(t)=sin(t)-u'(t), u'(0)=2, u(0)=1}, u(t));
```

$$\mathrm{ode}(\{u'(0) = 2,\ u(0) = 1,\ u''(t) + u'(t) - \sin(t)\},\ u(t))$$

```
U_sol:=solve(U)
```

$$\left\{4 - \frac{\cos(t)}{2} - \frac{\sin(t)}{2} - \frac{5\,e^{-t}}{2}\right\}$$

```
plot(U_sol, #G, t=-1..13, #L)
```

Figure 8-4. *MuPAD tools used to solve a second order ODE*

Example 6: Responding to No Analytical Solution

Given $\ddot{y}+3\dot{y}^3-|y|\mathrm{y}=2$, $y(0)=0$ and $\dot{y}(0)=0$, this is a second order non-homogeneous and non-linear ODE.

```
[Y:=ode({y"(t)+3*(y'(t)^3)-y(t)*abs(y(t))=2,y(0)=0,y'(0)=0},y(t))
```

$$\mathrm{ode}(\{y'(0) = 0,\ y(0) = 0,\ y''(t) + 3\ y'(t)^3 - |y(t)|\ y(t) - 2\},\ y(t))$$

```
[Ysolution:=solve(Y)
```

$$\mathrm{solve}(\mathrm{ode}(\{y'(0) = 0,\ y(0) = 0,\ y''(t) + 3\ y'(t)^3 - |y(t)|\ y(t) - 2\},\ y(t)))$$

No solution is computed in this exercise. When MuPAD cannot compute an analytical solution of a given ODE problem, it rewrites a given problem formulation and produces no analytical solution expression.

Laplace Transforms

Solutions of linear ordinary differential equations with constant coefficients can be evaluated by using the Laplace transformation. One of the most important features of the Laplace transforms in solving a differential equation is that the transformed equation is an algebraic equation. It will be used to define a solution to the given differential equation. In general, the Laplace transform application to solving differential equations can be formulated in the following way.

Let's consider the nth order derivative of $y^n(x) = f(t)$. The Laplace transform of $y^n(x)$ is as follows:

$$\mathcal{L}\left\{\frac{d^n y}{dt^n} = f(t)\right\} => s^n Y(s) - s^{n-1} y^{n-1}(0) - s^{n-2} y^{n-2}(0) -$$

$$-sy'(0) - y(0) = F(s) \qquad \text{(Equation 8.6)}$$

Or

$$s^n Y(s) - \sum_{i=1}^{n} s^{n-1} y^{i-1}(0) = F(s) \qquad \text{(Equation 8.7)}$$

In Equations 8.6 or 8.7, if we substitute constant values of initial conditions at $t = 0$ given as $y(0) = a_0$; $y^{'}(0) = a_1$; $y^{''}(0) = a_2$; …; $y^{n-2}(0) = a_{n-2}$; $y^{n-1}(0) = a_{n-1.}$

Now, we can rewrite the expression (Equation 8.6 or 8.7) as follows:

$$\mathcal{L}\left\{\frac{d^n y}{dt^n} = f(t)\right\} => s^n Y(s) - s^{n-1} a_0 - s^{n-2} a_1 - s^{n-3} a_2 - \ldots - s\, a_{n-2} - a_{n-1} = F(s) \qquad \text{(Equation 8.8)}$$

Subsequently, we first solve for `Y(s)`, take the inverse Laplace transform from `Y(s)`, and obtain the solution `y(t)` of the nth order differential equation.

The general procedure for applying the Laplace and the inverse Laplace transforms to determine the solution of differential equations with constant coefficients is as follows.

1. Take the Laplace transforms from both sides of the given equation.
2. Solve for `Y(s)` in terms of `F(s)` and other unknowns.

3. Take the inverse Laplace transform of the found expression in order to obtain the final solution of the problem.

Note that in Step 3, we should also break the expression from Step 2 into partial fractions in order to use tables of the inverse Laplace transform correspondences. A schematic view of the Laplace and inverse Laplace transforms is given in the flowchart shown in Figure 8-5.

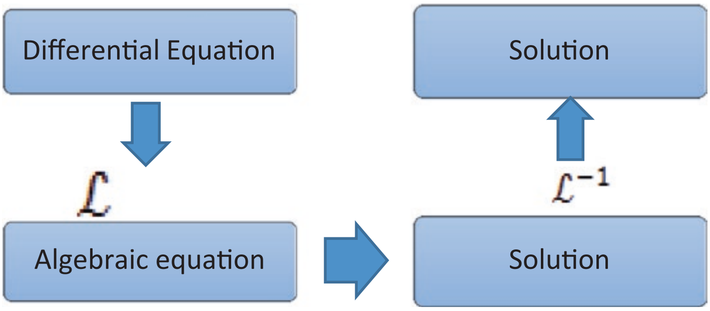


***Figure 8-5.** Flowchart of solving ODE with Laplace transform and its inverse*

Example 1: First Laplace Transform

Let's consider a second order non-homogeneous differential equation:

$$\frac{d^2y}{dt^2}+A\frac{dy}{dt}+C=e^{nt},\ dy(0)=k,\ y(0)=m.$$

Now, by applying the steps depicted in the flowchart of the Laplace and inverse transforms from the flowchart, we write the Laplace transform of the given problem in explicit steps.

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}+A\frac{dy}{dt}+C=e^{nt}\right\}=>L\left\{\frac{d^2y}{dt^2}\right\}+\mathcal{L}\left\{A\frac{dy}{dt}\right\}+\mathcal{L}\{C\}=\mathcal{L}\left\{e^{nt}\right\} \quad \text{(Equation 8.9)}$$

$$\mathcal{L}\left\{e^{nt}\right\}=\frac{1}{s-n} \quad \text{(Equation 8.10)}$$

So,

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\}=s^2Y(s)-dy(0)-s*y(0)=s^2Y(s)-k-s*m \quad \text{(Equation 8.11)}$$

$$\mathcal{L}\left\{A\frac{dy}{dt}\right\}=A*\mathcal{L}\left\{\frac{dy}{dt}\right\}=A*\left(s*Y\left(s\right)-y\left(0\right)\right)=A*s*Y\left(s\right)-m \qquad \text{(Equation 8.12)}$$

$$\mathcal{L}\{C\}=C \qquad \text{(Equation 8.13)}$$

By plugging Equations 8.10, 8.11, 8.12, and 8.13 back into Equation 8.9, we obtain the this expression:

$$s^2Y\left(s\right)-k-sm+AsY\left(s\right)-m+C=\frac{1}{s-n} \qquad \text{(Equation 8.14)}$$

We solve Equation 8.5 for `Y(s)` to determine the following:

$$Y\left(s\right)=\frac{1+\left(sm+\left(k+m-C\right)\right)\left(s-n\right)}{\left(s-n\right)\left(s^2+As\right)}=-\frac{Cn-Cs-kn-mn+ks+ms+ms^2}{s\left(A+s\right)\left(n-s\right)} \qquad \text{(Equation 8.15)}$$

From the expression of `Y(s)` in Equation 8.15, we can split this into partial fractions and take the inverse Laplace transform of both sides. We obtain Equation 8.16, which is the *y*(*t*) of the given differential equation:

$$y\left(t\right)=\frac{e^{nt}}{An+n^2}-\frac{Cn-kn-mn+1}{An}+\frac{Cn-kn-mn-A\left(k-mn-C\right)+1}{Ae^{At}\left(A+n\right)} \qquad \text{(Equation 8.16)}$$

The built-in `laplace()` function of the Symbolic Math Toolbox is used to evaluate the Laplace transform of any algebraic expression or differential equation. Likewise, the `ilaplace()` function of the Symbolic Math Toolbox is used to compute the inverse of the evaluated Laplace transformed `s` domain expression. These two functions handle all transformations by breaking up the partial fraction procedures automatically, and then compute an analytical solution of a given ODE exercise.

LAPLACE/ILAPLACE

As mentioned, `laplace/ilaplace` are based on the Laplace and inverse Laplace transforms, which are built-in function tools of the Symbolic Math Toolbox. The general syntax of `laplace/ilaplace` is as follows:

```
F=laplace(f)
F=laplace(f, t)
F=laplace(f, var1, var2)
```

and

```
f=ilaplace(F)
f=ilaplace(F, s)
f=ilaplace(F, var1, var2)
```

Example 2: Using LAPLACE

Given $x(t) = \sin(2t)$, the Laplace transform of $x(t)$ is computed with the following command syntax.

```
>> syms t
>> xt=sin(2*t); Xs=laplace(xt)
Xs =
2/(s^2 + 4)
```

Given: $y(t) = \sin(Kt)$

```
>> syms t K
>> yt=sin(K*t); Ys=laplace(yt)
Ys =
K/(K^2 + s^2)
```

Example 3: A Final LAPLACE

$$y(x) = ax^3 + b.$$

```
>> syms x a b
>> yx=a*x^3+b; Ys=laplace(yx)
Ys =
(6*a)/s^4 + b/s
```

We can also obtain the `t` variable domain instead of `s`.

```
 >> yx=a*x^3+b; Yt=laplace(yx, x, t)
Yt =
(6*a)/t^4 + b/t
```

The `ilaplace()` function syntax and implementation are exactly the same for `laplace`. Let’s look at several ODE exercises to see how to use `laplace`/`ilaplace` and compare their evaluated solutions to the ones obtained with `dsolve`.

Example 4: Comparing LAPLACE/ILAPLACE with DSOLVE

Let's solve $\dot{y}+2y=0, y(0)=0.5$ with laplace/ilaplace and dsolve. The following script (ODE_Laplace.m) shows the solution.

```
% ODE_Laplace.m
clearvars; clc; close all
% Step #1. Define symbolic variables' names
syms t s Y
ODE1='D(y)(t)=-2*y(t)';
% Step #2. Laplace Transforms
LT_A=laplace(ODE1, t, s);
% Step #3. Substitute ICs and initiate an unknown Y
LT_A=subs(LT_A,{'laplace(y(t),t, s)','y(0)'},{Y,0.5});
% Step #4. Solve for Y (unknown)
Y=solve(LT_A, Y);
display('Laplace Transforms of the given ODE with ICs'); disp(Y)
% Step #5. Evaluate Inverse Laplace Transform
Solution_Laplace=ilaplace(Y);
display('Solution found using Laplace Transforms: ')
pretty(Solution_Laplace)
% Step #6. Compute numerical values and plot them
t=0:.01:2.5; LTsol=eval(vectorize(Solution_Laplace));
figure, semilogx(t, LTsol, 'ro-')
xlabel('t'), ylabel('solution values')
title('laplace/ilaplace vs dsolve ')
grid on; hold on
% Compare with dsolve solution method
Y_d=dsolve('Dy=-2*y', 'y(0)=0.5'); display('Solution with dsolve')
pretty(Y_d); Y_sol=eval(vectorize(Y_d));
plot(t,Y_sol, 'b-', 'linewidth', 2), grid minor
legend('laplace+ilaplace', 'dsolve'), hold off; axis tight
```

After executing the ODE_Laplace.m script, the following output is obtained.

```
Laplace Transforms of the given ODE with ICs
1/(2*s + 4)
```

```
Solution found using Laplace Transforms:

  exp(-2 t)
  ---------
      2
Solution with dsolve

  exp(-2 t)
  ---------
      2
```

The plot of solutions shown in Figure 8-6 clearly displays perfect convergence of solutions found with `laplace/ilaplace` and `dsolve`.

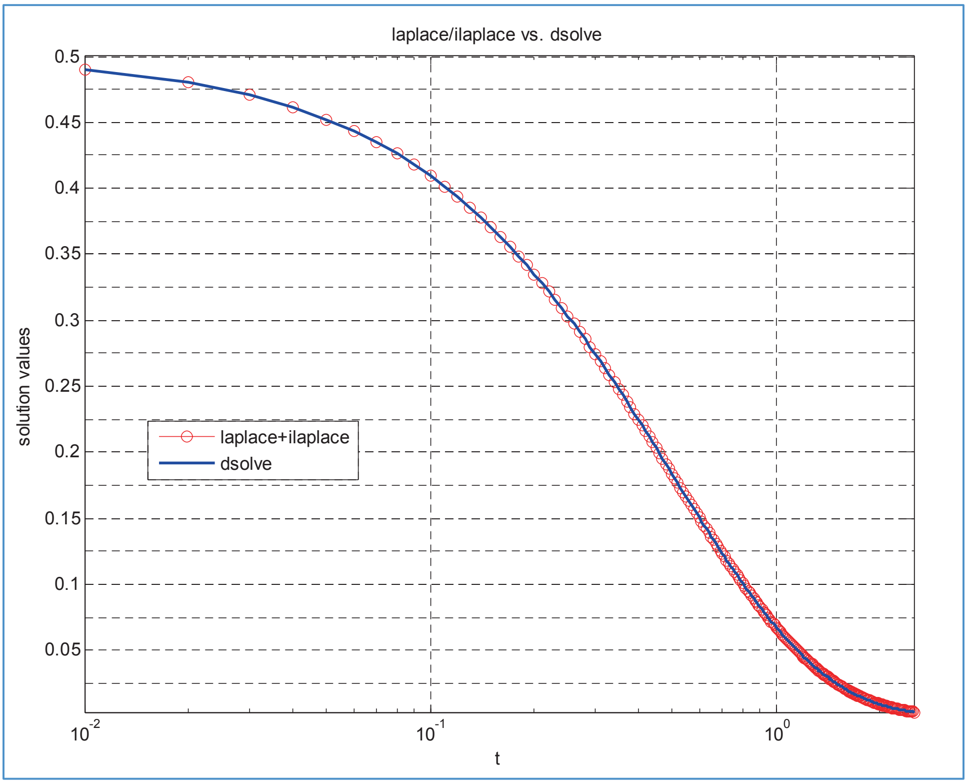


Figure 8-6. *The problem* $\dot{y}+2y=0, y(0)=0.5$ *solved with laplace/ilaplace and dsolve*

Example 5: Convergent Answers

Given $\ddot{y}+\dot{y}=\sin t, y(0)=1, \dot{y}(0)=2$, here is the solution script (Laplace_vs_Dsolve.m) of this second-order non-homogeneous ODE with laplace, ilaplace, and dsolve.

```
% Laplace_vs_Dsolve.m
% %%   LAPLACE TRANSFORMS
% Given: y"+y'=sin(t) with ICs: [1, 2] for y(0) & y'(0).
clearvars, clc, close all
syms t s Y
ODE2nd='D(D(y))(t)+D(y)(t)-sin(t)';
% Step 1. Laplace Transforms
LT_A=laplace(ODE2nd, t, s);
% Step 2. Substitute ICs and initiate an unknown Y
LT_A=subs(LT_A,{'laplace(y(t),t, s)','y(0)','D(y)(0)'},{Y,1,2});
% Step 3.  Solve for Y unknown
Y=solve(LT_A, Y);
display('Laplace Transforms of the given ODE with ICs'); disp(Y)
Solution_Laplace=ilaplace(Y);
display('Solution found using Laplace Transforms: ')
pretty(Solution_Laplace); t=0:.01:13;
LTsol=eval(vectorize(Solution_Laplace));
figure, plot(t, LTsol, 'ro-'); xlabel('t'),
ylabel('solution values')
title('laplace/ilaplace vs. dsolve: ddy+dy=sin(t)'); hold on
% dsolve solution method
Y=dsolve('D2y+Dy=sin(t)', 'y(0)=1, Dy(0)=2', 't');
display('Solution with dsolve:  ');
pretty(Y); Y_sol=eval(vectorize(Y));
plot(t,Y_sol, 'b-', 'linewidth', 2); grid minor
legend('laplace+ilaplace', 'dsolve'); hold off
```

The computed analytical solutions are as follows:

```
Laplace Transforms of the given ODE with ICs
(s + 1/(s^2 + 1) + 3)/(s^2 + s)
```

```
Solution found using Laplace Transforms:

      cos(t)   sin(t)   5 exp(-t)
  4 - ------ - ------ - ---------
        2        2          2
Solution with dsolve:

      cos(t)   sin(t)   5 exp(-t)
  4 - ------ - ------ - ---------
        2        2          2
```

From the plot displayed in Figure 8-7, it is clear that the solutions found via the Laplace transforms (`laplace/ilaplace`) and the `dsolve` functions converge perfectly well. Both functions evaluate the same analytical solution of a given ODE.

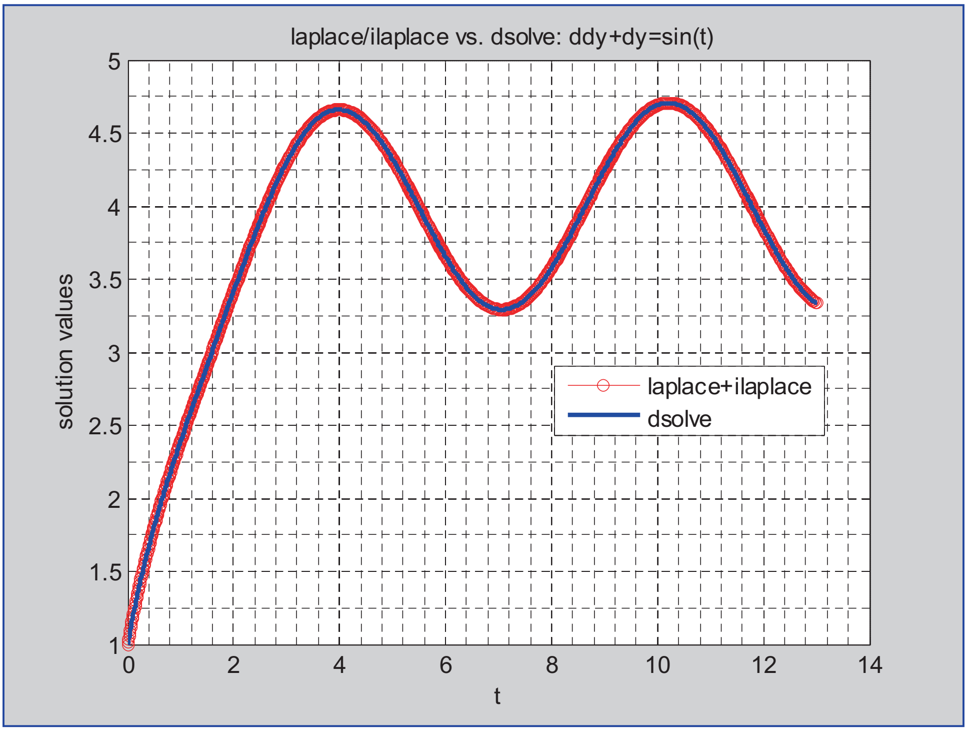


***Figure 8-7.** Numerical solutions of* $\ddot{y}+\dot{y}=\sin(t), y(0)=1,\ \dot{y}(0)=2$

Example 6: No Analytical Solution

Given a second order non-homogeneous and non-linear ODE $2\ddot{y}+3\dot{y}^3-|y|\cos(t)=2$, $y(0)=0$ and $\dot{y}(0)=0$. Here is the solution script (Lap_inv_Lap.m) with the Laplace and inverse Laplace transforms:

```
% Lap_inv_Lap.m
clearvars, clc, close all
syms t s Y
ODE2nd='2*D(D(y))(t)+3*(D(y)(t))^3-cos(100*t)*abs(y(t))-2';
% Step 1. Laplace Transforms
LT_A=laplace(ODE2nd, t, s);
% Step 2. Substitute ICs and initiate an unknown Y
LT_A=subs(LT_A,{'laplace(y(t),t, s)','y(0)','D(y)(0)'},{Y,0,0});
% Step 3.  Solve for Y unknown
Y=solve(LT_A, Y);
display('Laplace Transforms of the given ODE with ICs'); disp(Y)
Solution_Laplace=ilaplace(Y);
display('Solution found using Laplace Transforms: ')
pretty(Solution_Laplace)
```

The Lap_inv_Lap.m script produces the following output in the command window:

```
Laplace Transforms of the given ODE with ICs
(4 + s*laplace(abs(y(t)), t, s - 100*i) + s*laplace(abs(y(t)), t, s +
100*i) - 6*s*laplace(D(y)(t)^3, t, s))/(4*s^3)
Solution found using Laplace Transforms:

                 /                3              \
                 | laplace(D(y)(t) , t, s)       |
       3 ilaplace| -----------------------, s, t |
   2             |            2                  |
  t              \           s                   /
  -- - ------------------------------------------- +
  2                         2
```

```
        / laplace(|y(t)|, t, s - 100 i)       \
ilaplace| -----------------------------, s, t |
        |               2                     |
        \              s                      /
----------------------------------------------- +
                       4

        / laplace(|y(t)|, t, s + 100 i)       \
ilaplace| -----------------------------, s, t |
        |               2                     |
        \              s                      /
-----------------------------------------------
                       4
```

This output means that no analytical solution is computed explicitly with `laplace/ilaplace`, just like with the `dsolve` function tools.

Example 7: Demonstrating Efficiency and Effortlessness

Given a second order non-homogeneous ODE $2\ddot{y}+3\dot{y}-2y=g(t)$, $y(0)=0$ and $\dot{y}(0)=0$, where $g(t)$ is a forcing function that is discontinuous and defined by

$$g(t)=u_2(t)-u_{10}(t)=\begin{cases}5, & 2\le t<10,\\ 0, & 0\le t<2 \text{ and } t\ge 10.\end{cases}$$

The Laplace transform of the given equation is:

$$\mathcal{L}\{2\ddot{y}+3\dot{y}-2y\}=\mathcal{L}\{u_2(t)-u_{10}(t)\}$$

$$2s^2Y(s)-2s*y(0)-2*\dot{y}(0)+3sY(s)-y(0)-2Y(s)=\frac{5\left(e^{-2s}-e^{-10s}\right)}{s}$$

$$Y(s)=\frac{5\left(e^{-2s}-e^{-10s}\right)}{s\left(2s^2+3s-2\right)}$$

Note that e^{-2s}, e^{-10s} are explained with time delays in the system output signals; in other words, -2 and -10 mean 2 and 10 seconds of time delays. 5 is the magnitude of the Heaviside (step) function.

The formulation Y(s) is the solution of the differential equation in the s domain, but we need it in the time domain. Thus, we need to compute its inverse Laplace transform: $\mathcal{L}^{-1}\{Y(s)\}=y(t)$. By employing ilaplace(), the next short script (Lap_4_non_homog.m) is created. It solves the given problem and computes its analytical and numerical solutions.

```
% Lap_4_non_homog.m
syms t s
F=5*(exp(-2*s)-exp(-10*s))/s; Y=2*s^2+s+2;
TF=F/Y; TFt=ilaplace(TF); pretty(TFt); Sol=vectorize(TFt);
t=linspace(0, 20, 400); S=eval(Sol); plot(t, S, 'bo-');
grid minor
title('Differential Equation with Discontinuous Forcing Fcn')
grid on, xlabel('time'), ylabel('y(t) solution'), shg
```

After executing the script, the next solution plot is obtained along with the solution formulation, as shown in Figure 8-8.

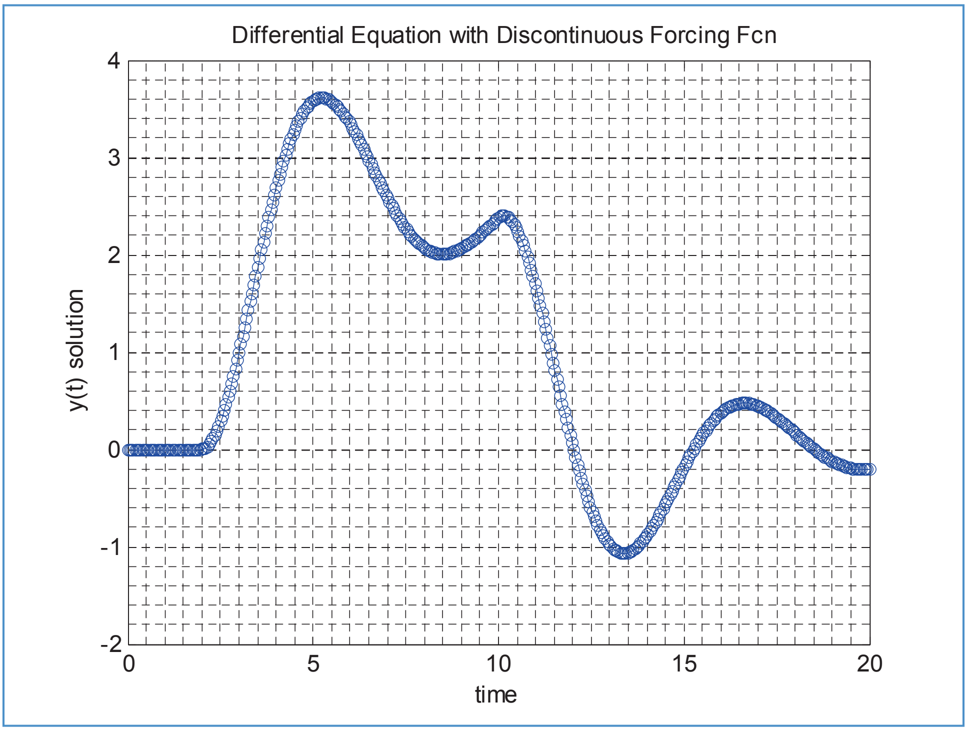


Figure 8-8. *Simulation of the second order non-homogeneous ODE subject to the discontinuous forcing function*

```
>> pretty(TFt)
                      /                /             1/2         \       \
                      |    /       t \ |           15    sin(#1) |       |
                      | exp| 5/2 - - | | cos(#1) + ------------- |       |
                      |    \       4 / \                15       /       |
  5 heaviside(t - 10) | ------------------------------------------ - 1/2 |
                      \                     2                            /

                        /                /             1/2         \       \
                        |    /       t \ |           15    sin(#2) |       |
                        | exp| 1/2 - - | | cos(#2) + ------------- |       |
                        |    \       4 / \                15       /       |
     5 heaviside(t - 2) | ------------------------------------------ - 1/2 |
                        \                     2                            /

  where

             1/2
           15    (t - 10)
     #1 == --------------
                 4

             1/2
           15    (t - 2)
     #2 == -------------
                 4
```

From the last exercise, the following points can be drawn. Using the Laplace transforms (laplace/ilaplace) to compute analytical solutions of non-homogeneous ODEs subject to external forcing functions, which are discontinuous, is relatively easy, fast, and effortless. Such exercises are found very often in particular within control engineering problems. Moreover, it must be noted that the Laplace and inverse Laplace transforms (laplace/ilaplace) are straightforward to implement in solving ODEs. The solutions of ODEs found with them match the ones found by dsolve() perfectly well. As mentioned, many ODEs cannot be solved analytically with the laplace/ilaplace and dsolve functions. Thus, numerical methods are often the only option.

References

[1] http://tutorial.math.lamar.edu/Classes/DE/Definitions.aspx, viewed September, 2013.

[2] Gear, C.W., *Numerical Initial-Value problems in Ordinary Differential Equations,* Prentice-Hall, Englewood Cliffs, N.J. (1971).

[3] Potter M. C., Goldberg J.L., Aboufadel E.F., *Advanced Engineering Mathematics,* 3rd Edition, Oxford University Press, (2005).

[4] Boyce W.E., Diprima R.C., *Elementary Differential Equations and Boundary Value Problems,* 7th Edition, John Wiley & Son, Inc, (2003).

[5] Hairer E., Norsett S. P., Wanner G., *Solving ordinary differential equations I: Non-stiff problems,* 2nd Edition, Berlin: Springer Verlag, ISBN 978-3-540-56670-0, (1993).

Index

S. Eshkabilov, *Beginning MATLAB and Simulink*, https://doi.org/10.1007/978-1-4842-5061-7